

## Simulation of Heat and Mass Transfer Between Bare Soil and Atmosphere

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**ABSTRACT:** A physically based mass and heat transfer model is presented and the theoretical results were compared with the experimental data measured under field conditions. The validity of a set of mass and heat balance equations as well as numerical schemes was discussed through sensibility analyses. In order to predict the evaporation from a bare soil, the model and simulation technique were applied to some practical data collected from a test site in southern France.

**Keywords:** heat and mass transfer, bare soil, drying, atmosphere, numerical simulation, field measurement

### 1. INTRODUCTION

During two last decades the irrigation plans in many semi-arid and arid areas of the world are still realized and designed. This went together with a strong increase in groundwater exploitation. Such cases have been found in the oil-rich countries of the Middle East and North Africa where the water requirement called for drilling activities of productive water wells. After only a few years it is, however, evident that the recession of groundwater resources was not fully understood and assessed. In many instances the accuracy achieved in enumerating various essentials of water and heat balance, and particularly of evaporation, can't be accepted as satisfying for the feasibility assessment.

Evaporation is an important element in hydrologic cycle and it plays a key role in water resource problems in semi-arid and arid regions. In order to plan the long term use of water resources in those regions, it is indispensable to have a good estimate of amount of evaporation from a bare soil ground.

Owing to the progress accomplished during the last decades in field of heat and mass transport in capillary porous bodies<sup>1)</sup>, an attempt in this paper is concerned to testing the possibility of theoretical modelling for numerating the evaporation from a bare soil. The model is rooted in an idea that the evaporation from soil surface occurs through unsaturated zone above groundwater table by heat and mass balance at global atmosphere. Accordingly, the evaporation from a bare soil can be enumerated by a set of heat and mass transport equations in unsaturated zone after putting boundary conditions at soil surface and groundwater table. The numerical scheme and simulation technique are also applied to field measurement data.

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## 2. MATHEMATICAL MODELLING AND BOUNDARY CONDITIONS OF HEAT AND MASS TRANSFER BETWEEN BARE SOIL AND ATMOSPHERE

### 2 - 1. Mathematical Modelling

The equations used for describing the heat and mass transfer in capillary porous bodies have been derived by Philip and de Vries in 1957<sup>2)</sup> and by Luikov in 1966<sup>3)</sup>. The governing equations with respect to heat and mass are obtained from a continuum concept of porous body and averaging approach of transport over a representative volume. The application of the equations to drying phenomena and restrictions imposed on them have been studied by Whitaker (1977)<sup>4)</sup>. The mass and energy balance equations in unsaturated zone for one - dimensional transport are :

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} (a_m \frac{\partial \theta}{\partial z} + a_d \frac{\partial T}{\partial z} - \frac{\rho_e}{\rho_o} k), \quad (1)$$

$$(\rho c)^* \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \{ (\lambda^* + \rho_o L a_{do}) \frac{\partial T}{\partial z} + \rho_o L a_{mo} \frac{\partial \theta}{\partial z} \}, \quad (2)$$

in which  $t$ : time,  $\theta$ : moisture content (mass of water per unit volume of dried porous material),  $T$ : temperature,  $\rho_e$ : density of liquid water,  $\rho_o$ : density of dried porous material,  $k$ : permeability of unsaturated porous material,  $\lambda^*$ : apparent thermal conductivity of unsaturated porous material,  $L$ : latent heat of water vaporization,  $z$ : vertical axis,  $a_m$ ,  $a_d$ ,  $a_{mo}$ , and  $a_{do}$ : total isothermal mass diffusivity coefficient, total non - isothermal mass diffusivity coefficient, isothermal mass diffusivity coefficient of vapor, non - isothermal mass diffusivity coefficient of vapor, respectively. The coefficients  $a_m$ ,  $a_d$ ,  $a_{mo}$ , and  $a_{do}$  depend upon both water content and temperature, and the permeability of unsaturated porous material depends on water content and temperature.  $\lambda^*$  and  $(\rho c)^*$  are related to water content.

In applying Eqs. (1) and (2) to drying problems, particular attention will be given to ~~three dominant forces~~ in moisture movement: (1) capillary force resulting in saturation and thermally induced effect, (2) molecular diffusion due to local vapor pressure, and (3) gravity. The possible application of the theoretical model has been tested and evaluated by several authors <sup>5),6),7)</sup> through numerical analyses and laboratory experiments. They proved that the equations are correctly adopted to describe most phenomena involved in the simultaneous heat and mass transfer in unsaturated porous materials. The discretization of Eqs. (1) and (2) for realizing numerical computation can be done by the FEM <sup>8)</sup>.

### 2 - 2 Boundary Conditions and Macroscopic Equations of Mass and Energy

When a thermodynamic system as shown in Fig.1 is considered, the evaporation from a bare soil may occur as a coupled heat and mass transport phenomenon between underground and atmosphere. Upon the accomplishment of numerical simulation by Eqs. (1) and (2), two boundary conditions at surface ( $z=0$ ) and groundwater table ( $z=z_N$ ) must be imposed as follows:

$$\left. \begin{aligned} \text{Mass flux } W &= -\rho_o (a_m \frac{\partial \theta}{\partial z} + a_d \frac{\partial T}{\partial z} - \frac{\rho_e}{\rho_o} k), \\ \text{Heat flux } G &= -\{ (\lambda^* + \rho_o L a_{do}) \frac{\partial T}{\partial z} + \rho_o L a_{mo} \frac{\partial \theta}{\partial z} \}, \end{aligned} \right\} \quad \text{for } z=0, t \geq 0 \quad (3)$$

$$\left. \begin{aligned} \text{Water content at groundwater table } \theta &= \theta_{sat} = \text{const.} \\ \text{Temperature at groundwater table } T &= T_N(t). \end{aligned} \right\} \quad \text{for } z=z_N, t \geq 0, \quad (4)$$

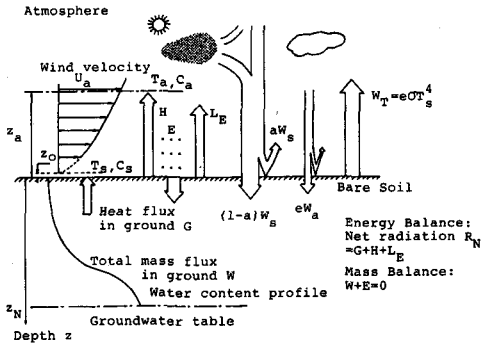


Fig.1 Energy Balance Between  
Bare Soil and Atmosphere

In order to fulfill the coupled heat and mass analysis for such a system as shown in Fig.1, two macroscopic balance equations with respect to water mass and heat energy must be given as follows<sup>9)</sup>: Total energy balance at surface ;

$$R_N - G - H - L_E = 0, \quad (5)$$

Total water mass balance at surface ;

$$W + E = 0. \quad (6)$$

In Eq.(5) the net radiation  $R_N$  is defined as,

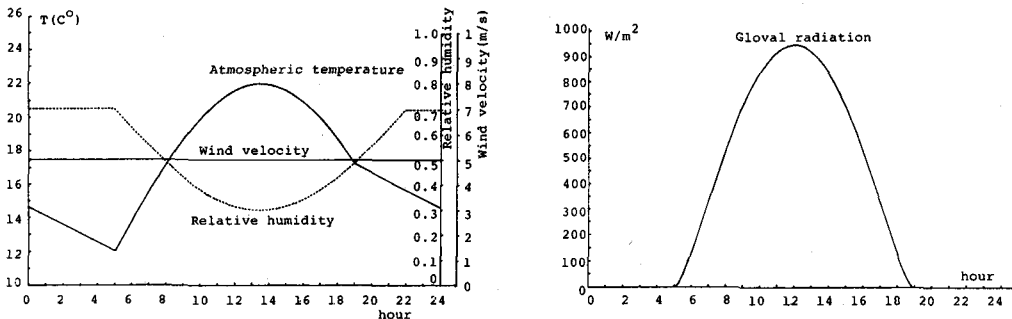
$$R_N = (1-a)W_s + eW_a + e\sigma T_s^4, \quad (7)$$

in which  $a$  : albedo,  $W_s$  : solar radiation near surface,  $e$  : radiation emissivity at surface,  $W_a$  : atmospheric radiation,  $\sigma$  : Stephan's constant and  $T_s$  : temperature at surface. The sensible heat flux  $H$  is proportional to the temperature difference between surface temperature  $T_s$  and atmospheric one  $T_a$ :  $H = h(T_s - T_a)$ ;  $h$  : heat transfer coefficient. The latent heat flux  $L_E$  is similarly defined by  $L_E = La(C_s - C_a)$ ,  $C_s$  : specific humidity at surface,  $C_a$  : specific humidity at atmosphere,  $a$  : mass exchange coefficient, and the mass flux  $E$  at soil surface is  $E = a(C_s - C_a)$ . Several coefficients such as  $h$ ,  $a$  and  $L$  can be determined by empirical and theoretical formulas<sup>10)</sup>.

### 3. ESSENTIALS OF PHENOMENA AND SENSIBILITY ANALYSIS

Essentials of heat and mass transfer in unsaturated zone can be numerically examined by using Eqs. (1) - (7) and auxiliary relations. Fig.2 (a) and (b) shows some adopted change patterns of temperature, wind velocity, relative humidity and global radiation at the height of 2.0m from a sand ground surface. The numeration of  $H$ ,  $E$  and  $L_E$  is done by a semi-empirical theory which was substantially derived by Obukhov (1971)<sup>11)</sup>, and aerodynamic roughness  $z_0$ , albedo  $a$  and radiation emissivity  $e$  are 1mm, 0.2 and 0.98, respectively. The atmospheric radiation of long wave  $W_a$  is computed by Brutsaert's formula (1975)<sup>12)</sup>. The temperature of groundwater is  $T_w = 15^\circ\text{C}$ .

To have a better understanding of physical process in unsaturated zone two computed results of energy balance with different groundwater depth are shown in Figs.3 and 4. According to both results, the net radiation  $R_N$  increases with higher groundwater depth since the temperature at surface  $T_s$  lowers by an effect of latent heat.



(a) Profiles of Temperature, Wind  
Velocity and Relative Humidity

(b) Profile of Global Radiation

Fig. 2 Atmospheric Conditions for Numerical Sensibility Analysis

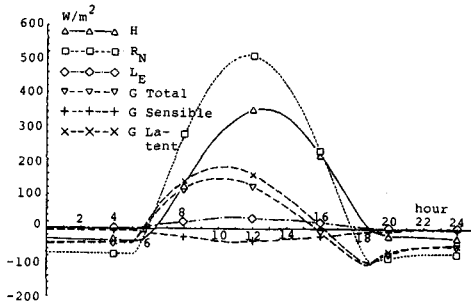


Fig.3 Energy Balance at Surface  
(Groundwater table at 1.5m)

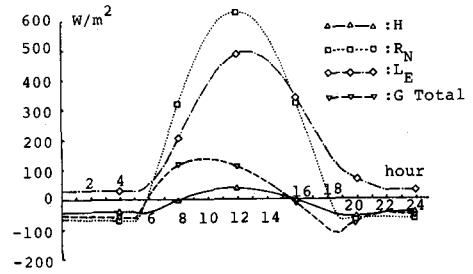


Fig.4 Energy Balance at Surface  
(Groundwater table at 0.5m)

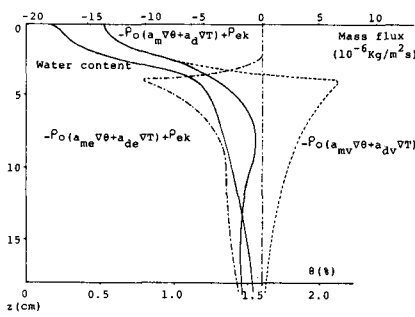


Fig.5 Water Content and Mass  
Flux Profiles at Midday  
(Groundwater table at 1.5m)

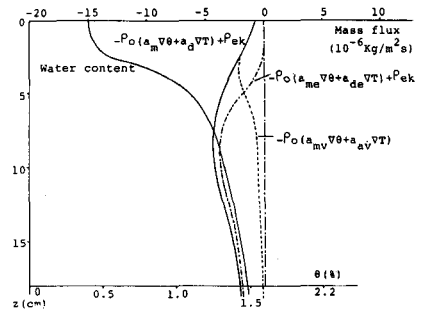


Fig.6 Water Content and Mass  
Flux Profiles at Midnight  
(Groundwater table at 1.5m)

In addition, the contribution of latent and sensible heat to total heat flux  $G$  is inverted each other. The water content and mass flux profiles with depth at midday and midnight are shown in Figs.5 and 6. It can be found that a zone of lower water content appears near ground surface in both cases, and the development of the dried zone is accelerated by temperature gradient  $\nabla T$ . The thermally induced mass flow plays an important part in moisture deficit by evaporation.

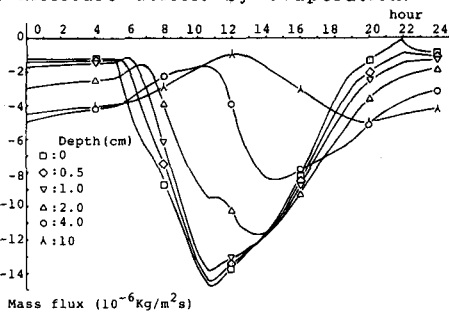


Fig.7 Variation Curves of Total  
Mass Flux with Depth  
(Groundwater table at 1.5m)

Fig.7 shows several variation curves of total mass flux  $G$  at different depth in a day. The computed results disclose that the atmospheric conditions at ground surface does not reach so deeper place, and the variation of mass flux during the daytime is not the same with that in the night because the temperature gradient and the relative humidity change with time. It is clear, however, that the thermally induced flow caused by a sharp gradient of temperature strongly contribute to rebuilding of mass flux transport near soil surface.

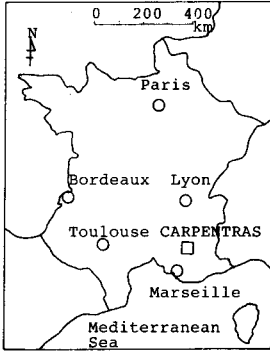


Fig.8 Location of Field Measurement  
(Carpentras, Umucluse District, France)

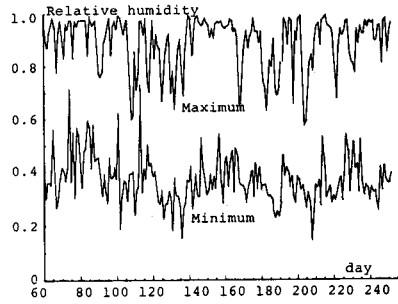


Fig.9 Observed Minimum and Maximum  
Relative Humidity at 2.00m  
(Carpentras, 1979)

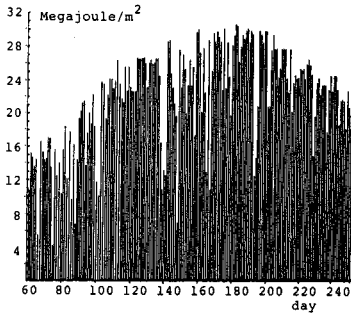


Fig.10 Observed Global Radiation  
(Carpentras, 1979)

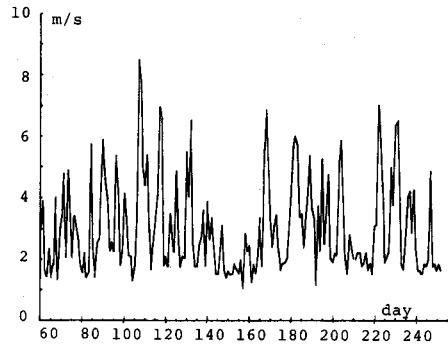


Fig.11 Observed Mean Wind Velocity at 2.00m  
(Carpentras, 1979)

#### 4. COMPARISON OF NUMERICAL SIMULATION WITH FIELD MEASUREMENT AT CARPENTRAS. (Umucluse District in France)

The simulation scheme used Eqs. (1) and (2) with a set of boundary conditions is applied to some typical results obtained from a test field close to Carpentras City in Fig.8. Available data in the period of 1th March, 1979 to 9th September, 1979 at la Station de la Météologie Nationale were: (1) Mean wind velocity, temperature and relative humidity (every three hours), (2) Extreme values of temperature and relative humidity at minimum and maximum, (3) Global radiation (every hour), (4) Exposure time of sunshine (every day), (5) Cloudy time (every three hours), (6) Rainfall intensity and duration (every day) and (7) Daily evaporation and temperature at surface.

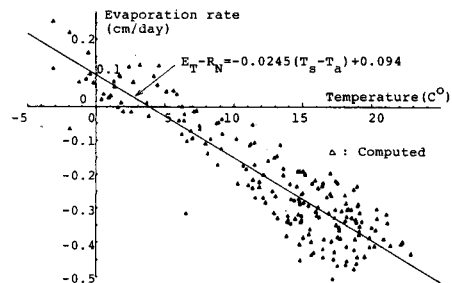


Fig.12 Comparison of Observed Net  
Evaporation Rate with Simulated  
One for Clayey Bare Soil (after 13hr)

Some of observed results with minimum and maximum relative humidity, global radiation and mean wind velocity are shown in Figs.9,10 and 11. Similar numeration with sensibility analysis were carried out by using all of the observed data at atmosphere and groundwater table. In Fig.12 many computed values of net evaporation high in day (cm/day), which is divided by  $\rho L$ , are compared with a simplified equation which was proposed by Jackson et al. (1977)<sup>13)</sup> and further developed by Itier & Riou (1982)<sup>14)</sup>. The latent heat flux  $E_r$  and  $R_n$  in the simplified equation denote a daily latent heat flux and daily net radiation, respectively, and  $T_s - T_a$  is the temperature difference between surface and atmosphere. It can be found that all computed values agree with the simplified equation within correlation coefficient 0.91.

## 5. CONCLUSION

Modelling of a macroscopic evaporation process sometime includes some complex problems to solve it in all of real phenomena. During the last decades a great deal of improvement in scientific knowledge with fundamental mechanism and modelling technique would have been made in the field of drying science. Owing to the progress made in the field, it is now possible to achieve a good numerical simulation of evaporation through unsaturated zone under natural conditions. An innovative attempt in this paper proves the extension of basic knowledge. It may be given as a conclusion that the evaporation from a bare soil ground takes place in imposed balance of water and heat transport within unsaturated zone, and also the computed results are in accord with those of field measurement. As emphasized by many soil physicists and hydrologists, a challenging problem in future remains in development of more reliable and accurate measurement techniques. This paper was completed as a part of jointed study between Institut de Mecanique des Fluides de Toulouse, CNRS and Hydrosience and Geotechnology Laboratory, Saitama University during his stay of invited scientist, S.Bories, in Japan. The authors would like to express their appreciation to the fellowship of the Japan Society for the Promotion of Science.

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