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A Numerical Investigation of the Effect of a Slight Terrain Slope on the Urban Heat Island

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The terrain slope is a real-world feature. In this study, the effect of a slight terrain slope on the flow structure is studied in connection with the problem of the urban heat island by a proposed two-length scale $k-\varepsilon$ model. Findings indicate that even a slight slope can cause appreciable change in the flow structure. The degree of alteration depends on both the slope and the temperature difference between the rural and urban area

Keywords: terrain slope, two-length scale $k-\varepsilon$ model, heat island, water vapor

1. Introduction

The most frequently observed and best documented climatic effect of urbanization is the increase in surface temperature of the urban area. This phenomenon has come to be known as the urban heat island. It is similar to that produced by sea/land breezes in the sense that these phenomena are caused by differential heating and cooling. However, the mechanism of the former is more complex than that of the latter. Numerical simulations of the heat island phenomenon have been made over the last two decades. Those studies have revealed many aspects of the heat island phenomenon. However, the authors are aware that the effect of a slight terrain slope on the structure of heat islands has been much neglected. The aim of this study is to examine the influence of a slight terrain slope on the flow structure by means of numerical simulation.

The central problem in simulating the heat island phenomenon is the parameterization of turbulence. Today a wide range of turbulence models is available from simple parameterization to sophisticated high-order closures. The higher-order closure models seem to deal satisfactorily with many problems, but at the price of huge computing time due to many additional equations needed for higher-order modelling. In order to get satisfactory results economically, a two-length scale $k-\varepsilon$ model is proposed, in which two different lengths, one for mixing and one for dissipation are used. It is believed that this two-length scale model can account for the effect of stratification better than the standard $k-\varepsilon$ model. Meanwhile, the computing cost of this two-length scale $k-\varepsilon$ model is almost the same as the standard one.

Because the principal goal of the present study is simply to investigate the effect of a slight slope on the flow structure, the flows considered in this study is non-rotating, which implies that the natural phenomena of interest are small enough or fast enough for the Coriolis forces to be negligible compared with the buoyancy and inertial forces. Meanwhile, no phase-change is taken into consideration, and the lower boundary of the flow domain is aerodynamically smooth. Although the flow situation is idealized, it has many of the important features of the real-world case.

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2. Governing Equations and Model Description

2.1 Basic assumptions

In this study, it is assumed that the Boussinesq approximation is applicable, which implies that one considers the fluid to be incompressible except for the buoyancy term. Most meteorological flows with a vertical scale $H < 1$km, a horizontal scale $L \ll 12$km and a wind speed $V \ll 100$m/s can be treated in this way. Meanwhile, the concept of virtual temperature is used to introduce the specific humidity into the momentum equations.

2.2 Basic equations for momentum, heat and water vapor

In a cartesian system of coordinates(x_i, i=1 to 3), where g_i represents the three components of gravity g, the basic equations resulting from Reynolds averaging process are following:

$$\frac{\partial U_i}{\partial x_i} = 0 \tag{1}$$

$$\begin{aligned}\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} &= -\frac{1}{\rho}\frac{\partial P}{\partial x_j} + \frac{\partial}{\partial x_j}(\nu \frac{\partial U_i}{\partial x_j} - \overline{u_i' u_j'}) \\ &\quad - g_i(\frac{T}{T_\infty} - 1) - 0.61 g_i (q - q_\infty)\end{aligned} \tag{2}$$

$$\frac{\partial T}{\partial t} + U_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}(\alpha \frac{\partial T}{\partial x_j} - \overline{u_j' T'}) + S_h \tag{3}$$

$$\frac{\partial q}{\partial t} + U_j \frac{\partial q}{\partial x_j} = \frac{\partial}{\partial x_j}(k_q \frac{\partial q}{\partial x_j} - \overline{u_j' q'}) + S_q \tag{4}$$

where $\vec{U} = U_i (i = 1, 2, 3)$ is the velocity, T is the temperature and q is the specific humidity which is defined as the mass of water vapor per unit mass of moist air.

2.3 Turbulence closure by a two-length scale model

For turbulent fluxes of momentum, heat and water vapor, a gradient approach is taken:

$$-\overline{u_i' u_j'} = \nu_t (\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}) - \frac{2}{3} k \delta_{ij} \tag{5}$$

$$-\overline{u_j' T'} = \frac{\nu_t}{\sigma_t}\frac{\partial T}{\partial x_j} \tag{6}$$

$$-\overline{u_j' q'} = \frac{\nu_t}{\sigma_q}\frac{\partial q}{\partial x_j} \tag{7}$$

ν_t is the turbulent or eddy viscosity. k is the the kinetic energy of fluctuating motion. It is a measure of the intensity of turbulence. σ_t, σ_q are called the turbulent Prandtal and Schmidt number and are taken to be 0.9 in this study. According to the Prandtl-Kolmogorov hypothesis:

$$\nu_t = C_k l_k k^{1/2} \tag{8}$$

where l_k is the mixing length scale of energy-containing eddies , and C_k is a numerical constant. Meanwhile, we have the Kolmogorov relation which expresses the dissipation rate of turbulence energy ε as a function of k, and the dissipation length scale l_ε:

$$\varepsilon = C_\varepsilon k^{3/2} / l_\varepsilon \tag{9}$$

A commonly used assumption is that the two length scales l_k, l_ε are equal which leads to the standard $k-\varepsilon$ closure method. But for strongly stratified flows, different length scales may need to be employed as will be explained below.

As the buoyancy effect becomes strong, large difference appears between vertical and horizontal eddy motions. Therefore, it can be expected that:

$$l_k = C_1(\overline{w'^2}/k)l_\varepsilon \tag{10}$$

In the particular case of neutral stratification,

$$l_k = l_\varepsilon$$

Then, $C_1 = (\overline{w'^2}/k)^{-1}_{neutral}$. That can be determined from experimental data[10]. In order to find out a relation between the two lengths, let us consider the simplified rate equation for $\overline{w'^2}$:

$$\frac{\partial \overline{w'^2}}{\partial t} = -\frac{\partial \overline{w'^3}}{\partial z} + 2\beta\overline{w'T_v'} - C_4 C_\varepsilon \frac{k^{1/2}}{l_\varepsilon}(\overline{w'^2} - \frac{2}{3}k) - \frac{2}{3}C_\varepsilon \frac{k^{3/2}}{l_\varepsilon} \tag{11}$$

where C_4 is Rotta's constant, and takes the value of 4. β is the buoyancy parameter, $\beta = g/T_0$. T_v is the virtual temperature. This simplified rate equation is derived from the exact rate equation in the following way. One first neglects the advection terms and molecular fluxes, then, the pressure terms which appear in the exact rate equation is modeled as "return-to-isotropy" terms following Rotta's suggestion.

From the above rate equation, by assuming stationarity and neglecting the diffusion term, we get:

$$\frac{\overline{w'^2}}{k} = \frac{2}{3}(\frac{C_4 - 1}{C_4}) + 2\frac{l_\varepsilon \beta \overline{w'T_v'}}{C_4 C_\varepsilon k^{3/2}} \tag{12}$$

Inserting the above expression into eq.(10), and by mathematical manipulation:

$$l_k = (1 + \beta\overline{w'T_v'} l_\varepsilon / C_\varepsilon k^{3/2}) l_\varepsilon \tag{13}$$

Substituting eqs(5),(6),(8) into the above equation, finally yields:

$$l_k = \frac{l_\varepsilon}{1 + \frac{\beta C_k k^{1/2}}{\sigma_t \varepsilon}\frac{\partial T_v}{\partial z}} \tag{14}$$

It can be easily seen that $l_k > l_\varepsilon$ under conditions of unstable stratification, and $l_k < l_\varepsilon$ under conditions of stable stratification. In this study, l_ε is determined from eq.(9), l_k is calculated from the derived relation (14) between l_k and l_ε.

The transport equations for k and ε are written as follows:

$$\begin{aligned} \frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} &= \frac{\partial}{\partial x_j}(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}) + \underbrace{\nu_t(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i})\frac{\partial U_i}{\partial x_j}}_{P} \\ &+ \underbrace{\frac{g_j}{T_\infty}\frac{\nu_t}{\sigma_t}\frac{\partial (T + 0.61 T_\infty q)}{\partial x_j}}_{G} - \varepsilon \end{aligned} \tag{15}$$

$$\frac{\partial \varepsilon}{\partial t} + U_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}) + \underbrace{c_{1\varepsilon}\frac{\varepsilon}{k}(P+G)(1 + c_{3\varepsilon}R_f)}_{generation-destruction} - c_{2\varepsilon}\frac{\varepsilon^2}{k} \tag{16}$$

The values of the constants in this two-length scale $k-\varepsilon$ model, except $c_{3\varepsilon}$, are shown in Table 1. The $c_{3\varepsilon}$ constant is chosen from Viollet[9].

$$c_{3\varepsilon} = \begin{cases} 1, & \text{if stable} \\ 0, & \text{if unstable} \end{cases}$$

Table 1: Values of the constants in the two-length scale k-ε model

c_ε	$c_{1\varepsilon}$	$c_{2\varepsilon}$	c_k	σ_k	σ_ε
0.166	0.144	1.92	0.55	1.0	1.3

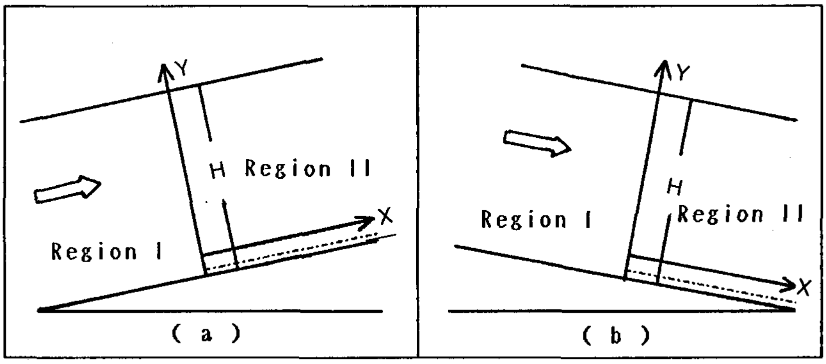

Figure 1: Flow configuration and coordinate system

3. Sloping effects on the urban heat island

3.1 Analysis

We consider an infinitely flat surface, sloped at a small angle α to the horizontal. In the case of upward slope(Fig.1a), the component of the buoyancy force along x direction is:

$$\beta \Delta T \sin \alpha$$

For unstable stratifications, ΔT is positive, consequently, the x component of the buoyancy force will produce an acceralation to the mean flow. For stable conditions, ΔT is negative, the x component of the buoyancy force will produce a deceleration. In the case of downward slope(Fig.1b), The opposite is true, it tends to minimize wind shear and turbulence production. In this study, the attention is focused on the positive terrain slope under unstable conditions.

3.2 Numerical procedure

The computational domain and the coordinate system used for the present computation is shown in Fig.1a. The surface temperature in the region I is lower than that in region II while the surface value of specific humidity in region I is higher than that in region II. Computational conditions in different runs are shown in Table 2. The subscripts w1,w2 in the Table 2 refer to the surface conditions in the region I and II respectively while the subscripts ∞,air refer to the free-stream conditions. The mean flow and turbulence model equations are solved with a conservative finite volume method on a non-uniform and staggered grid system. A grid system comprising 50×50 grid lines is employed in this study. Near the solid wall, the wall-function technique is employed which relates the streamwise velocity U, the kinetic energy k and the dissipation rate ε at the first grid point to the local friction velocity U_τ by

$$u_+ = \frac{U}{U_\tau} = \frac{1}{\kappa}\ln(Ey^+), \quad k = \frac{U_\tau^2}{\sqrt{c_\mu}}, \quad \varepsilon = \frac{U_\tau^3}{\kappa y}, \quad y^+ = \frac{U_\tau y}{\nu} \tag{17}$$

$$\phi_+ = \sigma_t(u_+ + P^*) \tag{18}$$

$$P^* = 9.24[(\frac{\sigma}{\sigma_t})^{3/4} - 1](1 + 0.28e^{-0.007\frac{\sigma}{\sigma_t}}) \tag{19}$$

where κ is the von Karman constant, E is a friction parameter. In the present study, $\kappa = 0.42, E = 9.8$. ϕ stands for temperature and humidity. When it refers to humidity, σ_t means the turbulent Schmidt number.

Table 2: Computation Conditions

Run	α	U_∞	T_{w1}	T_{w2}	T_{air}	Q_{w1}	Q_{w2}	Q_{air}
1	0.0(radian)	1.0(m/s)	300K	310K	300K	0.04	0.02	0.0
2	0.002	1.0(m/s)	300k	310K	300K	0.04	0.02	0.0
3	0.003	1.0(m/s)	300K	310K	300K	0.04	0.02	0.0
4	0.003	1.0(m/s)	300K	320K	300K	0.04	0.02	0.0
5	0.0	1.0(m/s)	300K	320K	300K	0.04	0.02	0.0

3.3 Results and discussion

The velocity distributions on different slopes have been calculated from the two-length scale model. They are compared with each other in Figs.2 and 3. Fig.2 shows the velocity distribution near the rural-urban interface in the region II. Fig.3 shows the velocity distribution far away from the interface. It can be concluded from these comparisons that the slope has a significant effect on the mean velocity profiles even if the surface is only slightly inclined. This is because the slope-induced term in the momentum equation of x direction, when added to the pressure gradient term, gives an effective pressure gradient near the ground. The maximum reduction of temperature caused by the slope in all these runs, is found to be approximately $0.7°C$. If the boundary condition is given cooling rate instead of prescribed surface temperature, it is expected that the intensity of heat island would be weakened due to the presence of the terrain slope. Fig.5 illustrates the calculated eddy viscosities by both the two-length scale model and the standard $k-\varepsilon$, which to some extent demonstrates the turbulence structure and the characteristic of the proposed two-length scale model. The presence of a slope increases the vertical mixing as seen in Fig.5. The increase in the vertical mixing could alleviate the undesirable heat island effect. In the Run 4 and 5, the temperature difference between the rural and urban area is higher as compared with other runs. As a result, the change in velocity profiles (Fig.4), as expected, is greater than that in other runs.

On the other hand, the authors have previously conducted the investigation of water vapor effect on the mean velocity profiles over a warmer flat horizontal surface. When the surface is slightly inclined, it is found from the present study that the water vapor effect can be enhanced if a slight slope is present. For example, the maximum relative deviation of velocity profiles caused by the presence of water vapor in Run 1 (unsloped case) is about 7% while in the Run 3 (sloped case), the maximum relative deviation caused by water vapor is doubled.

4. Conclusions

A two-length $k-\varepsilon$ closure method, replacing the eddy viscosity formula in the standard $k-\varepsilon$ model, $\nu_t = c_\mu \frac{k^2}{\varepsilon}$ by eq.(8), has been applied to the numerical study of slope effect on the flow structure over a heat island. Numerical results reveal that even slight terrain slopes–the real world feature but a much neglected aspect in previous studies, can strongly affect the mean flow and turbulence structures. In comparison with the unsloped case, The alteration is appreciable. A slight positive slope intensifies the vertical mixing, which is beneficial to our human being. On the basis of obtained results, we would like to hypothesize that some discrepancies between the computational results and the observation data, reported in the literature, may partly be explained by the influence of a slightly inclined surface.

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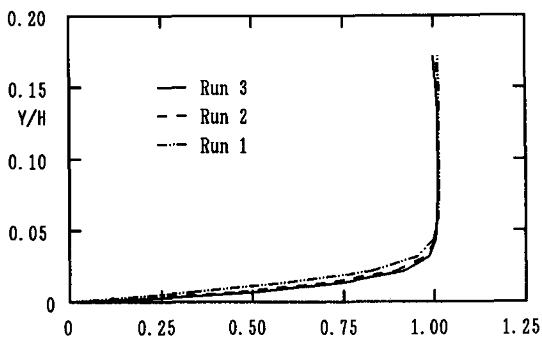

Fig. 2 Comparison of Velocity Profiles at X/H=2.16

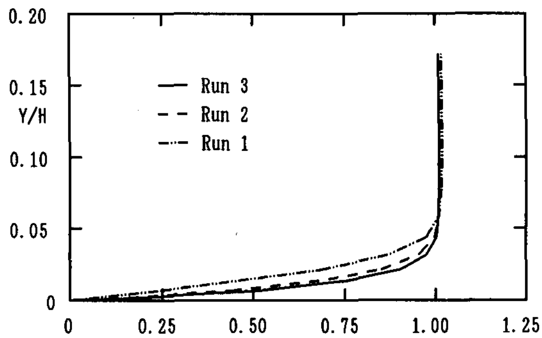

Fig. 3 Comparison of Velocity Profiles at X/H=3.8

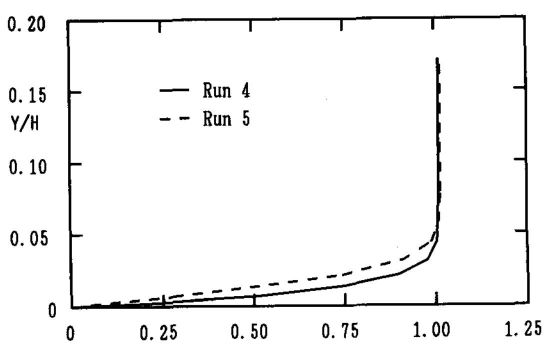

Fig. 4 Comparison of Velocity Profiles at X/H=2.16

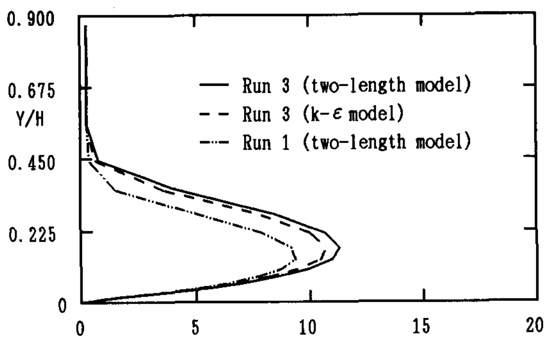

Fig. 5 Comparison of Eddy Viscosities at X/H=3.8