

BEHAVIOR OF UNSATURATED FLOW IN LAYERED SOILS AND ITS APPLICATION TO DRAINAGE PROBLEMS

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I - INTRODUCTION

In the last two decades there has been a notable interest in investigating the flow of water into partially saturated soils, generally named the unsaturated zone. A considerable amount of work has been carried out on this regard, however, most of the papers reported so far were accomplished for homogeneous soils solely, (e.g., Klute¹⁾, Youngs²⁾, Vauclin et al³⁾). Theoretical-based studies on layered soils have been reported satisfactorily by Hanks and Bowers⁴⁾(one-dimensional), Freeze⁵⁾(two-dimensional), Morales et al⁶⁾(two-dimensional), all by FDM, and Frind et al⁷⁾ using FEM(three-dimensional). Experimental research in the field has been performed by Rogowski et al⁸⁾ examining the transient response of a layered sloping soil to natural rainfall.

This paper presents the results obtained in hydraulic models of infiltration into layered soils with a set of drains. Moreover, a theoretical model is developed to numerically simulate the experimental results. In the first part of the study, the results in a two-dimensional mathematical-hydraulic case are presented while in the second part a three-dimensional laboratory test is described.

II EXPERIMENTAL MODEL

II-1 Real Soil Structure and its Conception in Laboratory

Porous media making up aquifers and the soil structure as a whole are seldom homogeneous. Hence, it can be stated with well accuracy that non-homogeneity is a property inherent to most natural environments. Schematically a real soil structure is shown in Figure 1.

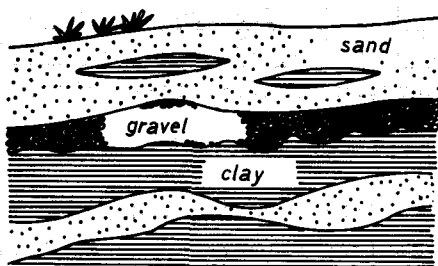
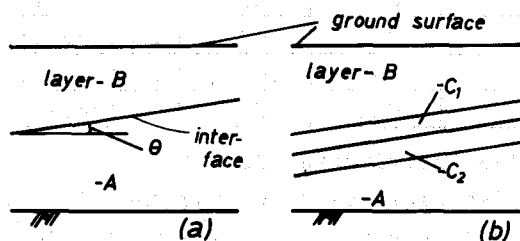


Fig.1, Typical Soil Structure



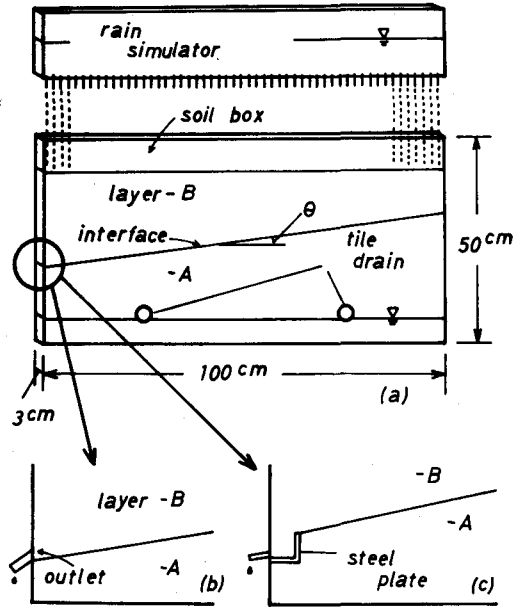
two-layer model; multilayer model
Fig.2, Modelization of Layers (Lab.)

In this work, a layered system was adopted to treat the problem of the typical nonhomogeneous medium. To control satisfactorily the parameters involved, the present authors used two experimental models: a) two-layer and, b) multilayer-model, as indicated schematically in Fig.2. The model a) will be discussed particularly, for the sake of simplicity in the numerical analysis.

II-2 Experimental Apparatus and Study Cases

The experimental apparatus used in this study is schematically shown in Fig.3. It mainly consists of two sections: the rainfall simulator and the soil-box. The latter allows for the arrangement of the soil strata bounded by the common interface. In the lower soil, a pair of drains are fixed and, besides that, one outlet is provided on the left side wall in order to measure the lateral flow in the distance of the interface.

Three experimental conditions reported here are summarized in Table 1. Coarse sand (CS) and coarse glass beads were used for layer B and A, respectively. The properties of the filling materials are presented in Table 2 below. As indicated in Table 1, the soil layers for the Case 4 were arranged in reverse position with respect to the other cases, in order to verify the significance of the role of capillary pressure at the interface.



b) type-1 outlet; c) type-2 outlet

Fig.3, Experimental Apparatus

Table 1, Experimental Cond.

Case	Layer-B	Layer-A	θ	outlet
1	CS	CGB	3	type-1
2	CS	CGB	2	type-2
3	CS	CGB	1	type-2
4	CGB	CS	1	type-2

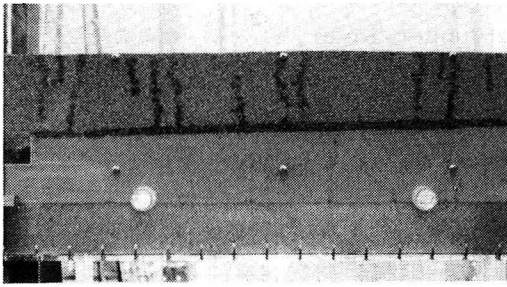
Table 2, Soil Properties (Layers)

Soil	k	Cap. Rise	Gr.S.	Por.
CS	0.48	2.3-3.3cm	1.7mm	0.34
CGB	1.09	0.3-0.4	2.5	0.34

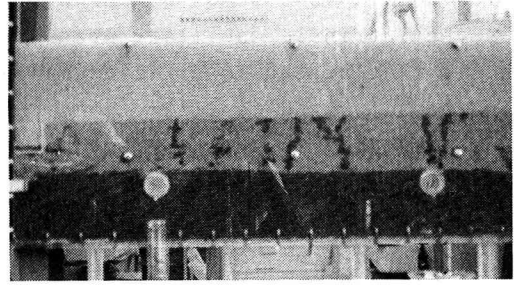
k: perm, cm/s; Gr.S.: grain size, mm.
Por.: porosity

II-3 Experimental Results

Two pictures revealing the flow response within the layered system are shown in Fig.4. They illustrate the general flow pattern in cases 3 and 4, from which the discrepancy between two cases is clearly visualized. In Fig.4(a), the water particles tend to flow mainly along the interface



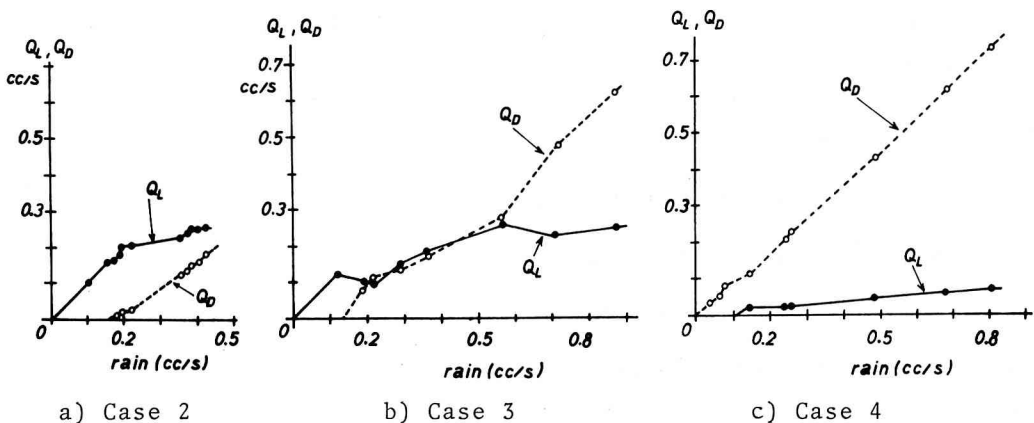
a) Case 3



b) Case 4

Fig.4, Experimental Flow Response Through Pictures

while in Fig.4(b), the flow path is directed into the lower soil with an insignificant contribution to lateral flow. Therefore, the two flow components arisen will be designated as Q_L and Q_D for lateral and downward flow, respectively. Figure 5 shows the relationship between Q_L , Q_D and R (rain intensity). From the figure it follows that low rain intensity induces lateral flow in the initial stage of groundwater runoff at the interface, with relatively small amount of Q_D , as exposed clearly in cases 2 and 3. For high rain intensity, a dominant response of Q_D over Q_L is observed. As for the Case 4, Q_D accounted for nearly all the flow drainage discharge, and Q_L on the contrary, was considerably low. Therefore, it can be stated that among the phenomena observed in the various cases, the selective mechanism of the whole rain infiltration into lateral and downward flow components is one of the most noteworthy features at the interface in layered soils. This fact has its basis on the difference in capillary pressure working between two layers at the interface.



a) Case 2

b) Case 3

c) Case 4

Fig.5, Relationship between Drainage Discharge and Rain intensity

III- MATHEMATICAL MODEL

III-1 Mechanism of Flow

For the analytical process, the potential distribution ϕ_i within the

flow domain is assumed as shown in Fig.6, where h_u is the intrinsic capillary pressure of upper layer, h_l is that of the lower layer and h_c , the pressure in the saturated capillary zone. Since $h_u > h_l$, no downward flow is anticipated until ϕ_{i+} surpass ϕ_{i-} where the subscripts $i+$ and $i-$ refer to the points immediately above and below the interface. Such a condition at the boundary explains the generation of lateral flow in early stage through the interconnected pores, being held by capillary forces along the interface. This mechanism concerns the first three cases indicated above and it plays an influential role for the flow in an unsaturated zone.

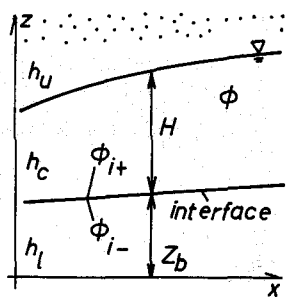


Fig.6, Flow domain

III-2 Governing Equations

In this work, the inhomogeneity of the medium is considered as inhomogeneity of type-2 (J.Bear⁹), which presumes that each subdomain(layers A and B) enclosed by boundaries of discontinuity, is homogeneous in itself.

The problem is classified into two stages: a)one-dimensional analysis for the stage of a quasi-uniform lateral flow, and b)two-dimensional analysis for the stage after downward flow component appears. Accordingly, for the first stage of flow, the following relation holds,

$$\lambda \frac{\partial \phi}{\partial t} = k \frac{\partial}{\partial x} \left(H \frac{\partial \phi}{\partial x} \right) + Q \quad \dots (1)$$

$$\phi = H + z_b$$

where, λ and k are the effective porosity and hydraulic conductivity of upper soil, respectively. Q is the source term and H , z_b are identified in Figure 6.

For the second stage, the flow in the "held water" can be expressed by the following equation:

$$k \frac{\partial^2 \phi}{\partial x^2} + k \frac{\partial^2 \phi}{\partial z^2} + Q = 0 \quad \dots (2)$$

On the water surface, the following relation should be satisfied,

$$\frac{\partial \eta}{\partial t} + u_s \left[\frac{\partial \eta}{\partial x} \right] = w_s \quad \dots (3)$$

where, u_s , w_s are the velocity components on the water surface. η regard to the relative position of the water level. u_s and w_s are determined by the following equations:

$$u_s = \frac{k}{\lambda} \frac{\partial \phi_s}{\partial x} - \left[\frac{\partial \eta}{\partial x} \right] w_s ; \quad w_s = \frac{k}{\lambda} \frac{\partial \phi_s}{\partial z} \Big|_{z=\eta} \quad \dots (4)$$

The above-noted equations were solved making use of Finite Difference Schemes and applying the analytical procedure previously reported by Mo-

rales et al⁶⁾ and Watanabe and Tamai¹⁰⁾.

III- Numerical Results and Comparison

Figure 7 shows the numerical and experimental drainage discharges (Q_L, Q_D) at different rain intensities, for Case 1. Similar behavior was noticed through the experimental data of Cases 2 and 3, shown before in Figure 5. Thus, it is found that the analytical results well accord with the experimental ones, and the small discrepancy is due to slight dis-

turbances in the soil condition. Figure 8 indicates the manifestation of the water surface level above the interface and the location where downward flow occurs. The laboratory model demonstrated similar pattern and in fact, Q_D appeared near the left boundary wall. In Fig.9, the transient behavior of the drainage discharge is presented for 36mm/hr and 27mm/hr. The effect of rain intensity on the runoff pattern is obvious. Both figures are obtained for Case 1.

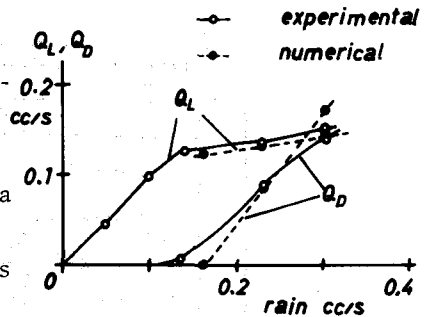


Fig.7, Experimental and Numerical data (compar.)

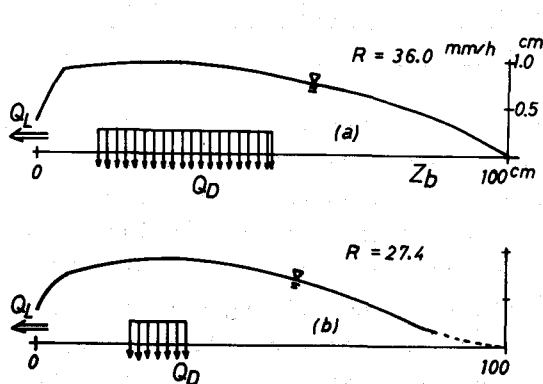


Fig.8, Shape of the Water Surface above the interface

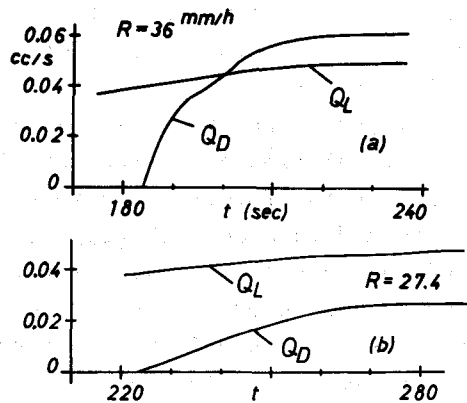


Fig.9, Transient behavior of outflow rate

IV- THREE-DIMENSIONAL LABORATORY MODEL

IV-1 Outline

The experimental model reported so far has been performed on a two-dimensional domain and tested satisfactorily through the mathematical model presented. However, as all groundwater flow in nature is to some extent three-dimensional, an additional model is suggested fulfilling such requirements of dimensional character. Furthermore, a drainage system is proposed for its application in practice.

The experimental model is shown in Fig.10. As formerly, a rain simulator and soil-box are employed. The latter allocates the soil strata and drainage system. Two trenches symmetrically located produce a fixed water head and the excess of flux is to be released through the outlets a-

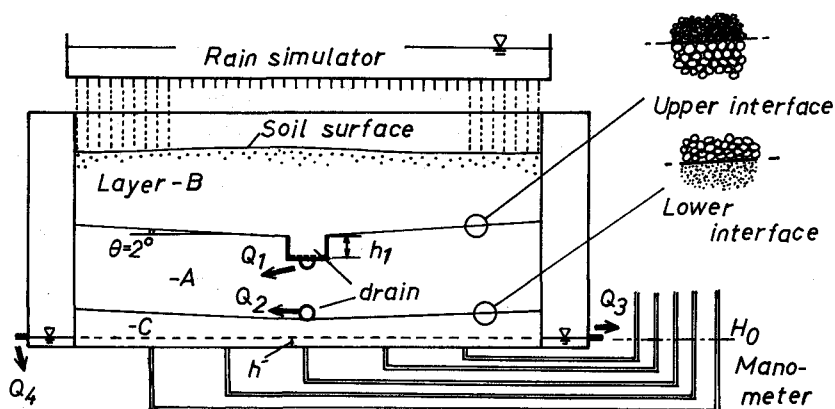


Fig.10, Three-dimensional Experimental Model
dimensions: 100×50×50 cm.

ffixed each side. Additionally, any variation in the stationary water table will be recorded by means of the manometers attached to the apparatus. A pair of drains are provided within the

Table 3. Soil Prop.

Layer	Gr. size	Perm.	Cap. rise	Poros.
B -FS	0.42mm	0.075	6.00 cm	0.37
A -CS	1.70	0.480	2.3-3.3	0.34
C -SS	0.18	0.024	19.50	0.39

soil layers. The uppermost, a canal-pipe drain system, allows to measure the lateral flow above the interface, which corresponds to "lateral drainage" in the former two-dimensional model. The depth of drain, h_1 , is determined to exceed the capillary pressure difference in the layers. The lower drain will discharge water as soon as the stationary water table reaches its inferior level. The properties of the filling materials are described in Table 3. Coarse sand (CS) is used for the middle layer, fine sand (FS), for the uppermost layer and, standard sand (SS) for the lowermost. The order in magnitude of hydraulic conductivity is $k_{CS} > k_{FS} > k_{SS}$.

IV-2 Experimental Results

Experimental runs were conducted for two different heights in the cut-off-wall of upper drain. The criterion to do so is based on the difference in capillary forces working at the interface or, it was adopted a drain wall with $h_1 \geq h_B - h_A$. As a first trial, $h_1 = 3\text{cm}$ was used and afterwards it was increased to 6 cm. In the former case, $h_1 = h_B - h_A$. Such values refer to the capillary pressure head of layers B and A disclosed heretofore (Table 3). Hereinafter the experimental results obtained for the latter height will be discussed. Fig.11 shows the variation of drainage discharges (Q_1 , Q_2 , Q_{3+4}) with the applied rain intensities. As the water starts to percolate into the soil for step-wise increase in rain in-

tensity, two stages are observed.

1. Initial Stage:

a) on the upper interface,

$$\phi_{i+} < \phi_{i-} , \quad Q_1 \text{ exists} , \quad Q_D (=Q_2+Q_3+Q_4)=0$$

As time goes on, a saturated capillary zone is developed and the entire flow discharge is released indefinitely through the upper outlet.

b) on the lower interface,

The local conditions remain unchanged, which means that no flow is occurring along or across the interface. Such condition was sustained from 4mm/hr up to 49mm/hr in rain intensity. Thereinafter the flow pattern no longer behaves in that manner.

2. Second Stage:

a) at certain points on the upper interface,

$$\phi_{i+} > \phi_{i-} , \quad Q_D \text{ starts to increase but, } Q_1 > Q_D$$

The water particles flow laterally along the interface as well as flow downward into the middle layer, but the ratio Q_1/Q_D being always larger than unity. Such condition takes place for $R > 49\text{mm/hr}$.

b) on the lower interface,

$$\phi_{i+} > \phi_{i-} ; \quad k_A > k_C , \quad Q_2=0 , \quad Q_{3+4} \text{ occurs first.}$$

As time advances, $\bar{h} > H_0$, the stationary water table rises gradually until \bar{h} reaches the drain lower level. Thereinafter Q_2 appears indefinitely with Q_{3+4} .

Figure 12 shows the remarkable effect upon the groundwater runoff observed through the usage of a shorter drain wal. For step-wise increments of the amount of rainfall, the water flowing laterally along the upper interface(Q_1) is stored in the trench first. Q_1 appears in the initial stage but

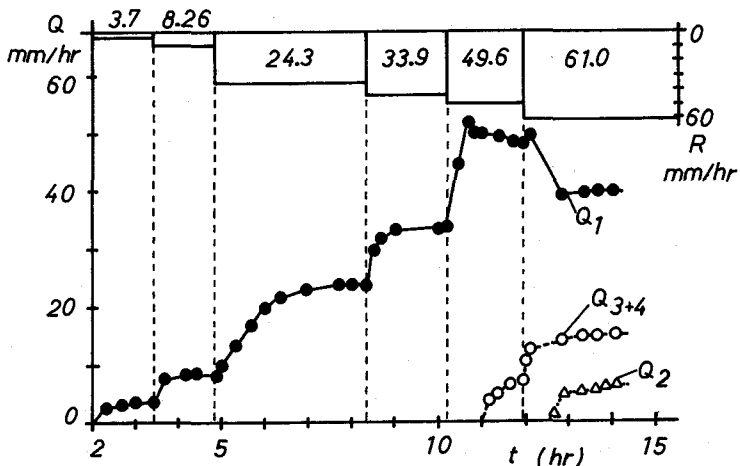


Fig.11, Drainage disch. vrs. elapsed time($h_1=6\text{cm}$)

since h_1 is nearly equal to h_B-h_A , soon after downward flow occurs anywhere on the interface, thus, lateral flow to the trench ceases. Therefore, the importance of trench-wall to drainage response is evident.

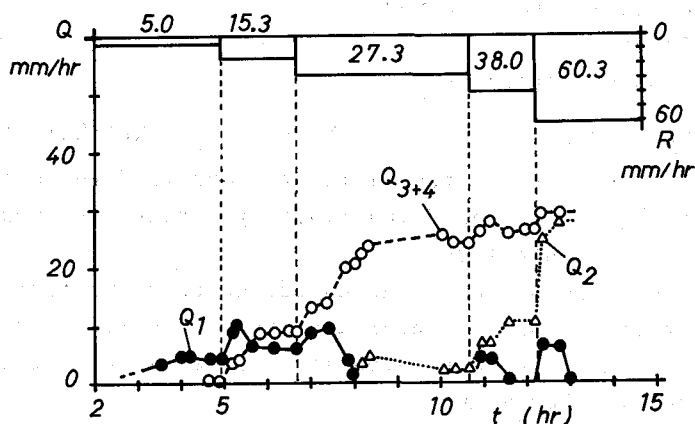


Fig.12, Drainage disch. vrs. elapsed time

V- CONCLUSIONS

In this work, the infiltration mechanism into an unsaturated flow domain was studied. The most remarkable phenomena can be summarized as follows:

1. The mechanism of the unsaturated flow into nonhomogeneous soil is widely affected by the capillary force at the interface between the layers.
2. Low rain intensity induces lateral flow in the initial stage of groundwater runoff in finer soils. In the three-dimensional model, such tendency is observed even at relatively high rain intensity.
3. The flow behavior in the unsaturated zone is characterized by a non-homogeneous pattern, in contrast to previous theories which assume uniformity in the advancement of wet front.
4. The mechanism of flow described here can be simulated by a mathematical model using a finite difference scheme in two-dimensional works.
5. The arrangement of layers plays an important role in an unsaturated flow if the capillary pressure is taken into account.
6. The drain system proposed and an appropriate arrangement of layers seem to have engineering applications such as recharge problem, infiltration of rainwater to retard surface runoff, stabilization of base flow and others.

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