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RUNOFF SIMULATION OF EXPERIMENTAL WATERSHED BASED ON THE CONCEPT OF VARIABLE SOURCE AREA

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SYNOPSIS

This paper studies primarily runoff simulation based on the concept of variable source area and the optimization technique for a mountain watershed located in Taiwan. The different hillslope lengths derived by virtue of kinematic wave theory, and the occurrence conditions related to the expanding and shrinking process between the overland and subsurface flow, are also introduced into the modeling procedure of runoff simulation. The proposed analytical method is finally applied to the study watershed in order to check its reliabilities.

Keywords : runoff simulation, variable source area, experimental watershed, kinematic wave theory, overland flow

1. INTRODUCTION

The main concept of the variable source area, as applied to forested land is that precipitation down to the surface infiltrates firstly the undisturbed soil, then migrates downslope, and maintains saturation or near saturation at lower slope positions. The lower hillslope positions contribute rapidly the subsurface flow to the stormflow, as the zone of saturated surface layer expands during a storm. The degree to which the lateral expansion of saturation is predominated by the soil-moisture, condition, thickness of surface layer, type of hyetograph,etc. A significant distinction between the variable source area (v.s.a.) concept and that of Hortonian overland flow is confirmed recently. It is the latter stands for a phenomenon that lies at one extreme, while the total'throughflow'domination lies at the other extreme.Throughflow depicts the flow mechanics of movement more than that of process, however, the v.s.a.concept incorporates the overall range of hillslope runoff process which is considered as a dynamic system, variable in time and space, and varying between the extremes mentioned above.

In this paper, the authors study on the runoff simulation for an mountain watershed where the traveling time of river channel is negligibly small compared with that of hillslope, based mainly on the dynamic process concept of v.s.a.. The kinematic wave equations of continuity and momentum conforming to the Manning and Darcy's rules are used for determining the occurence lengths of overland runoff and storm runoff. As a result, a proposed model with four parameters (surface roughness ; pemeability coeff. of surface layer ; standard deviation and mean value of surface layer thickness) is derived. These parameters are identified by the simplex optimization method.

2. MODELING OF RUNOFF PROCESS FOR A MOUNTAIN WATERSHED

When one applies the continuity equation of unsteady flow into a basin slope, the equation and sketch may be expressed in Eq.(1) and Fig.1, respectively.

$$\frac{\partial(\lambda h)}{\partial t}+\frac{\partial q}{\partial x}=r_e \cos\theta \qquad (1)$$

where r_e =effective rainfall intensity ; h= depth of flow
λ =porosity (=1.0, for overland flow)
q =discharge per unit width in a hillslope
x =downstream distance measured from the starting estimated point along a basin slope
t =time ; θ =angle of a slope

Since the term of energy gradient is generally considered sufficiently greater than other terms in momentum equation, its equation can be simplified as

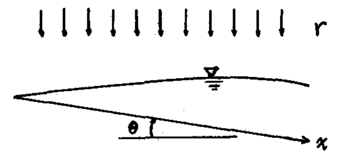

Fig.1 Schematic sketch of flow in basin slope

$$h = K q^{p} \qquad (2)$$

wherer K, p =constant. If Manning's formula is adopted, $K=(N/\sqrt{\sin\theta})^{p}$ and p =0.6. However, if one uses Darcy's rule, $K=\lambda/k\sin\theta$ and p=1.0, in which N =Manning's roughness coeff. in surface,
k =pemeability coeff. in surface layer.

Moreover, the total discharge at the outlet of watershed where the effect of the concentration time for a river channel can be neglected related to that of hillslope, may be expressed as

$$Q(t) = \sum q(\ell,t)\,\Delta b \qquad (3)$$

where $Q(t)$ = total discharge at the outlet for a given time t
Δb = width along a river channel with a slope length ℓ
$q(\ell,t)$ = runoff per unit width at the end of a hillslope, having a slope length (ℓ) at a given time t.

The value of ratio $\ell\Delta b/A$ may be considered as

$$\frac{\ell\Delta b}{A} = f(\ell)\,d\ell \qquad (4)$$

where A = basin area
ℓ = the estimated slope length at any given time, which can be evaluated by using the characteristic equation corresponding to Eqs.(1),(2) through the kinematic wave theory.
$f(\ell)$= probability density function of a slope length(ℓ).

Accordingly, the value of $f(\ell)d\ell$ is equal to the probability of the corresponding slope length.

Eliminating Δb from Eqs.(3) and (4), one gets

$$Q(t) = A\int_{0}^{\infty} q(\ell,t)\,\frac{f(\ell)\,d\ell}{\ell} \qquad (5)$$

We can evaluate numerically the values of $q(\ell,t)$ and ℓ expressed above for a given time by using the characteristic curve illustrated in Fig.2, in which total slope length ℓ is equal to the sum of ℓ_f and ℓ_s , which denote the occurrence length of subsurface and overland flow, respectively.

When the rainfall falls down to hillslope,the infiltration must occur, the flow depth in surface layer increases related to increasing rainfall.As soon as the depth of interflow is greater than thickness of surface layer, part of the interflow becomes overland flow. The area occurs on the surface flow (i.e. v.s.a.) varys with time, but the occurrence conditions of overland flow must satisfy the following two conditions.

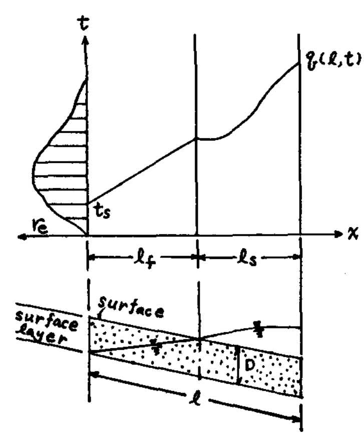

Fig.2 Graphic illustration of characteristic curve for the evaluation of slope length (ℓ)

(1) $$\int_{t_s}^{t} r_e(\tau)\cos\theta\,d\tau > \lambda D = \eta$$

(2) $\ell > k\,(t - t_o)\sin\theta$

where D = thickness of surface layer
η = effective depth of flow in surface layer
t_o = starting time of any characteristic curve
$t - t_o$ = concentration time for a given rainfall

From some past experiences obtained, we see that the slope lengths of mountain watershed are near close to the log-normal distribution with parameters of mean value and standard deviation which can be obtained from the basin map.

Based on the above assumption, and according to the probability principle, one gets

$$f(\ell)d\ell = f(x)dx \qquad (6)$$

where $X = \ell og\,\ell$; f(X)= density function of normal distribution with variable X. Eq.(6) can be rearranged as

$$\frac{f(\ell)\,d\ell}{\ell} = e^{-cx}\,\frac{1}{\sqrt{2\pi}\,\sigma_x}\,\exp\left[-\frac{1}{2}\left(\frac{x-\overline{x}}{\sigma_x}\right)^2\right]dx \qquad (7)$$

where c=2.3026, $\overline{X}$= mean value of X ; σ_x = standard deviation of X

Substituting Eq.(7) into Eq.(5), the total discharge at the outlet may be expressed finally as

$$Q(t) = A\int_0^{\infty} q(\ell,t)\,e^{-cx}\,\frac{1}{\sqrt{2\pi}\,\sigma_x}\,\exp\left[-\frac{1}{2}\left(\frac{x-\overline{x}}{\sigma_x}\right)^2\right]dx \qquad (8)$$

The integration term on the right side in Eq.(8), can be evaluated by means of numerical technique, in which the slope length determined by kinematic wave theory, is function of four parameters. i.e., surface roughness K; pemeability coeff. of surface layer k; mean value and standard deviation of surface layer: $\overline{\eta}$; σ_η , based on the hypothesis conforming to normal distribution. The optimal values of the above parameters can be identified by simplex method. Meanwhile,the following equation is also equated based on the probability relationship between the hillslope lengths and variable source area. namely,

$$\int_{\ell_o}^{\infty} g(\ell)\,d\ell = \frac{A_o(\eta)}{A} \qquad (9)$$

where $g(\ell)$=probability density function of ℓ_o
$A_o(\eta)$ =v.s.a filled with overland flow with respect to a assumed η value

From which, the total value of variable source area at any given time can be estimated from

$$A_o(t) = \sum_{0}^{\infty} A_o(\eta)\,f(\eta)\,\Delta\eta \qquad (10)$$

where $f(\eta)$ = density function of η , which is assumed to comply with the normal distribution, here.

Substituting Eq.(9)into Eq.(10), we get

$$A_o(t) = A\sum_{0}^{\infty}\sum_{\ell_o}^{\infty} f(\eta)\cdot g(\ell)\cdot\Delta\ell\cdot\Delta\eta \qquad (11)$$

Therefore, the variation of variable source area with respect to time may also be evaluated from Eq.(11).

3. A CASE STUDY TO AN EXPERIMENTAL WATERSHED

The analytical method as described above is applied to an experimental watershed, named Pi-Lu No.11 watershed, which was constructed in the upper middle part of Taiwan by Taiwan Forestry Research Institute. Pi-Lu experimental watershed includes two watersheds, i.e. Pi-Lu No.11 and Pi-Lu No.12, respectively. The records of daily rainfall and daily runoff of both watersheds were published by Taiwan Forestry Research Institute. However, about half year ago, a few of hourly rainfall and runoff data in Pi-Lu No.11 were supplied only to public for analysis; accordingly in the study, Pi-Lu No.11 was adopted for simulation. Pi-Lu No.11 occupies the basin area of 1.44 km^2 , and the outline of the watershed is shown in Fig.3, 4. The geologic feature belongs to tertiary slate and densely covered with broad-leaves trees. The watershed is equipped with one raingauge near central part and one flow gauge at the outlet.In addition, the slope lengths of the watershed are confirmed to be near close to the log-normal distribution with mean and standard deviation of 155m and 2.45m, respectively, as shown in Fig.5.

Here, the slope lengths were measured running orthogonally to the contours from the dividing ridge to the nearest stream.

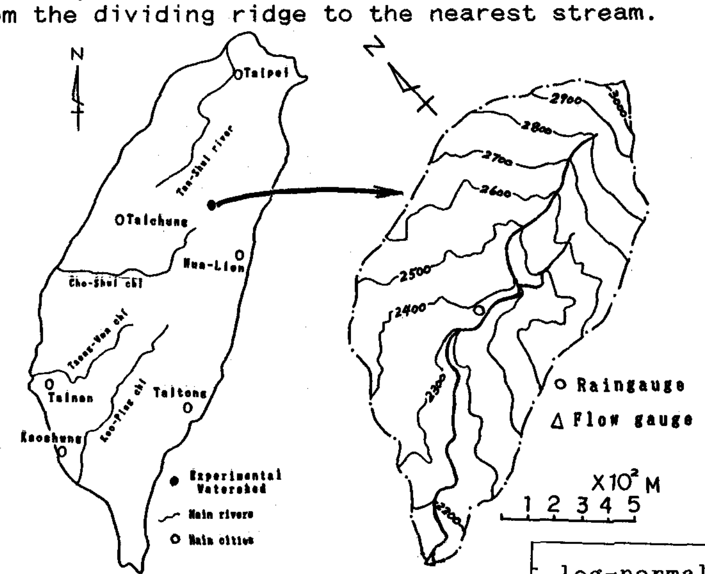

Fig.3 Topographic map and location of Pi-Lu No.11 watershed

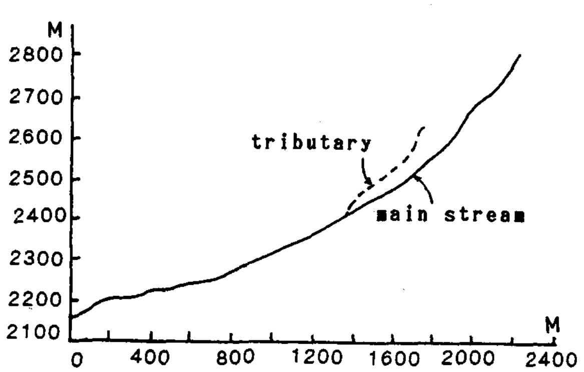

Fig.4 Profile of main stream in Pi-Lu No.11 watershed

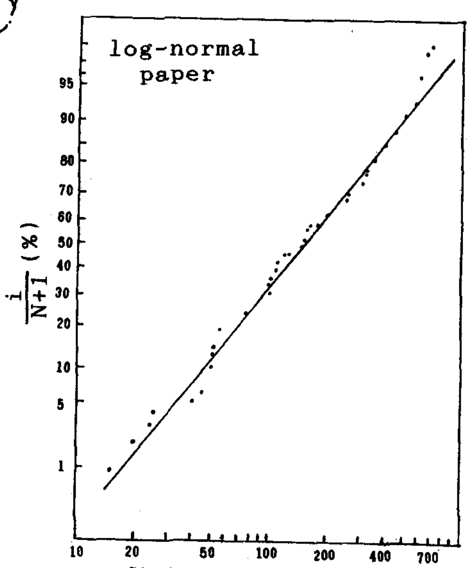

Fig.5 Distribution of slope length with (ℓ)

The first step of simulation process is to find the optimal parameters. Many optimal methods have been proposed. After some investigations ,the simplex optimal method was used finally, because it is more effective for the sake of the optimization.
A general form of non-linear equation with four variables is

$$f(\bar{\eta}, \sigma_\eta, K, k) \rightarrow \mathrm{Min}\sum(Q_i - Q_e)^2 \qquad (12)$$

Q_i : observed , Q_e : simulated

and the associated constraints are

$$\bar{\eta} \geq 0 \; ; \; \sigma_\eta \geq 0 \; ; K \geq 0 \; ; \; k \geq 0 \qquad (13)$$

Table.1 Physiographic factors of Pi-Lu No.11 Watershed (based on 1/5000 scale basin map)

items	quantity
Watershed Area (ha.)	144.0
Circumference of watershed (m)	5335
Main stream length (m)	2450
Mean watershed width (m)	587.8
Form factor	0.24
Compactness	0.80
Average hill slope (%)	70
Maximum elevation (m)	3060
Minimum elevation (m)	2105
Average altitude (m)	2530
Aspect	W

According to the simulation results, the simulated hydrographs show seemingly good agreement with the observed ones as shown in Fig.6,7.
The variations of dynamic areas occupied by the overland flow with respect to time are also plotted in the figures, which show that a proportional relation with the computed runoff values. The comparisons of the identified parameters with the total rainfall are also illustrated in Fig.8. The values of $\bar{\eta}$ and σ_η seem to be in the reasonable ranges.

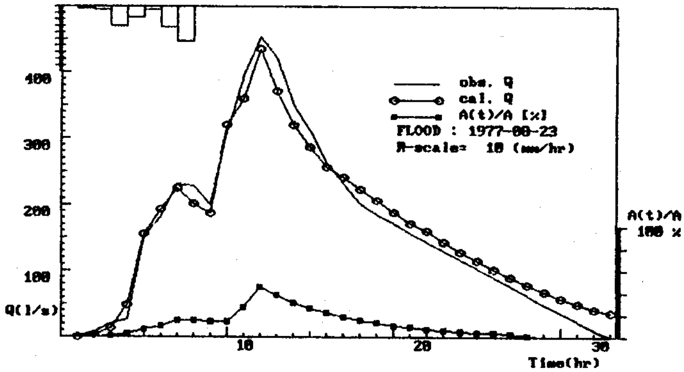

Fig.6 Computed and observed hydrograph ($\bar{\eta}$=0.34, σ_η=0.241,k=0.722,K=0.950)

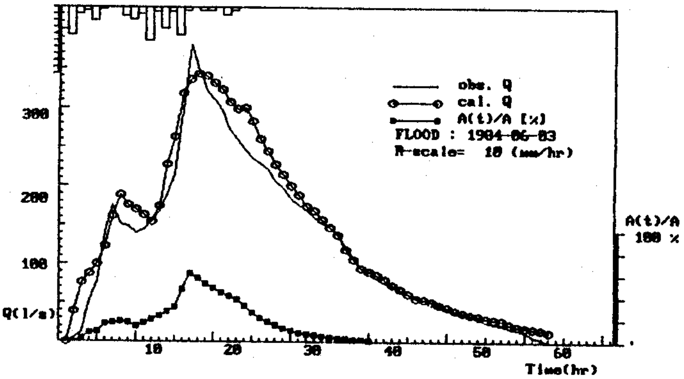

Fig.7 Computed and observed hydrograph ($\bar{\eta}$=0.63, σ_η=0.353,k=0.964,K=3.040)

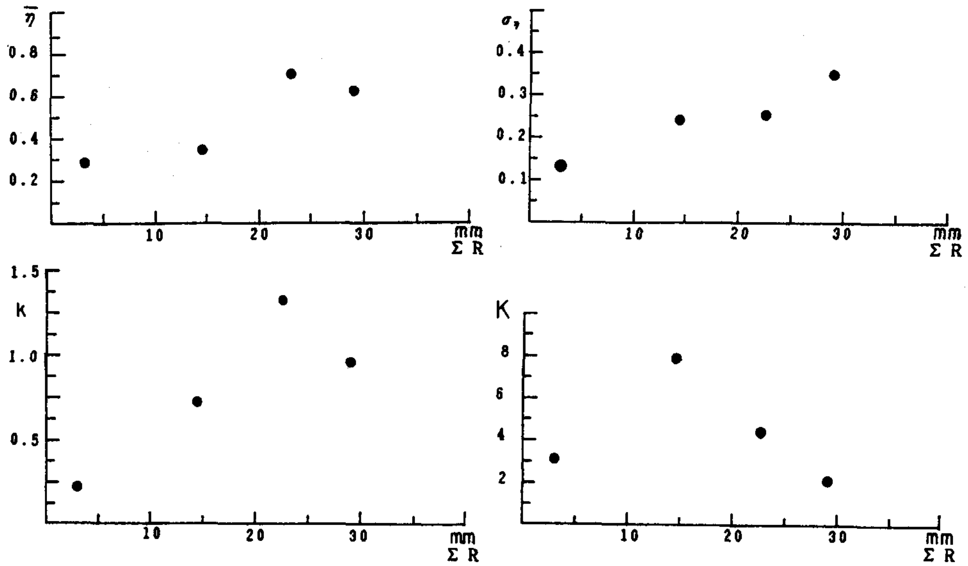

Fig.8 Comparison between the total rainfall and identified parameters

4. CONCLUSION

A runoff simulation method for a small mountain watreshed, based on the concept of variable source area, was proposed and cheched its applicabilities with recorded data of an experimental watershed. From the results, the proposed method is effective for the mountain watershed where the concentration time is negligibly small compared with that of the river. The hourly records which we can utilize were only four cases of flood. The parameters of surface layer seem to be in the reasonable ranges, but the variations of the total rainfall with respect to the parameters of K and k is not clear. We will continue this study later to find more definite relations of the watershed parameters, as the consequent data are provided.

Furthermore, if the infiltration rate which seeps from surface layer into bedrock is consirered as a variable, then the identified parameters becomes five. This case will also be studied and reported in another time.

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