⑽ Structural Vulnerability Analysis

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The purpose of structural vulnerability analysis is to enable the identification of the most vulnerable parts of a structure so that they may be suitably protected and monitored. A graph model is used to analyse various load paths and to define the new concepts of structural rings and clusters. A structure is represented at various hierarchical levels of definition in terms of clusters of interconnected structural rings. A failure scenario is described in terms of deteriorating events of structural rings.

Keywords: *Vulnerability, Robustness, Structural Ring, Structural Cluster, Failure* Scenario, Reliability

1. INTRODUCTION

This paper summarises a theory, the purpose of which is to identify the most vulnerable parts of a structure so that they may be suitably protected and monitored (Wu 1991). The systems approach used is designed to take advantage of computer based pattern recognition techniques. A graph model of a structure has been developed in order to analyse load paths. The model includes some new concepts, the single most important of which is a structural ring. A structural ring is a load path which is capable of resisting an arbitrary set of applied forces. A structure then can be represented at various hierarchical levels of definition in terms of clusters of interconnected structural rings. A failure scenario is described in terms of deteriorating events and a measure of the effort required is called damage demand. In the analysis of vulnerability three types of failure scenario are defined, the minimal, the maximal and the particular.

The emphasis of this analysis of vulnerability is not the usual one of analysing a structure under some given loading condition, rather it is to examine the vulnerability of a structure to any possible loading condition. This is accomplished by examining (i) the well-formedness of the structural rings at various levels of definition within a structure and (ii) those rings which are the most vulnerable or critical together with the actions which might cause failure.

2. HIERARCHICAL REPRESENTATION OF A STRUCTURE

2.1 Structural Ring

A structure can be regarded as a graph model $S=\{M, J\}$, which consists of two sets: a set of member objects $M=\{M_i \mid i=1,...,N_M\}$ and a set of joint objects $J=\{J_j \mid j=1,...,N_J\}$. Each member object is associated with at least two joint objects. A structural path is a sequence of connected members. A structural ring is a path which is either open or closed and either structurally over-stiff or just-stiff. It is therefore capable of resisting an arbitrary equilibrium set of applied forces given adequate strength.

In its most elementary form a ring consists of a series of members such as the pin-jointed triangle of Fig.1(a) and the portal frame of Fig.1(b) which includes the ground C_g as a special member linking the bases of the frame. Fig.1 also shows an abstract representation of a structural ring as a circle with joints and with arcs representing the members.

The well-formedness of a structural ring is a measure of its ability to resist loading from any arbitrary direction. The well-formedness depends on the orientation and stiffness of the members within the ring and the stiffness of the joints connecting the members in the ring. The quality of well-formedness of a joint contained in a ring is here defined as

$$qj = det(Djj) \qquad (2.1)$$

where Djj is the stiffness sub-matrix associated with members in the ring at a joint j and $det(D_{jj})$ is the determinant of the matrix D_{jj}. We then define the quality of well-formedness of a structural ring R as

$$q(R) = \Sigma_j \, q_j \; ; j=1,\dots,N(J_R) \qquad (2.2)$$

where $N(J_R)$ is the total number of joints in the ring R. Note that the determinant of D_{jj} is equal to the product of all eigengvalues that satisfy $det(D_{jj}-\lambda In) = 0$. Thus q_j is independent of the co-ordinate system used. Note also that q is not dimensionless but could be made so by normalising with respect to the optimum configuration for a given joint. It is recognised that the measure may need to be improved: units will not be quoted in this paper.

2.2 Structural Clusters

A structural cluster C^l at a level of definition l is defined as a sub-set $C^l=\{M^l, J^l\}$ of S in which the objects are, in some sense, more tightly connected to each other within the cluster than to other objects outside the cluster. A primitive cluster contains only one member and two end joints. The conventional way of describing a structure uses primitive clusters and is the lowest level of definition.

If clusters of members and joints are formed and the structure is defined in terms of those clusters then that forms the second level of definition. If clusters of the second level are formed then this becomes a third level of definition. This process can be repeated for even higher levels until at the top level where there is only one cluster, the whole structure. Thus the structure can be represented hierarchically as

$$S = S^0 = S^1 \dots = S^l = \{C^l i, J^l j \,/ i=1,\dots,N_C l; \, j=1,\dots,N_J l\} \qquad (2.3)$$

where $N_C l$ and $N_J l$ are the total number of clusters and joints within the structure at level of definition l.

The structural tightness of a cluster is a measure of connectivity. It depends on the number of structural rings within the cluster, the degree of overlap between them and the well-formedness of the rings. Thus the structural tightness of the cluster C^l is defined as

$$Q(C^l) = \Sigma_j \, det(D_{jj}) \,/\, N(J_C l) \; ; j=1,\dots,N(J_C l) \qquad (2.4)$$

where $N(J_C l)$ is the total number of joints in C^l.

2.3 Hierarchy Formation

The ground or foundation of the structure is effectively a very tight single cluster and must be considered specifically. In order to cluster the structure independently of the ground, the following algorithm does not include rings containing the ground until all the remaining rings not yet taken into the cluster, include the ground.

An algorithm for forming a hierarchical representation of a structure is

(1) Start at hierarchical level $l=0$, which is the lowest level of definition at which the members and joints of the basic structure are described

(2) At level 1 of build-up, identify all $N(R^l)$ structural rings in the structural model S^l ignoring the ground C_g unless all rings contain C_g. If no structural ring can be found then go to step (10), otherwise

(3) Calculate the well-formedness $q(R^l{}_i)$ for each of $N(R^l)$ rings according to Eq.(2.2) and rank them

(4) Choose the highest ranked ring not in a cluster as a forming cluster $C^l j$ and set $Q(C^l{}_j)=Q(R^l{}_i)$

(5) Assume an overlapping ring $R^l{}_i{}^*$ added in to the forming cluster $C^l j$, and calculate the new value of structural tightness $Q^*(C^l j)$ using equation 2.4.

(6) Include the ring $R^l{}_i{}^*$ in $C^l{}_j$ if $Q^*(C^l{}_j)>Q(C^l{}_j)$

(7) Repeat step (5) to (6) for all other overlapping rings

(8) Go to step (4) in order to form another cluster with $j=j+1$
(9) Go to the next hierarchical level of definition by $l=l+1$ and go to step (2)
(10) Stop.

The process of cluster formation produces a hierarchical model of a structure which is also a set of structural rings at each level of definition as

$$S^l = \{R^l{}_i \mid i=1,...,N(R^l)\} \quad (2.5)$$

where $N(R^l)$ is the total number of structural rings at the level of definition l.

The structure of Fig.2 is used as an example to illustrate the procedure of hierarchy formation. At the lowest level of definition (level 0) there are 4 structural rings with well-formedness values, from Eq.(2.2), as follows:

$$R^0{}_1 = \{M_1, J_1, M_2, J_2, M_6, J_3\} \qquad q(R^0{}_1) = 16.0$$
$$R^0{}_2 = \{M_2, J_2, M_3, J_4, M_5, J_1\} \qquad q(R^0{}_2) = 15.1$$
$$R^0{}_3 = \{M_1, J_3, M_4, J_4, M_5, J_1\} \qquad q(R^0{}_3) = 15.0$$
$$R^0{}_4 = \{M_3, J_4, M_4, J_3, M_6, J_2\} \qquad q(R^0{}_4) = 16.8$$

Note that rings such as $\{C_g, J_1, M_2, J_2\}$ are not included. Ring 4 has the highest well-formedness and is the seed for the cluster formation. Ring 1 is added to ring 4 and the structural tightness of the cluster so formed is calculated using equation (2.4). As Q increases the process is repeated adding in ring 2 and finally ring 3 to form cluster $C^1{}_1$. At this stage $Q(C^1{}_1) = 47.2$.

Since there are no more rings at this stage, we move on to the next level (level 1). Hence there is only one ring $R^1{}_1 = \{C_g, J_1, C^1{}_1, J_2\}$ including the ground cluster, it is the highest level of definition and the algorithm stops. The process is illustrated in Fig.3.

3. FAILURE SCENARIOS

3.1 Deterioration Hierarchy of Structural Rings

Fig.4 is a deterioration hierarchy for a structural ring (DHSR) and shows all of the possible ways in which a fully fixed ring can deteriorate into a mechanism. A rectangular portal frame is shown also in the figure as a practical example of a structure consisting of a single ring made of members which are primitive structural clusters and the ground.

A deteriorating event is the loss of a degree of freedom (DOF) within the ring. It may occur adjacent to a joint or inside a cluster causing it to become two separate clusters connected by a joint. If a ring deteriorates until it becomes a just-stiff ring then one more deteriorating event will cause it to become a mechanism.

3.2 String Patterns of Structural Rings

A structural ring can be characterised by a string pattern representing the DOF $d^l{}_{i,j}$ carried at each joint and by each cluster and where $d^l{}_{i,j}=1$ when the force can be carried and 0 otherwise and i is a joint at level l and j is u, v, θ in the conventional x-y co-ordinate system. Thus, a fixed joint at i is $J^l i = \{1,1,1\}$, a pin joint at i is $J^l i = \{1,1,0\}$. For further details see Wu 1991.

3.3 Failure Scenarios

A path through the DHSR is a scenario $F_h(R^l)$ of a ring R^l.

$$F_h(R^l) = \{R^l{}_k \mid k=1,...,m_h\} \quad (3.1)$$

where each $R^l{}_k$ is a deteriorated ring of R^l and there are m_h such rings in the scenario. In a scenario, the $R^l{}_k$ are ordered in the sense that $R^l{}_{k+1}$ is more deteriorated than $R^l{}_k$. A failure scenario is a scenario in which the final element is a mechanism. One or more deteriorating events are the result of actions which would cause the loss, in a ring, of the capacity to transmit a DOF. Those actions may be natural (e.g. wind, earthquake) or human (e.g. mistake, sabotage).

Therefore for a failure scenario there are series of deteriorating events $g^l{}_{i,j,k}$ occurring either in a joint or in a cluster, which can be represented by

$$F_h(R^l) = \{g^l{}_{i,j,k} / \; i=1,\dots,N(J_{R^l});\; j=u,v,\theta;\; k=1,\dots,m_h-1\}. \tag{3.2}$$

3.4 Damage Demand

The effort required to cause a given deteriorating event is called damage demand with respect to that event. The damage demand for a deteriorating event $g^l{}_{i,j,k}$ is defined as $e(g^l{}_{i,j,k})$ and is assumed to be directly proportional to the loss of the principal stiffness for a given DOF. The total damage demand for a failure scenario $F_h(R^l)$ is

$$E[F_h(R^l)] = \Sigma_i \, \Sigma_j \, \Sigma_k \, e(g^l{}_{i,j,k}) \;; i=1,\dots,N(J_{R^l});\; j=u,v,\theta;\; k=1,\dots,m_h-1 \tag{3.3}$$

3.5 Action Effect

An action may cause more than one deteriorating event. We define an action effect, adjacent to a joint i (or a member i) as

$$E_i = \{a_{i,u} \, , a_{i,v}, a_{i,\theta}\} \tag{3.4}$$

where $a_{i,j}$ $(j=u,v,\theta)$ is the action effect and is 1 when the DOF remains and 0 otherwise. For instance, $E_i=\{0,0,0\}$ is the effect of complete failure. The intersection of the string patterns of the first ring $R^l{}_1$ and the first action E_1, gives a deteriorated ring $R^l{}_2$. If this is not a mechanism then a second action E_2 can be considered in a similar manner. In general for k actions

$$F_h(R^l) = \{R^l{}_k \, / k=1,\dots,N_E\} \tag{3.5}$$

where $R^l{}_{k+1}=R^l{}_k \cap E_k$, and N_E is the total number of actions.

4. VULNERABILITY ANALYSIS

Among all of the possible failure scenarios for a structure three types of scenarios are of particular interest: (i) the minimal failure scenario, (ii) the maximal failure scenario, (iii) any particularly interesting failure scenario.

4.1 The Minimal Failure Scenario

The minimal failure scenario of a structural ring at level of definition l is the one in which the least damage demand is required to transform the structural ring into a mechanism. If a given ring can fail with little damage demand then it is a vulnerable ring. Thus minimal failure scenario at level l is

$$F_{min}(R^l) = min_h \, \{E[F_h(R^l)] \, / h=1,\dots,p\} \tag{4.1}$$

where p is the total number of possible failure scenarios for a ring R^l.

The algorithm for the formation of the hierarchy and hence the levels of definition results in the least well-formed and most vulnerable rings being formed last. The analysis of structural vulnerability therefore starts at the top of level of definition and works downwards in a sort of 'unzipping' search process.

Consider again the structure of Fig.2 and limit the possible actions $E_i(i=1,..,6)$ to "cutting one member". In this case all DOFs of the member are lost.

If $R^0{}_1 = \{M_1, J_1, M_2, J_2, M_6, J_3\} = \{(1,1,1),(1,1,0),(1,1,1),(1,1,0),(1,1,1),(1,1,0)\}$ then if M_1 is cut, $E_1 = \{(0,0,0),(1,1,1),(1,1,1),(1,1,1),(1,1,1),(1,1,1)\}$ and the new ring $R^0{}_1$ will be obtained as $R^0{}_1 \cap E1 = \{(0,0,0),(1,1,0),(1,1,1),(1,1,0),(1,1,1),(1,1,0)\}$.

Matching the deteriorated ring with the DHSR, we see that the ring becomes a mechanism. Thus the action E_1 triggers the failure scenario for $R^0{}_1$. The damage demand for this scenario is $E[F_1(R^0{}_1) = 2160$ (Eq.3.3). The same procedure also can be applied to each action and the other rings. The minimal failure scenario is the one where the least damage demand is required and can be shown to be $F_{min}(R^l) = E[\,F_5(R^0{}_1)\,] = 1620$, i.e. cut M_5 in the rings $R^0{}_1$ and $R^0{}_3$.

4.2 The Maximal Failure Scenario

A failure scenario may or may not cause a given cluster to be separated from a reference cluster (usually the ground). The separateness of a structural ring R^l with respect to that failure scenario is defined as

$$\gamma[F_h(R^l)] = \Sigma_i\, Q(C^l{}_i) \,/\, \Sigma_j\, Q(C^l{}_j) \tag{4.2}$$

where $\Sigma_i\, Q(C^l{}_i)$ is the total tightness of all clusters which are disconnected from the reference cluster $C^l{}_r$, and $\Sigma_j\, Q(C^l{}_j)$ is the total tightness of all clusters still connected with the reference cluster $C^l{}_r$.

The effective consequence of a failure scenario at a level of definition is defined as the ratio of the separateness to the total damage demand for that scenario and is therefore

$$\xi[F_h(R^l)] = \gamma[F_h(R^l)] \,/\, E[F_h(R^l)] \tag{4.3}$$

The maximal failure scenario at a given level of definition is then the one in which the smallest damage demand causes the largest number of clusters to be disconnected from a reference cluster and is therefore defined as

$$F_{max}(R^l) = max_h\, \{\xi[F_h(R^l)] \,/ h=1,...,p\}. \tag{4.4}$$

In order to find the maximal failure scenario, we look at the ring R^1 at level 1 where the system would loose its integrity without the reference cluster (ground cluster). Intersecting this ring and a set of actions, we may examine the possibility of the deterioration of the ring.

Now the cluster $C^1{}_1$ may be represented at level 0 as

$C^1{}_1 = \{R^0{}_l \,/ l=1,...,4\} = \{J_i, M_j \,/ i=1,...,4;\, j=1,...,6\}$
$= \{(1,1,0),(1,1,0),(1,1,0),(1,1,0),(1,0,0),(1,0,0),(1,0,0),(1,0,0),(1,0,0),(1,0,0)\}$.

If we cut M_1, action E_1, then the deteriorated cluster is

$C^1{}_1 = C^1{}_1 \cap E_1 = \{(1,1,0),(1,1,0),(1,1,0),(1,1,0),(0,0,0),(1,0,0),(1,0,0),(1,0,0),(1,0,0),(1,0,0)\}$

Since this does not cause collapse a second action may be to cut M_2, action E_2,

$E_1 \cap E_2 = \{(0,0,0),(0,0,0),(1,1,1),(1,1,1),(1,1,1),(1,1,1)\}$ resulting in

$C^1{}_1 \cap (E_1 \cap E_2) = \{(1,1,0),(1,1,0),(1,1,0),(1,1,0),(0,0,0),(0,0,0),(1,0,0),(1,0,0),(1,0,0),(1,0,0)\}$

which is a mechanism.

At level 1, the deteriorated ring $C^1{}_1$ at this level is described as

$R^1{}_1 = \{(1,1,0),(0,0,0),(1,0,0),(1,1,1)\}$. Now this ring is also a mechanism. The damage demand for this scenarios is calculated using Eq.3.3 as $2160+1700=3860$.

The same procedure for the other combinations of any two actions results in 15 failure scenarios. It is clear however that unless member 2 is cut the separateness from the ground will not be a maximum. Thus the 15 failure scenarios reduce to 5 and all produce an infinite separateness (46.8/0.0 using Eq.4.2). The maximal failure scenario will therefore be that scenario with a cut to member 2 together with a cut to the member taken from the set of members 1, 3, 4, 5, 6 which has minimum damage demand. Thus the maximal scenario is E_2 & E_5.

5. References

Wu X (1991) "Vulnerability Analysis of Structural Systems", *PhD Thesis*, University of Bristol, Bristol, UK.

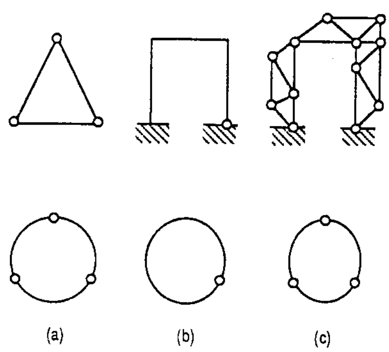
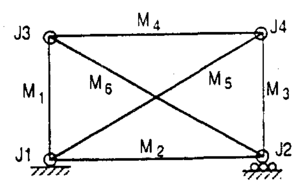


Fig 1. Examples of Structural Rings



Member	A (mm)	I (mm⁴)
1	80	2000
2	90	2100
3	82.5	2050
4	92.5	2150
5	100	2500
6	120	3000

E = 200kN/mm^2

Fig 2 Example Structure

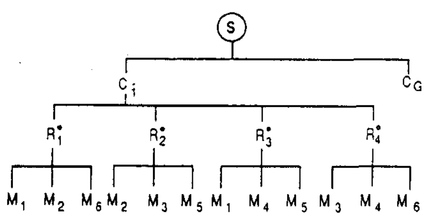


Fig 3 Cluster Formation

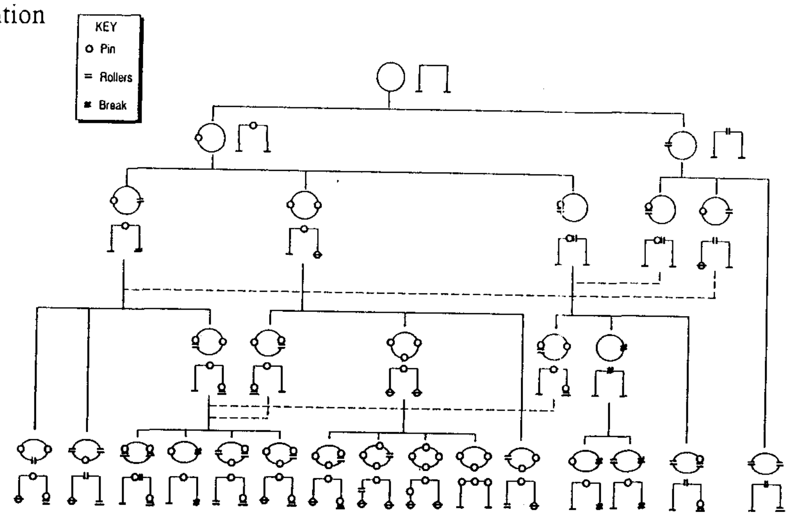


Fig. 4
Deterioration Hierachy
for a Structural Ring