A PHYSICALLY-BASED CONCEPTUAL MODEL FOR CONTINUOUS MODELLING OF HORTONIAN OVERLAND FLOW AND INFILTRATION UNDER VARIABLE RAINFALL

Katumi Musiake,Oka Yasumiti and Jin Lee
Institute of Industrial Science, University of Tokyo

A physically-based conceptual model for continuous modelling of Hortonian overland flow and infiltration under variable rainfall has been proposed and tested at a point, with the values generated by a complex finite-element infiltration model. The model has been shown to predict both the volumes and positions in time of overland runoffs, from a real 146 mm/day rainfall pattern quite well, for three simulation time intervals and three values of top-soil saturated hydraulic conductivity. An interesting feature of the model is its ability to give realistic determination of rainfall excess, compared to the methods currently available.
Keywords : conceptual model, Horton flow, continuous modelling, rainfall excess

1. INTRODUCTION

There are a variety of conceptual catchment models for rainfall-runoff modelling today. Most of the models proposed consist of a series of water storages or tanks together with some related parameters used to control the distribution of rainfall inputs through the tanks. The parameters used to control the flow of water through the tanks usually have no direct one-to-one correspondence with the reality they are supposed to model. This can usually be seen in the use of a coefficient to determine the surface runoffs from rainfall inputs, in the currently available lumped urban streamflow models. In order for conceptual rainfall-runoff modelling to progress, there is a need to develop conceptual models which take into account each of the hydrological processes acting in a catchment. The models' formulations should have a direct one-to-one correspondence with reality.

This paper presents a physically-based conceptual model for modelling Hortonian overland flow and infiltration under variable rainfall on a continuous basis. The model's formulations have a direct one-to-one correspondence with the reality it is modelling. The model is particularly applicable for urban catchments where Hortonian overland flow predominates. Since the model is a conceptual one, its performance was evaluated with the values generated from a complex finite-element simulation at a point at process-level time scale.

2. THE CONCEPTUAL MODEL FOR THE HORTON MECHANISM FOR OVERLAND FLOW

Since the Horton mechanism is a process-level phenomenon, we cannot expect to model it physically in a conceptual model. This is because of the typically larger time intervals used in conceptual models. Instead, we should aim at a conceptualization of the effects of the mechanism, in generating overland flow over the time interval used in conceptual models. The conceptualization should retain the main characteristics of the phenomenon, to ensure one-to-one correspondence with reality. Since the main characteristics generating the phenomenon are the saturated hydraulic conductivity of the soil surface, the rainfall rate and the duration that the rainfall takes place at that rate, we should aim at a conceptualization that incorporates these elements.

Since the time intervals used in conceptual models, are usually an hour or more, we should aggregate all effects due to process-level phenomena occurring at a time resolution higher than the models' time intervals, into that of the time intervals. For the discussion that follows, we shall adopt the hourly time interval as representative of the models' time intervals. Hence, aggregations should be applied to the following event-based information occurring within an hour.

(a) Since one of the necessary conditions for the occurrence of Horton overland flows is that the rainfall rates have to be greater than the saturated hydraulic conductivity (KS) of a soil surface, we need to get the durations of occurrence of rainfall rates, higher than the value of KS, for each rainfall event within an hour.

(b) Also, we need to get the amount of rainfall for each rainfall event within an hour, that can effectively contribute to the Horton overland flow. This amount is controlled by the KS value of a soil surface, and will be less for a higher value of KS and more for a lower value.

The above two event-based derived values, together with the surface wetness state of the soil, at the time of the occurrence of the event, controls the amount of the total event rainfall that can possibly become Horton overland flow. For our hourly time scale, we need to aggregate all these event-based values into an hourly-based value. Similarly, we have to aggregate for an equivalent hourly surface-wetness state for the soil, to replace the various event-based surface-wetness states of the soil. By doing these aggregations, we can transform the event-based Horton overland flow phenomenon, into an hourly-based one, having characteristics and effects similar to that of the individual events.

2.1 Aggregation for the effective rainfall duration and amount

In order to get the above described event-based rainfall durations and amounts, we need high resolution rainfall data. Then, based on the predominant value of KS for the soil that we intend to model, we decide on the 'cut-off' rainfall rate to be used for the extraction of the effective rainfall durations and amounts. The 'cut-off' rainfall rate is equal in value to the soil's KS value.

Figure 1 shows how the effective rainfall durations and amounts, including the amounts for surface wetting, are determined for a 'cut-off' KS of 12 mm/hr, for each time interval of the model. For time intervals A and C, the effective rainfall durations are T2 and T3 respectively, and the rain falling within T2 and T3 are their respective rainfall amounts. Within the time interval A, the rain falling within the time T1 will be used for increasing the wetness state of the soil surface at the beginning of the time interval A, whereas time interval C has no surface wetting amount. Since the intensities of the rainfall events within the time interval B are smaller than 12 mm/hr, they are aggregated within the time interval, and used only for wetting the soil surface between the time intervals A and C.

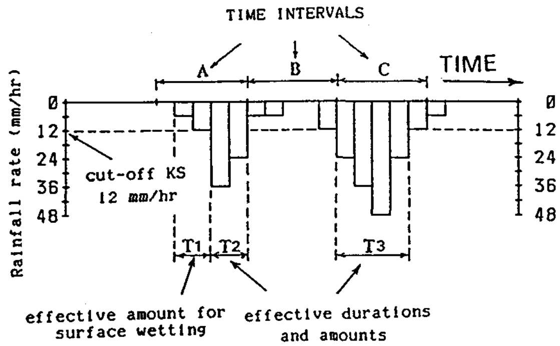

Fig.1 Aggregation for the effective rainfall durations and amounts

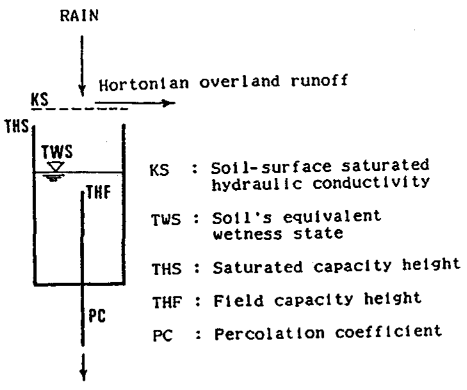

Fig.2 Conceptual tank for representing soil-surface wetness

2.2 Conceptualization for soil-surface's wetness state

To model on a continuous basis, we need a formulation in the model to keep track of the wetness state of the soil surface, over the simulation time intervals. For our model, we shall use the variation of the water level in a tank to act as the conceptual equivalent of the surface-wetness state of the soil. Figure 2 shows the structure of the conceptual tank. The saturated capacity (THS) and field capacity (THF) heights of the tank, are dependent on the moisture-retention properties of the soil-surface layer, and on the thickness of the top-soil layer to be represented by the tank. The moisture-retention properties of the soil can be derived from its moisture-suction curve. The percolation coefficient is a value that depends on the model's simulation time interval.

For an urbanized catchment, the properties in the top 50 cm of the soil are usually quite different from those below it, because of compaction. This has been investigated by Musiake and Oka (1988). Two figures (3 and 4) from their paper shall be discussed here. First, the effects of compaction on the moisture-retention properties of the top soil of an urban catchment was shown by them using a 3-phase distribution diagram.

Second, the depth of influence of evaporation in the soil was described in a figure showing the daily changes in the observed hydraulic potential profiles over six rainless days. From the two figures, we have the physical basis for determining the thickness of the top-soil layer to be represented by the conceptual tank. It can be taken as 25 centimeters, the approximate depth of influence of evaporation, represented by line E-E of Figure 3 of the paper.

In order to derive the moisture-retention parameters of THS and THF, we need to define the concept of saturated (Θ_s), field capacity (Θ_f) and equilibrium (Θ_e) soil moisture content. The saturated moisture content is the maximum percentage of water that a soil can hold. The field capacity is the stable value of the soil moisture content after a rainfall event, and is assumed to be associated with a pF value of 2. The pF index is a measure of soil moisture suction, and is defined as the logarithm of the suction pressure measured in centimeters of water. The equilibrium soil moisture content is the value of soil moisture where the hydraulic conductivity is effectively zero, and is assumed to be associated with a pF value of 3. Figure 3 shows a soil moisture suction curve for a top soil in an urban catchment. With the values of Θ_s, Θ_f and Θ_e determined from the soil moisture-suction curves of the top-soil layer (Figure 3), we can get the physically-based conceptual values of THS and THF by multiplying the chosen soil thickness (25 cm) with (Θ_s-Θ_e) and (Θ_f-Θ_e) respectively. The values are 27.5 cm and 21.5 cm for THS and THF respectively.

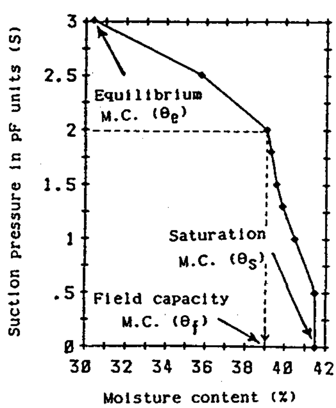

Fig.3 Soil moisture-suction curve for a top soil in an urban catchment

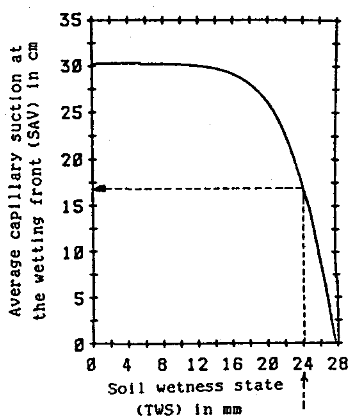

Fig.4 Average capillary suction at the wetting front (SAV) versus TWS

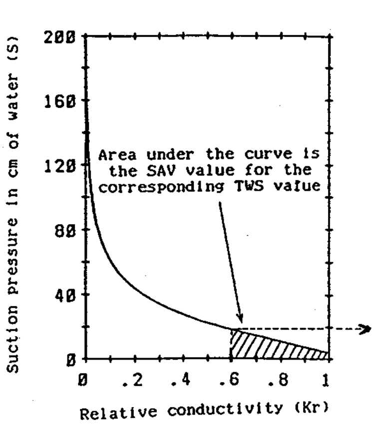

Fig.6 Soil suction-relative conductivity curve for determining SAV values

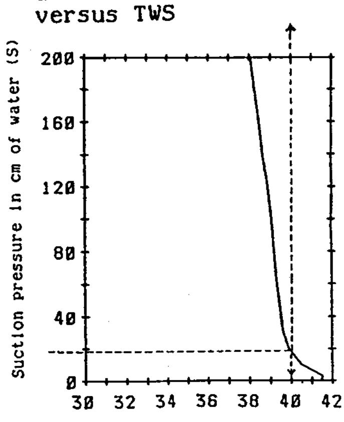

Fig.5 Part of figure 3 with suction pressure in centimeters of water

2.3 Conceptualization for time to ponding

At the process-level, the time taken from the start of rainfall to the time overland flow occurs, is known as the time to ponding. This time is dependent on two factors. They are the wetness state of the surface soil, and the average rainfall rate for the rainfall event. It is shorter for a wetter surface because of the lesser amount of water required to bring the surface to the saturation state. Also, it is shorter for a higher average-event rainfall rate because of the shorter time it takes to meet the surface saturation moisture-deficit. Horton flow will not occur if the duration for a particular rainfall rate is less than the time to ponding. For our hourly model, we have to conceptualize for an equivalent structure to represent the process-level time to ponding.

Mein and Larson (1973) gave a formulation that can predict the volume of infiltration into a soil prior to surface runoff, for steady rainfall. The formulation is as follows :-

$$FS=SAV*MD/(I/KS-1) \qquad \text{for } I>KS \;\ldots\ldots\; (1)$$

where MD=moisture deficit, I=rainfall intensity, SAV=average capillary suction at the wetting front, KS=saturated hydraulic conductivity and FS=cumulative infiltration up to surface saturation.

In their formulation, they proposed that SAV be determined from the suction (S) versus conductivity (K) relationship for the soil using the following equation :-

$$SAV = \int_{0.01}^{1} SdKr,$$ where Kr is the relative conductivity (K/KS).

The value of SAV is then simply the area under the S-Kr curve between Kr=0.01 and Kr=1. Since they were dealing with steady rainfall, they have taken SAV to be a constant.

The above formulation by Mein and Larson can be modified for use in the conceptual model, for the prediction of the amount of infiltration for surface saturation and the time to ponding. The modification involves making SAV and MD dependent variables of the soil wetness state (TWS). This is necessary since we are modelling on a continuous basis, which requires the soil-wetness state to change continuously. The modified formula for use in the conceptual model is as follows :-

$$FS=SAV*SMDF/(RATE/KS-1) \qquad \text{for } RATE>KS \;\ldots\ldots\; (2)$$

where SAV=capillary suction at the wetting front determined from Figure 4, $SMDF=(TWS/THS)*(\Theta_s-\Theta_e)$, RATE=RA/DR, the equivalent rainfall intensity for the model time interval, RA=effective rainfall amount for the time interval, DR=effective rainfall duration for the time interval and, FS and KS as defined previously. The time to ponding (TP) is then equal to (FS/RATE).

Figure 5 shows part of the soil moisture-suction curve of Figure 3, with the scale for the suction pressure transformed from pF units to cm of water. Figure 6 shows the relative conductivity versus suction curve for the given soil, used for determining the SAV values. The relationship between SAV, the area under the S-Kr curve and TWS, the wetness state of the soil can then be clearly seen from Figures 4,5 and 6.

2.4 Model's computation

The model's computation takes place in two stages for each time interval. The first stage involves the computation for the Horton overland flow. The second stage updates the wetness state of the soil. The two stages of the computation are illustrated in Figures 7 and 8 respectively. For the computation in the first stage, the soil wetness state (TWS) is first wetted by the surface wetting amount in the time interval, if any. If the value of TWS is greater than THS after surface wetting, then the time to ponding (TP) is taken to be zero. Otherwise, the amount of infiltration up to surface saturation (FS) is determined using equation 2 above, and the value of TP computed. The value of TP is then compared with the effective rainfall duration (DR) and the value of the Horton overland flow (QHO) computed accordingly as shown in Figure 7. For the computation in the second stage, the amount of infiltration into the soil (QINF) is computed. This amount is used to update TWS for the next simulation time interval. The computation for this stage is shown in Figure 8.

3. EVALUATION OF MODEL PERFORMANCE

In order to check the performance of the model, an hourly simulation was carried out at a point with a 24-hour real rainfall pattern as shown in Figure 9. The rainfall data used have a resolution of 2.5 minutes. The volumes of the overland flows generated

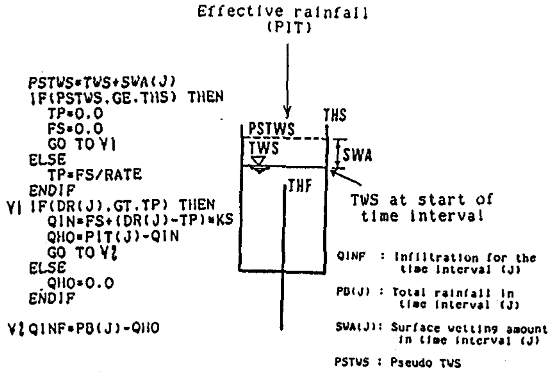

Fig.7 Model computation - Stage I
(Horton overland flow)

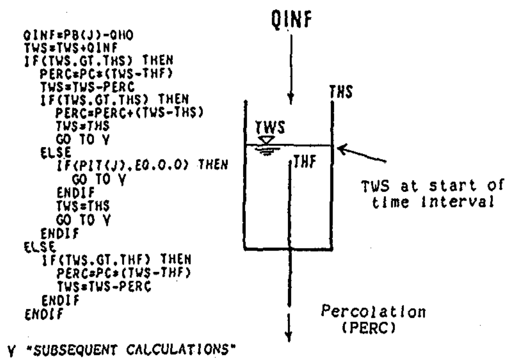

Fig.8 Model computation - Stage II
(Updating soil wetness state

Table 1 - 24hrs total surface runoffs
for the two models
(hourly intervals)

KS (mm/hr)	INITIAL CONDITION	CONCEPTUAL MODEL	FEM MODEL	PERCENTAGE DIFFERENCE
3	WET	99.42	94.58	- 4.9
	DRY	96.48	94.47	-2.1
6	WET	75.55	82.39	+9.1
	DRY	73.78	82.29	+11.53
12	WET	46.73	51.87	+11.0
	DRY	45.0	51.70	+14.9

Table 2 - 24hrs total surface runoffs
for different simulation
time intervals

KS (mm/hr)	FEM VALUES	30 MINUTES		ONE HOUR		TWO HOURS	
		VALUES	% DIFF	VALUES	% DIFF	VALUES	% DIFF
3	96.48	94.31	-2.20	94.47	-2.10	103.69	+7.50
6	73.78	80.32	+8.90	82.29	+11.53	83.82	+13.60
12	45.00	50.53	+12.30	51.70	+14.90	52.57	+16.80

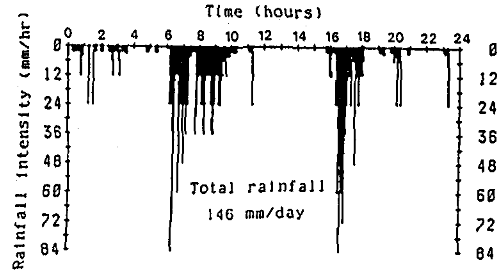

Fig.9 Real rainfall pattern used
for evaluation of model's
performance

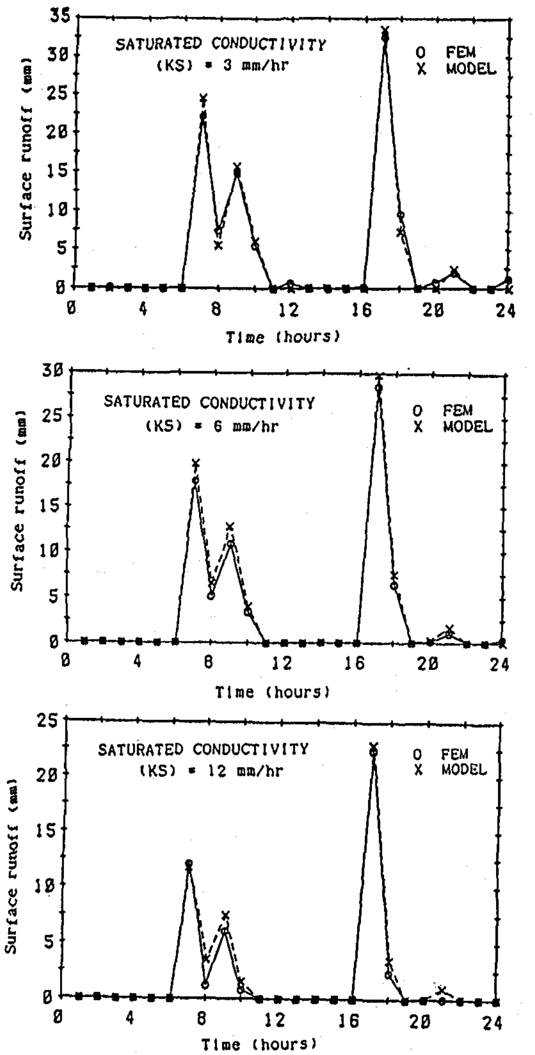

Fig.10 Comparison of surface runoffs
generated by the two models

by the model were then compared with the volumes generated from a process-level simulation using a finite-element infiltration model (FEM). The model is based on the Richards' equation for flow in the unsaturated zone, and details of the equation together with the finite-element modelling can be found in Musiake and Oka (1988).

Table 1 compares the 24 hours total surface runoff generated by both the FEM and the conceptual model for the two extreme initial conditions and three KS values - 3,6 and 12 mm/hr. It can be seen from the table that the % error in the conceptual model's prediction varies from 2 to 15 %. Also, the % error increases with increase in the KS value. Since the model operates on a continuous basis, the effects of the initial conditions on the total simulated flows are minimal, as can be seen in the table. Hence, for the discussion that follows, we shall use only the results for the dry initial conditions.

Figure 10 compares the hourly values of the surface runoff generated from the FEM with the conceptual model, for the three different KS values. It can be seen that, overall the model predicts the continuous surface runoffs quite well.

In order to see how the model's predictions is affected by a change in the simulation time intervals, simulations were carried out at 30-minutes and two-hourly time intervals, and their results compared with the FEM values. Table 2 gives the comparison of the 24 hours total surface runoffs generated by the model at the three different time intervals, with the FEM values. It can be seen that generally, the model's prediction improves with a smaller simulation time interval and KS value. It was also found that, overall the model predicts the continuous surface runoffs reasonably well at all three time intervals. Also, it was seen that the shape of the surface runoff hydrographs at the two-hourly time interval is quite different from those at the other two time intervals. In addition, it was noticed that the prediction at the time of 8 hours was not so good for all three simulation time intervals. The prediction becomes worse at larger values of KS. The reason for this and also for the earlier observation of deterioration in model's performance at higher KS value, is most probably due to the quicker draining of the soil at the process-level, for high KS values. This characteristic is difficult to model using a linear tank concept.

The shape of the two-hourly time interval surface runoff hydrographs shows that the two-hourly time interval is too large to capture the main characteristics of the rainfall pattern. However, the shapes of both the 30-minutes and hourly time intervals hydrographs show that the two time intervals can capture the rainfall pattern quite well.

4. CONCLUSIONS

It has been shown above that a physically-based conceptual model for the process-level Horton mechanism of surface runoffs, can be constructed from aggregations of its effects at the process-level. The model has been shown to perform well for three simulation time intervals and saturated hydraulic conductivities.

The model has several noteworthy features. First, its structure has been conceptualized through the aggregation of the effects of processes occurring at real-time. This enables the model structure to maintain one-to-one correspondence with the reality it is modelling, and thus, facilitates understanding of the model's behaviour at different time scales. Second, the model is able to predict surface runoffs from any real rainfall pattern on a continuous basis. This feature will enable a more realistic determination of rainfall excess, than the currently available methods, such as the Φ index and Horton methods. Finally, the conceptual nature of the model will facilitate its incorporation into any conceptual urban catchment model, for better prediction of the surface runoff components. However, the nature of the model necessarily restricts its application to small urban catchments where the model's assumptions of uniformity in rainfall intensity pattern and soil surface properties can be maintained.

REFERENCES

Mein, R.G. and C.L. Larson (1973), "Modelling Infiltration during a Steady Rain", Water Resources Research, Vol.9, No.2, pp. 384-394

Musiake,K. and Y. Oka (1988), "Field Observations and Numerical Simulations of Moisture Behaviour in the Unsaturated Zone - The Case of Evaporation in an Urbanized Field Plot", Proceedings of the 6th Congress - Asian and Pacific Regional Division, International Association for Hydraulic Research, Kyoto, Japan, Vol.1, pp.79-86