Field Estimation of Saturated Conductivity Using Borehole Test :Effect of Unsaturated Flow and Soil Anisotropy

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Constant head borehole test is analyzed using a numerical model of the Richards equation. Steady state analysis is used to develop an efficient model scaled with respect to soil conductivity, thus allowing it to be used for the estimation of soil conductivities from field test results. Analytical formulae based on free-surface assumptions and those taking soil capillary effects into account are assessed by comparing with the numerical model results. The steady state analysis method is extended to the estimation of conductivities in uniform anisotropic soils. Field tests and results are discussed.
KEYWORDS: Borehole test, Richards' equation, Field Conductivity

1. INTRODUCTION

Field estimation of soil saturated conductivity is required in a number of civil engineering applications such as in the construction of canals, dams and infiltration (recharge) systems. In recent years, the development of distributed physically based hydrological models such as 'SHE' has drawn increased attention to the estimation of field conductivity required in such modelling. Borehole infiltration tests are used to estimate the field saturated conductivities at arbitrary depths above the groundwater. In this test a borehole is drilled and filled with water. Outflow rate required to maintain a constant prescribed water head within the borehole is measured, and from this rate, the soil saturated conductivity (K_0) is estimated.

The test had been limited by theoretical approximations used in the derivation of formulae used for the estimation of K_0. Traditionally the flow from borehole test had been described using the free-surface concept(Stephens and Neuman 1982a) where the flow is assumed to take place inside an envelope known as free surface, outside of which the soil is assumed to be dry. Recent investigations have clearly identified that flow from a small well to unsaturated soil to be a three-dimensional infiltration process where a stationary saturated bulb is formed in the vicinity of the wetting surface, while unsaturated flow continue to expand out side of the saturated front (Stephens and Neuman 1982b, Reynolds et al. 1983, Herath and Musiake 1987). Reynolds et al (1985) and Philip (1985) proposed two analytical solutions for the borehole test which incorporate the soil hydraulic properties and the saturated-unsaturated flow mechanism into consideration. However, in order to derive closed form solutions both approaches make the assumption that flow due to gravity and capillarity can be separated. In addition, all of the above analyses assume the soil to be uniform and isotropic. The soil anisotropy cannot be taken into account in those solutions.

Musiake and Herath (1988) showed that saturated conductivities in both horizontal and vertical directions can be estimated from the borehole test by numerically solving the Richards' equation and employing inverse solution procedure. However, this solution is computationaly extensive requiring a powerful computer and not suitable for general application. This paper discusses the estimation of K_0 from borehole test using steady state numerical analysis of Richards' equation without resorting to iterative solutions and the extension of the method to anisotropic soils. The numerical solutions are compared with approximate analytical solutions of Reynolds and Philip and field experiments and results are discussed.

2. NUMERICAL MODEL DESCRIPTION

2.1 PROBLEM FORMULATION

As mentioned, flow from a borehole is an unsteady process, never achieving a steady state. However, the infiltration rate become constant due to the formation of a finite stationary saturated bulb. By estimating this saturated front, using a quazi-steady state imposed on the flow domain by fixed boundary conditions, it is possible to estimate the final infiltration rate from a test. A finite difference numerical model was developed, due to the simplicity of source geometry and flow domain, to solve the Richards' equation for steady state given by,

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$$\nabla \bullet [\, k(\varphi)\,(\varphi - z)] = 0 \qquad (1)$$

where φ is the soil water suction and $k(\varphi)$ is the soil conductivity at suction φ. As the flow can be considered symmetric, only two dimensional analysis is required. By expressing the conductivity as,

$$k(\varphi) = K_0\, kr(\varphi) \qquad \text{and} \quad \gamma = K_{0h} / K_{0v}$$

where $kr(\varphi)$, the relative conductivity is assumed to be isotropic and K_{0h} and K_{0v} are the saturated conductivities in the horizontal and vertical directions respectively, eq(1) in cylindrical coordinates can be expressed as,

$$\frac{1}{r}\frac{\partial}{\partial r}[\, \gamma\, kr(\varphi)\, r \frac{\partial \varphi}{\partial r}] + \frac{\partial}{\partial z}[\, kr(\varphi) \frac{\partial}{\partial z}(\varphi - z)] = 0 \qquad (2)$$

Once the final pressure distribution is obtained from (2), infiltration from the borehole(Q) can be obtained from Darcy's equation applied to borehole bottom and the wall as,

$$\frac{Q}{K_{0v}} = \int_{A=0}^{A=\pi a^2} kr(\varphi) \left.\frac{\partial(\varphi - z)}{\partial z}\right|_{z = z_b} dA + \int_{z=z_t}^{z=z_b} 2\pi a \gamma \;\; kr(\varphi) \left.\frac{\partial(\varphi)}{\partial r}\right|_{r = a} dz \qquad (3)$$

The simulation domain is shown in the Fig. 1 with relevant notation. Equation (2) is to be solved subject to the boundary conditions given as,

$0 <= z <= z_n$	$x = x_n$	$\varphi = \varphi_n$ or $\frac{\partial \varphi}{\partial x} = 0$
$z = z_n$	$0<=x<=x_n$	$\varphi = \varphi_n$
$z = 0$	$x > a$	$\varphi = \varphi_n$
$z = z_b$	$x <= a$	$\varphi = h$
$z_t <= z <= z_b$	$x = a$	$\varphi = z_b - z_t$
$0 <= z < z_b$	$x = a$	$\partial\varphi/\partial x = 0$
$z_b <= z <= z_n$	$x = 0$	$\partial\varphi/\partial x = 0$

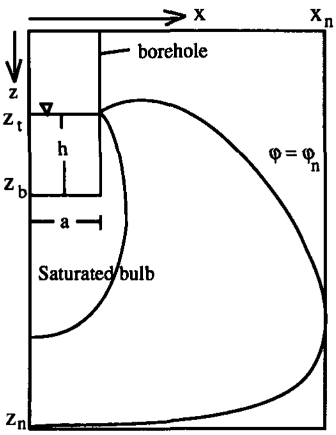

Fig. 1 Model domain used for numerical simulation

where, x_n and z_n are the model boundaries in x and z directions, z_t and z_b are the depth to the water surface and the bottom of the borehole, 'a' is the radius and h is the water head. The x = 0 grid defines the center line of the source. Both boundary conditions at the extreme end of the domain, $x = x_n$, resulted in identical saturated bulb extent and hence the same final infiltration rates.

2.2 SOLUTION PROCEDURE

Due to the non linearity of the equation(2), it has to be solved by an iterative process. In each iteration $kr(\varphi)$ function has to be updated for most recent solution of φ. These iterative updating of $kr(\varphi)$ is termed as the outer-solutions. Within each of these iterations, φ values of the model domain are solved for the given boundary conditions. These iterations are termed as the inner-solutions. The termination condition of the outer solutions taken is,

$$\text{Mass Balance} = |\,(Vt - Vb)\,| \times 100 / Vt \; < 2\%$$

where, Vt = Infiltration from source and Vb = total outflow from model boundaries. At different levels of iterations two types of iterative schemes are used for the upgrading of the $kr(\varphi)$ given by,

a) $kr(\varphi)_{n+1} = kr(\varphi_n)$ and

b) $kr(\varphi)_{n+1} = kr(\varphi_n) + \partial kr/\partial\varphi_n \cdot d\varphi$

where n is the iteration number. ADI scheme is used for the solution of the set of linear equations which result from the discretization of Eq.(2) . Here also two schemes are used. One of them is used to stabilize the iterative procedure and the other is used to accelerate the convergence towards the final solution. The first one used is the Peaceman-Rathford iterative scheme with cyclic acceleration parameters, and the second one is a variant of it in which the splitting of the coefficient matrix is done so that all the contributions of the current node are taken as unknowns. With the second scheme, the solution at the end of an iteration is under relaxed. The switching between the two schemes were done automatically by monitoring the error terms in the individual cells as well as the rate of reduction of the total mass balance of the simulating region.

2.3 DISCRETIZATION

At the beginning, setting up of the grids is done in the program using a grid generating algorithm. In the solution of Eq.(2) it is necessary to use very small spatial discretization near the spreading water front due to the rapid changes of $kr(\varphi)$. As it is dynamic, (i.e. the location of the water front changes with the iterations until the final correct position is obtained as solution), it become necessary to have a large number of grids if static meshes are used. Also depending on the geometry of the infiltration facility, number of grids required would vary. To avoid these problems, a mesh moving algorithm is used in the simulation routine. This routine is triggered if there are fluctuations in the mass balance decrease during the computations. In this case the solution from the last iteration is discarded and the computation is resumed after redistribution of the grids and interpolation of the prior solution to the new grid nodes. In the grid re-distribution the method described by Dane and Mathis(1981) is used. The criterion they have used for a one dimensional situation is adapted to the present two dimensional case by applying it to the strips in vertical and horizontal directions.

3. COMPARISON BETWEEN ANALYTICAL FORMULAE AND NUMERICAL SOLUTION

Some closed form solutions for borehole test were compared with the results of numerical simulation to check the applicability of various assumptions made in the derivation of these solutions. Two of the solutions assume free surface type flows, and the remaining two, although considers unsaturated-saturated flow domain, employ linearization using the approximation of conductivity in the form of $kr(\varphi) = e^{-\alpha\varphi}$ and other assumptions with regard to the flow. As all these are for isotropic mediums, γ was taken as 1 in the numerical model. The analytical solutions are described in Table 1.

Table 1. Borehole formulae tested and their characteristics

Name	Equation	Major Assumptions
Glover (1953)	$Q/K_0 = C\,a\,h$ where $C = \dfrac{2\pi(h/a)}{\mathrm{Sinh}^{-1}(h/a) - 1}$	Derived for freesurface conditions. Based on an analytical solution for a vertical line source whose strength is zero at the top and varies linearly with response to gravity. Disregards outflow through the open bottom. Uniform isotropic medium.
Nasberg-Terletskaya (1951-1954)	$\dfrac{Q}{K_0} = \dfrac{h^2}{0.423\,\mathrm{Log}[2\,h/r]}$	Assumes free-surface flow. Derivation similar to Glover's except for inclusion of an image sink point to account for a free surface. Uniform isotropic medium.
Reynolds (1985)	$\dfrac{Q}{K_0} = \dfrac{1}{C}[2\pi h^2 + \pi C r^2 + 2\pi h\,\phi_m]$ where $\phi_m = \int_{\varphi_i}^{0} kr(\varphi)\,d\varphi$ φ_i is the initial suction, and $C = 4\left\{\dfrac{1}{2}\sinh^{-1}\dfrac{h}{2a} - \sqrt{\left(\dfrac{a}{h}\right)^2 + \dfrac{1}{4}} + \dfrac{a}{h}\right\}$ corresponding to half source solution	Includes unsaturated flow effects through linearization. Uniform isotropic medium. Coefficient C is estimated by representing a well by series of point sources along the well axis to provide a line source, or by numerically solving the Laplace equation applied to a well whose dimensions are normallized with respect to radius, a.
Philip (1985)	$\dfrac{Q}{K_0} = a^2\sqrt{H^2 - 1}\ \times$ $\left[\dfrac{4.117\,H\,(1 - H^{-2})}{\ln\left(H + \sqrt{H^2 - 1}\right) - \sqrt{1 - H^{-2}}} + \dfrac{4.028 + 2.517\,H^{-1}}{A\ln\left(H + \sqrt{H^2 - 1}\right)}\right]$ where $H = h/a$ and $A = (\alpha\,a)/2$	Includes unsaturated flow effects through linearization. Uniform isotropic medium. Replaces the borehole with a prolate-spheroid to derive the solution. Assumes gravity and capillary components of flow are separable. Model is derived by equating the flow from the borehole into the saturated bulb with the unsaturated flow leaving the saturated zone.

The Q/K_0 curves for borehole test with a=11 cm were estimated for average Kanto Loam, Sand, Loam and Clay soils. The soil moisture properties for the average curves are shown in Fig. 2. These curves represent median of representative shallow soil samples, some measured at the Tokyo University and others taken form the Mualem's catalogue(1976), for each group. For the ease of numerical simulation, the data are represented by the analytical functions. Moisture-suction relation is represented by the equation,

$$\theta = \frac{\alpha(\theta_0 - \theta_r)}{\alpha + [\ln(\varphi)]^{\beta}} + \theta_r \qquad (8)$$

where θ = moisture content, θ_0 = saturated moisture and θ_r = residual moisture. The conductivity-suction relation is represented by the equation,

$$kr(\varphi) = \frac{\alpha 2}{\alpha 2 + \varphi^{\beta 2}} \qquad (9)$$

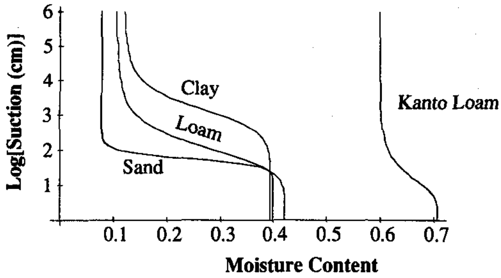

Fig. 2 Representative moisture suction relations.

where $\alpha 2$ and $\beta 2$ are arbitrary parameters. Table 2 lists the parameters for each of the soils. The conductivity curves were generated by the $kr(\varphi) = Se^n$ model with n determined according to the model proposed by Mualem(1978).

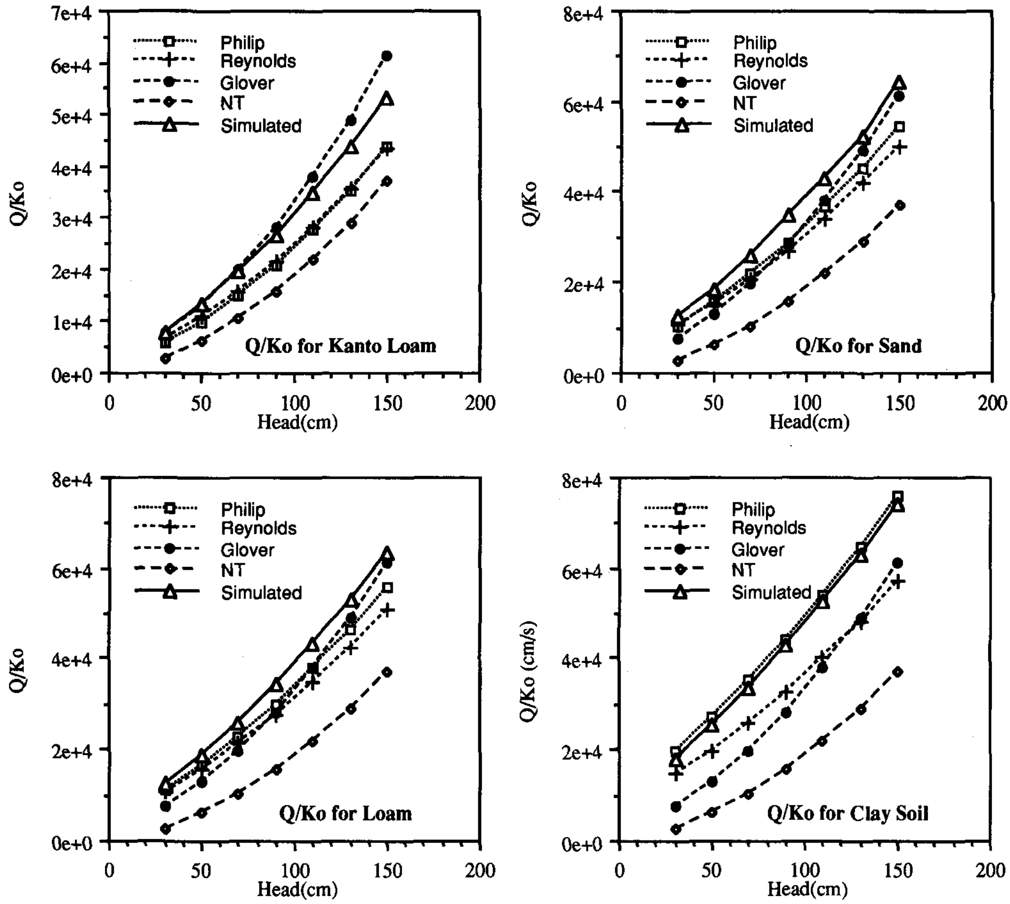

Fig. 3 Q/K_0 curves from various formulas compared with simulated results (a = 11 cm borehole)

Fig. 3 shows the Q/K_0 curves for the four formulas listed above together with the numerical simulation results for the soil properties listed in Table 2. Initial suction of the soil was assumed to be constant at 100 cm. It can be seen that Nasberg-Terletskaya formula always overestimate conductivity, while the performance of Glover solution depend on the type of soil. Philip's and Reynolds model agree closely except for clay soil. Out of the analytical formulae Philip's solution's consistency makes it the best candidate for quick estimates.

Table 2. Parameters for the Representative soils

Soil Name	θ_0	θ_r	β	α	$\alpha 2$	$\beta 2$
Kanto Loam	0.707	0.598	3.92	72.8	156	2.34
Sand	0.400	0.077	16.95	$1.752\ 10^{10}$	$4.34\ 10^{13}$	8.42
Loam	0.422	0.104	5.56	6451	$4.5\ 10^{4}$	3.11
Clay	0.394	0.120	9	$6.576\ 10^{7}$	$1.72\ 10^{4}$	2.3

4. ESTIMATING SOIL CONDUCTIVITY IN ANISOTROPIC SOILS

The Q/K_{0v} for different γ and water heads for the average Kanto Loam soil properties (Table 2 1st row) are shown in Fig. 4. These values are also for a 22 cm diameter borehole as in section 2. In the figure each curve represent Q/K_{0v} values for one water head, with x axis representing Log of γ and y axis representing $\mathrm{Log}(Q/K_{0v})$. If the soil anisotropy is not known, for a given water head several combinations of vertical and horizontal saturated conductivities can give the same final infiltration rate. These combinations of conductivities are represented by each curve for a particular water head.

If the soil formation can be assumed to be uniform, using two field tests, conducted at two different water heads, common anisotropy value can be estimated from the two Q/K_{0v} vs. γ curves of the test. This can be easily done by using a chart in the form shown in Fig. 5. Here one of the tests is taken as the standard (assumed to be 50 cm head test) and the y axis is Q/Q_s. Once the anisotropy (γ) is estimated, numerical simulation using the model for Eq(2) is used to estimate the correct Q/K_{0v} value, or it can be approximately estimated from the curves in Fig. 4. From the resulting Q/K_{0v} value, K_{0v} and hence K_{0h} (= γ K_{0v}) can be estimated.

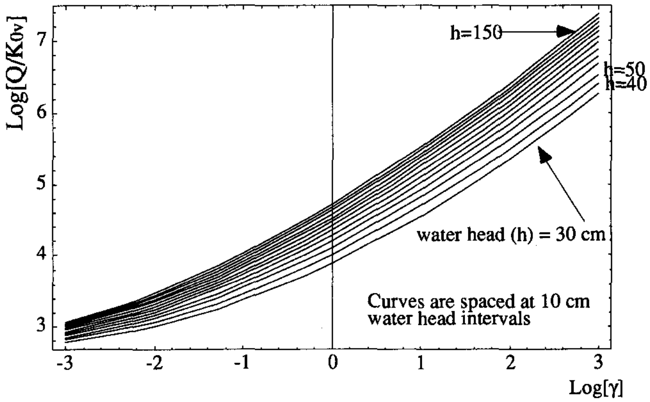

Fig. 4 Effect of anisotropy on the borehole discharge. Each curve represent the discharge for a single water head for 22 cm diameter borehole in Kanto Loam

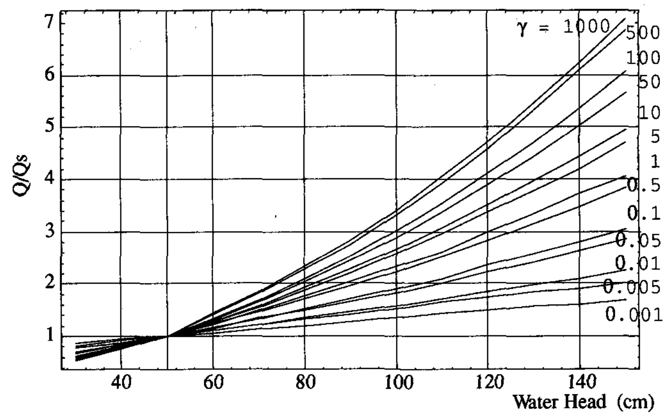

Fig. 5 Q/Qs (h=50 cm, d=22 cm) curves for borehole test in Kanto Loam

5. FIELD TESTS AND RESULTS

Borehole tests were conducted in Nagayama catchment in western suburbs of Tokyo metropolitan area and in Chiba prefecture , both areas consisting of a Kanto Loam top soil layer. The experimental set up is shown in Fig. 6. In these tests following observations were made. The discharge from the borehole is affected by the state of the infiltrating surface. If the borehole become muddy or clogging occur, then the discharge is reduced, and hence the conductivity is underestimated. Borehole tests of 10 cm and 22 cm diameter were carried out for various water heads. It was found that 22 cm diameter test is better for cleaning the infiltrating surface and obtaining stable results.

The test results show a dependency of K_0 on water heads for low water heads. The infiltration rates for boreholes with 10 cm or lower heads can be as low as 1/3 to 1/5 of the high water head test results.

When the water head is above 30 cm, the estimated conductivity was consistent. It was suspected that complete saturation does not occur at low water heads. To clarify this, some tests were carried out in which infiltration rates were measured in the same borehole for different water heads, starting with higher water head. In this case it was found that K_0 values were consistent for 100 cm, 50cm and 20 cm water heads at 0.0016, 0.0014 and 0.0012 cm/s respectively, for that particular site.

The borehole tests conducted in Chiba experimental station resulted in near isotropic parameters. Discharge from 50 cm head 22cm diameter borehole was found to be 51 cc/s. From the values of Fig.3 for 50 cm head Q/K_0 can be obtained and hence K_0 computed. The following values were obtained for different formulas. Philip 5.16×10^{-3} , Reynolds 4.73×10^{-3} , Glover 3.96×10^{-3} , Nasberg-Terletskaya 8.27×10^{-3}, and 3.86×10^{-3} cm/s from simulation curve. These results show Glover formula to give excellent results, but at different a/h ratios its performance become very poor.

Two tests conducted in Nagayama catchment at 50 cm head and 100 cm head for 22 cm diameter borehole resulted in infiltration rates $Q_{50} = 39.93$ cc/s and $Q_{100} = 73.61$ cc/s. From these values $Q_{100}/Q_{50} = 1.84$ and from Fig. 4, $\gamma \sim 0.07$. Using this value saturated conductivities in horizontal and vertical directions are estimated as,

$K_{0v} = 1.4 \times 10^{-2}$ cm/s and $K_{0h} = 0.98 \times 10^{-3}$ cm/s

These values closely agree with a separate analysis made using unsteady state parameter optimizing technique on the 50 cm head test (Musiake and Herath 1988) which resulted in the values of $K_{0v} = 1.14 \times 10^{-2}$ cm/s and $K_{0h} = 1.03 \times 10^{-3}$ cm/s.

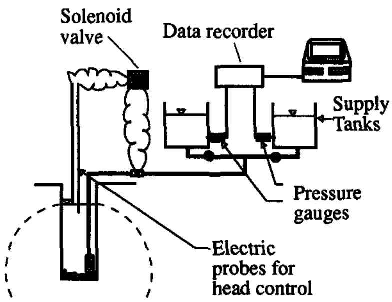

Fig 6. Field Apparatus for the borehole test

6. CONCLUSIONS

For the accurate field estimation of saturated hydraulic conductivity, analysis which take soil hydraulic properties into account is required. Steady state numerical simulation is fast and Q/K_0 values can be computed for given dimensions within a few minutes in a PC class machine. In the absence of such models, Philip's model provides good results especially when $h/a > 20$. Using the method outlined in this paper, it is possible to estimate the soil conductivities which can be used to estimate infiltration from trenches and wells adequately (Herath and Musiake, 1987) and thus should also be possible to use in the analysis of seepage from dams and canals etc.

The tests have also shown that complete saturated flow does not occur at low water heads. This means where ponding water heads are low, such as during rainfall, the conductivities to be used should be less than those estimated from borehole tests. In such situations it may be necessary to estimate the parameters from tests which imitate the process of interest more closely.

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