

## ANALYSIS OF SCOUR PROCESS DOWNSTREAM OF A SLUICE

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The evolution of local scour with time, caused by a submerged hydraulic jump downstream of an apron, is investigated on the basis of the sediment continuity equation as well as the experimental evidence that the configuration of scour holes in the progressive stage possesses the properties of geometric similarity. A semi-theoretical analysis of one aspect of the phenomena of the problem is presented. Calculated results are compared with available data.

*Keywords: Local scour, Submerged jump, Hydraulic structure.*

### 1. INTRODUCTION

The localized scour of alluvial beds near hydraulic structures has been studied extensively as a consequence of the occurrence of severe damage. Such erosion could be caused by local flow acceleration, secondary flow, high velocity jets, and other causes. One of the typical and the simplest possible situations of such phenomenon is the local scour downstream of a sluice. In this case, the sediment inflow rate at the upstream boundary is less than the capacity of the flow to transport sediments in the region where the local scour takes place.

When water is issued from a sluice, a hydraulic jump, which develops like a submerged wall jet on a rigid apron before it encounters a mobile bed, is formed. The velocity of such a wall jet is often very high. In some cases the high local shear stress may exceed the critical shear stress for incipient motion of the bed material even over the loose bed downstream of the apron. As a result, in many practical situations a large-scale hole may be observed. Since the jet plunges into and develops along the scour hole, the introduction of an erodible bed downstream of a rigid apron causes the flow characteristics of the jet to alter significantly: as seen, for example, by Saito<sup>1)</sup>, Chatterjee and Ghosh<sup>2)</sup>, and Hassan and Narayanan<sup>3)</sup>. The internal flow structure of such a wall jet along the scour hole is found to be substantially different from the one over a rigid flat apron. Fig.1 shows schematically the various factors involved in the scouring process along with the coordinate system to be used.

The problem of local scour is an extremely complex phenomenon. This is not only primarily because of the complex interaction between the sediments and the flow properties within a scour hole, but also because of the flow characteristics within a scour hole itself are not easy quantified. This complex nature of the scouring process involved makes theoretical analyses extremely difficult, so that studies dealing with local erosion are mostly based on empirical results. However, some interesting analytical attempts have been made to

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quantify the process of local scour under various degrees of approximations and assumptions. For instance, Tsuchiya & Iwagaki<sup>4)</sup> attempted a similarity analysis, considering bulk sediment balance within scour holes. Saito<sup>1)</sup> investigated the evolution of the scouring process along loose boundaries, taking the characteristics of both a wall jet and sediment movement under non-equilibrium condition into account. However, a general analytical answer to this problem has not yet been found.

In this paper, the clear water scour downstream of an apron is considered. Taking an analogous approach to the one taken by Tsuchiya & Iwagaki<sup>4)</sup> and utilizing available experimental data, a simple semi-theoretical approach to predict the evolution of scour hole with elapsed time is proposed.

## 2. DIMENSIONAL CONSIDERATION

In order to evaluate the scour process in a systematic manner, it is of importance to first consider the various factors affecting to the problem of scour eroded by wall jets issuing from a sluice. With reference to Fig.1 and in light of the dimensional analysis, the general relationship expressing the phenomenon may be given by

$$f\left(\frac{z}{d_s}, \frac{x}{d_s}, \frac{Ut}{d_s}, \frac{U}{v_s}, \frac{v_s d_s}{v}, \frac{L_p}{D}, \frac{D}{d_s}, \frac{h_0}{d_s}, s, \sigma\right) = 0 \quad (1)$$

where,

- $z$  = scour hole depth;
- $x$  = horizontal axis;
- $t$  = elapsed time;
- $D$  = sluice opening;
- $U$  = flow velocity at the sluice;
- $L_p$  = length of apron;
- $h_0$  = tailwater depth;
- $d_s$  = sediment size;
- $v_s$  = sediment velocity =  $(sgd_s)^{1/2}$ ;
- $s$  = submerged specific weight =  $\rho_s/\rho - 1$ ;
- $\rho_s$  = density of sediment particles;
- $\rho$  = density of water;
- $\nu$  = kinematic viscosity of water;
- $\sigma$  = geometric distribution of sediment diameters about the mean;
- $g$  = acceleration of gravity

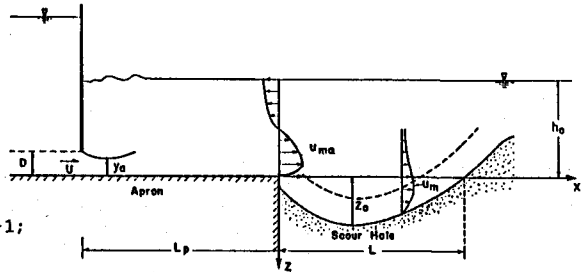


Fig.1 Definition Sketch

Consider the evolution of the geometry of a scour hole of a loose bed made of a uniform sediment in time, the important characteristics length scale of the eroded bed in the progressive stage are the maximum scour depth  $z_0$  and the length  $L$  of the scour hole. Rewriting Eq. (1) with  $z = z_0$  and  $x = L$ , one can find

$$f_1\left(\frac{z_0}{d_s}, \frac{L}{d_s}, \frac{Ut}{d_s}, \frac{U}{v_s}, \frac{v_s d_s}{v}, \frac{L_p}{D}, \frac{D}{d_s}, \frac{h_0}{d_s}, s\right) = 0 \quad (2)$$

Since it is not feasible to include all the possible variables affecting the scouring process in the analysis, it is necessary to exclude some of the variables in Eq. (2). In general, the parameter  $s$  may be considered constant and  $v_s d_s/\nu$  is not significant because the effect of viscosity is considered negligible in the scour process. Furthermore, restricting the problem to include only the local scour due to a deeply submerged hydraulic jump downstream of a sluice, namely, there is no pronounced tailwater effect. The variable  $h_0/d_s$  can be omitted. Eq. (2) can be reduced to

$$f_2\left(\frac{z_0}{d_s}, \frac{L}{d_s}, \frac{Ut}{d_s}, \frac{U}{v_s}, \frac{L_p}{D}, \frac{D}{d_s}\right) = 0 \quad (3)$$

It is well known that scour profiles in the progressive stage exhibit certain geometrical similarities if relevant parameters such as the maximum scour depth  $z_0$  or scour length  $L$  are used for scaling the shape of a scour hole. Of particular importance is the fact that the nondimensional scour hole profile remains approximately constant with time in the progressive stage. However, a similar profile scaled by a single reference length scale (i.e.  $z_0$  or  $L$ ) appears to depend on other conditions such as the sizes of sand, the sluice gate opening, the length of the apron and the upstream flow velocity. Saito<sup>1)</sup> demonstrated

that a unique similarity profile exists if scour hole profiles are non-dimensionalized by  $z_0$  in the vertical direction and by  $L$  in the longitudinal direction. Rajaratnam<sup>5)</sup>, on the other hand, employed  $z_0$  and the longitudinal distance  $x_m$  from the end of the apron to the position of maximum scour depth. These studies indicate that in order to collapse scour profiles under different conditions into a single profile, it is necessary to use different scalings in the vertical and longitudinal directions. We will now examine further the validity of similarity scour profiles. The experimental results of Tsuchiya & Iwagaki<sup>4)</sup>, Saito<sup>1)</sup>, Rajaratnam<sup>5)</sup>, and Hassan & Narayanan<sup>3)</sup> are used for this purpose. Among these works, Saito and Rajaratnam conducted experiments without aprons, whereas Tsuchiya & Iwagaki and Hassan & Narayanan with aprons. It may be seen from Fig.2 that except for the region in the vicinity of sluices or the toe of aprons, these data show good collapse of data regardless of the experimental conditions, indicating that the maximum scour hole depth  $z_0$  and the scour length  $L$  can be taken as pertinent vertical and longitudinal reference dimensions.

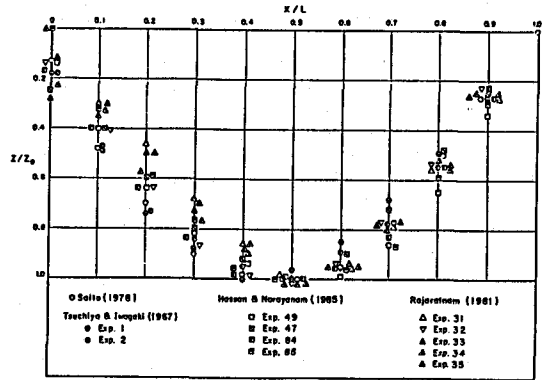


Fig.2 Similarity Profiles of Scour Hole

With reference to Fig.2, a fairly simple polynomial expression is obtained to describe approximately the dimensionless scour hole profile.

$$Z = 2.06X^2 - 2.79X + 0.12 \quad \text{for} \quad 0 \leq X \leq 0.5 \quad (4a)$$

$$Z = 2.71X^2 - 1.97X + 0.69 \quad \text{for} \quad 0.5 \leq X \leq 1 \quad (4b)$$

It can be expected that there is a unique relationship between scour length and maximum scour depth, so that one of these two variables will become redundant. Thus, Eq. (3) can be reduced to

$$\frac{z_0}{d_s} = f_3 \left( \frac{U_t}{d_s}, \frac{U}{v_s}, \frac{L_p}{D}, \frac{D}{d_s} \right) \quad (5)$$

3. MAXIMUM SCOUR DEPTH-TIME FUNCTION

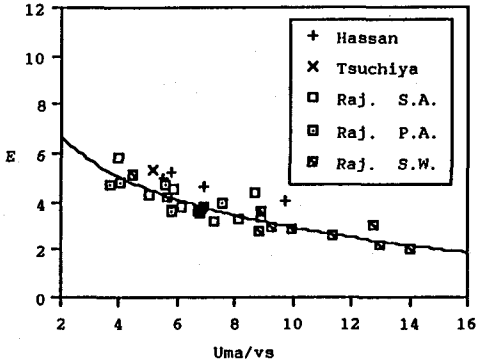


Fig.3 Relationship Between E and  $U_m a/v_s$

Herein the dimensionless parameter E, which expresses the maximum depth-length ratio of a scour hole, is introduced

$$E = \frac{L}{z_0} \quad (10)$$

The one-dimensional sediment continuity equation is given by

$$(1 - p) \frac{\partial z}{\partial t} - \frac{\partial q_s}{\partial x} = 0 \quad (6)$$

where:  $q_s(s, t)$  = the sediment transport rate by volume;  $p$  = the bed porosity

In view of the foregoing considerations on the geometrical similarities of scour holes, the following new independent variables  $X$  and  $Z$  are defined as

$$X = \frac{x}{L} \quad (7)$$

$$Z = \frac{z}{z_0} \quad (8)$$

With use of the independent variables  $X$  and  $Z$ , the sediment continuity equation is transformed into

$$\frac{1}{(1 - p)L} \frac{dz_0}{dt} dq_s = Z dX \quad (9)$$

As seen in Fig.2, the geometry of scour holes possesses a similar shape. This implies that the parameter E is invariant with time and with position of maximum scour depth. We will now examine the dependence of E on the other parameters by examining existing experimental data. Of the several factors that affect the parameter E on the scouring process, the most significant may be the sizes of sediment and the upstream flow conditions. These two conditions are parameterized into  $D/d_s$  and  $U/v_s$ . In cases where scours occur downstream of aprons, the maximum flow velocity  $u_{ma}$  and characteristics length scale  $\delta_a$  at the end of the apron can be used instead of U and D, respectively. Time-invariant similarity profiles of scour holes are found to depend mainly on the parameter  $u_{ma}/v_s$  (or  $U/v_s$ ). No systematic dependence of E on other parameters is recognized. Plotted in Fig.3 is the relationship between E and  $u_{ma}/v_s$  (or  $U/v_s$ ), where it can be seen that the parameter E is an apparent function of  $u_{ma}/v_s$ . E decreases monotonously as  $u_{ma}/v_s$  increases. The following empirical relation may be deduced:

$$E = 8.25 - 2.30 \ln(u_{ma}/v_s) \dots\dots\dots (11)$$

As the dimensionless scour profile is invariant with time. Hence, Eq.(9) can be integrated with respect to X from X= 0 to 1.0 under the boundary condition of  $q_s(0) = 0$ , i.e. clear water scour.

$$q_s(L) = K_1 E (1 - p) z_0 \frac{dz_0}{dt} \dots\dots\dots (12)$$

$K_1$  is the parameter dependent only on the shape of the non-dimensional scour hole

$$K_1 = \int_0^1 Z dX \dots\dots\dots (13)$$

Examining available data obtained under different experimental conditions, it is found that the values of  $K_1$  is about constant and vary from 0.639 to 0.597 with the average value 0.618.

Since in clear water scour no sediment is transported over the apron to the downstream end of a sluice, the phenomenon of sediment transport is essentially non-equilibrium. The imbalance between the sediment inflow rate and the sediment transport capacity causes bed scouring. A certain distance is required before the alluvial system reaches an equilibrium state. The non-equilibrium sediment transport rate  $q_s(x)$  is, in general, given by

$$n(x) \frac{dq_s(x)}{dx} = q_{se}(x) - q_s(x) \dots\dots\dots (14)$$

where  $q_{se}$  denotes the equilibrium sediment transport rate and  $n(x)$  the step length, which is assumed to be expressed in the following simple form:

$$n(x) = j d_s \dots\dots\dots (15)$$

Integration of Eq. (14) with the boundary condition  $q_s(0) = 0$  yields

$$q_s(x) = \exp\left(-\frac{x}{n}\right) \int_0^x q_{se} \frac{1}{n} \exp\left(\frac{x}{n}\right) dx \dots\dots\dots (16)$$

Eqs. (12) and (16) yield the governing equation to describe the maximum scour depth-time of the scouring process at the end of the scour holes

$$K_1 E (1 - p) z_0 \frac{dz_0}{dt} = \exp\left(-\frac{L}{n}\right) \int_0^L q_{se} \frac{1}{n} \exp\left(\frac{x}{n}\right) dx \dots\dots\dots (17)$$

#### 4. NONDIMENSIONAL SCOUR DEPTH-TIME FUNCTION

Defining the dimensionless sediment transport rate  $\theta$  as

$$\theta = \frac{q_s}{d_s v_s} \dots\dots\dots (18)$$

Herein the following form of the equilibrium sediment transport formula is adopted:

$$\theta_e = K \theta^m \left[ 1 - \frac{\theta_c}{\theta} \right] \dots\dots\dots (19)$$

where  $\theta$  denotes a Shields parameter defined by

$$\theta = \frac{\tau_b}{\rho v_s^2} \dots\dots\dots (20)$$

where  $\tau_b$  denotes the local bed shear stress,  $\theta_c$  is the critical Shields stress, and K and m are constants to be calibrated from experimental data.

To describe the combined effect of shear stress and gravity, a modified Shields parameter

$\theta^*$  proposed by Fredsoe<sup>6)</sup> may be used.

$$\theta^* = \theta - 0.1 \frac{dz}{dx} \dots\dots\dots (21)$$

Introducing the dimensionless time  $t^*$  and the dimensionless maximum scour depth  $z_{0^*}$  as

$$t^* = \frac{u_{ma} t}{dx} \dots\dots\dots (22)$$

$$z_{0^*} = \frac{z_0}{d_B} \dots\dots\dots (23)$$

where  $u_{ma}$  is the maximum velocity at the end of apron

The nondimensional maximum scour depth-time function is formulated as follows,

$$\frac{dz_{0^*}}{dt^*} = \frac{1}{K_1(1-p)j} \left( \frac{v_B}{u_{ma}} \right) \exp\left(-\frac{Ez_{0^*}}{j}\right) \int_0^1 \theta_0 \exp\left(\frac{Ez_{0^*}X}{j}\right) dX \dots\dots\dots (24)$$

## 5. HYDRAULIC CONDITIONS

The length of an apron differs from one case to another, affecting the hydraulic conditions at the end of the apron. In order to analyze the scour hole downstream of the apron, the influence of the apron on the hydraulic conditions must be taken into account.

The following empirical relationships obtained by Hassan & Narayanan<sup>3)</sup> for the maximum velocity  $u_{ma}$  and the distance  $\delta_a$  between the plane of zero velocity on the boundary to the plane where the mean velocity is  $u_{ma}/2$  in the outer layer are used:

$$\frac{u_{ma}}{u} = 3.83 \left( \frac{L_p}{D} \right)^{0.5} \dots\dots\dots (25)$$

$$\frac{\delta_a}{D} = 0.5 + 0.065 \left( \frac{L_p}{D} \right) \dots\dots\dots (26)$$

For calculation of sediment transport in the scour hole, it is necessary to evaluate the shear stress acting over the bed during the development of the scour hole. From such a point of view, the hydraulic conditions within scour holes were explored experimentally by Saito<sup>1)</sup>, and Hassan & Narayanan<sup>3)</sup>, and others. An empirical equation for maximum velocity decay  $u_{mh}$  along a scour hole may be deduced from Hassan & Narayanan's data, which were obtained from five fixed scour hole profiles

$$\frac{u_{mh}}{u_{ma}} = \frac{1}{2} \left[ 1 + \exp(-0.23(x/\delta_a)^{1.55}) \right] \dots\dots\dots (27)$$

A friction coefficient  $C_b$  is defined by

$$C_b = 2 \frac{\tau_b}{\rho u_{mh}^2} \dots\dots\dots (28)$$

## 6. MODEL SIMULATION

The proposed semi-theoretical model was applied to simulate the scour process using the experimental data of Hassan & Narayanan. In the initial stage, the scour hole shapes do not follow similarity profiles. However, the time required for the development in this stage is much shorter than that of the progressive stage, hence the model was applied from the initiation of the scouring process, i.e.  $t = 0$ .

Calculations were carried out using the modified Hamming's predictor-corrector method to solve Eq. (24). The hydraulic conditions at the end of the apron were calculated by using Eqs. (25) and (26). The bed porosity  $p$  was calculated by the empirical equation proposed by Komura<sup>7)</sup> with a submerged specific weight of sediment equal to 1.6; the coefficient for step length of  $j = 100$ ; and the friction coefficient for bed shear stress  $C_b = 0.0109$ , (Schwarz & Cosart<sup>8)</sup> proposed for a turbulent wall jet over flat bed). A comparison between Hassan & Narayanan's data and the calculated results of the proposed model are shown in Figs. 4 and 5. In Fig. 4, comparison of calculated temporal rate of maximum scour depth and experimental data is shown. In Fig. 5, comparison of generated temporal rate of the scour hole profile is shown.

## 7. CONCLUSIONS

Local scour downstream of a sluice was investigated on the basis of the transformed sediment continuity equation with the aid of the similarity of scour hole profile. A semi-

theoretical model was proposed to predict the evolution of the scour process due to a deeply submerged hydraulic jump. The model was used to generate the scour process of the experimental data of Hassan & Narayanan<sup>3)</sup>. Eq. (24) along with the recommended values of  $K=3.0$ ,  $m=2.5$  and  $C_b=0.0109$  can be used to predict reasonably well the complicated phenomenon of the scour process. The proposed method can be used to predict the scouring problem in the range of:  $4.0 < u_{ms}/v_s < 14.0$ ,  $2.7 < D/d_s < 28.6$ , and  $8.0 < L_p/D < 95.8$

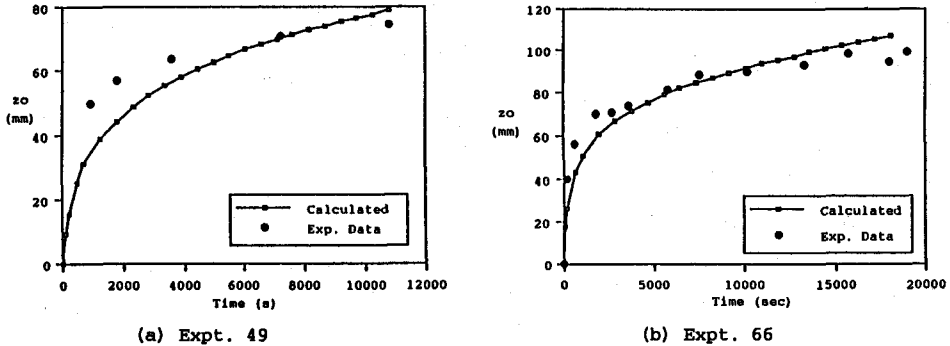


Fig.4 Comparison Between Calculated Results and Hassan & Narayanan's Data

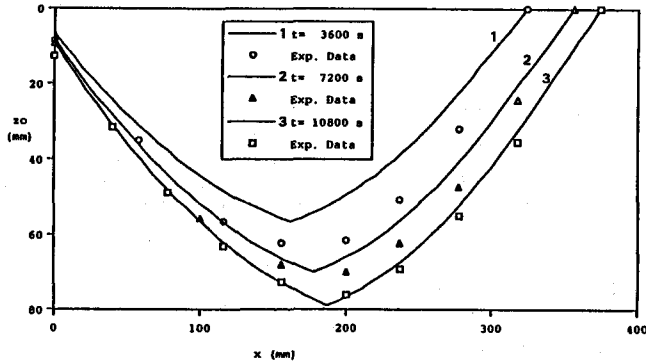


Fig.5 Generation of the Scour Process of Expt. 49

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