THE EVALUATION OF STATIC AND DYNAMIC FRICTIONAL PROPERTIES OF ROCK DISCONTINUITIES FROM TILTING AND STICK-SLIP TESTS

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In this study, the authors describe tilting and stick-slip experiments on various natural rock discontinuities and saw-cut planes to determine their static and dynamic frictional properties. Natural rock discontinuities involve schistosity planes in quartzite, green-schist, cooling planes in andesite, granite, saw-cut planes of Ryukyu limestone, Motobu limestone, andesite and basalt from Mt. Fuji, dolomite from Kita-Daitojima, grano-diorite from Ishigaki and Inada granite. Furthermore, these experimental results are compared with each other to discuss what method is appropriate to determine could be used for determining static and dynamic frictional properties of rock discontinuities. static and one-way dynamic shearing experiments and their implications in practice are discussed.

Key Words : Rock discontinuity, static, dynamic, friction property, tilting test, stick-slip test.

1. INTRODUCTION

The static and dynamic friction angles of rock discontinuities such as joints, faults and intrinsic discontinuities such as bedding planes, sheeting joints, flow planes etc are quite important as they influence the initiation and post-sliding conditions in collapse, toppling, sliding, impacts of projectiles and meteorites[1,2,3)]

Earthquake is an instability problem of Earth's crust and it should be regarded as a subject of rock mechanics. Earthquake is caused by the varying crustal stresses and it is generally a product of slippage of major discontinuities such as faults and fracture zones[4)].

This study is concerned with the determination of static and dynamic frictional properties of rock discontinuities. The static and dynamic friction angles of rock discontinuities can be determined from tilting tests if the testing device is equipped with appropriate instruments such as non-contact laser transducers, accelerometers. The stick-slip testing device can also be used to frictional characteristics during the initiation and post-slip regime. The authors performed numerous tilting tests and stick-slip tests on the same rock discontinuities and evaluated the static and dynamic frictional characteristics of natural and saw-cut discontinuities associated with various rocks. The authors will report the outcomes of these experiments and investigate the correlations among the results of the titling tests and stick-slip tests. Natural rock discontinuities involve schistosity planes in quartzite, green-schist, cooling planes in andesite, saw-cut planes of Ryukyu limestone, Motobu limestone, andesite and basalt from Mt. Fuji, dolomite from Kita-Daitojima, grano-diorite from Ishigaki and Inada granite.

2. FRICTIONAL BEHAVIOUR OF DISCONTINUITIES

Dynamic shearing tests on rock discontinuities and interfaces are recognized as an important item of constitutive modeling of discontinuities and interfaces under static and dynamic conditions3,[5,6,7,8)]. When structures are constructed on/in rock masses having various kind geological discontinuities and possible ground motion estimations incorporating fault modeling, the determination of their dynamic shearing characteristics are necessary[1,10)]. Furthermore, dynamic conditions may be particularly of great significance in relation to the long-term stability of the structures and during earthquakes as well as in the science of earthquakes due to rate dependency of strength characteristics.

3. THEORY AND DEVICE OF TILTING TESTS

Tilting tests are known as a laboratory technique for measuring the friction angle in physics and illustrating the concept of friction[11,12)]. This technique is one of the most popular technique due to its simplicity and it is one of the most suitable technique to perform and to obtain the frictional characteristics of rock discontinuities and interfaces in-situ.

3.1 Theory of Tilting Tests

Let us assume that a block is put upon a base block with an inclination α as illustrated in Figure 1. The dynamic force equilibrium equations for the block can be easily written as follows:

For *s*-direction

$$W \sin\alpha - S = m\frac{d^2 s}{dt^2} \tag{1}$$

For *n*-direction

$$W\cos\alpha - N = m\frac{d^2n}{dt^2} \tag{2}$$

Let us further assume that the following frictional laws holds at the initiation and during the motion of the block[13] as illustrated in Figure 2: At initiation of sliding

$$\frac{S}{N} = \tan\phi_s \tag{3}$$

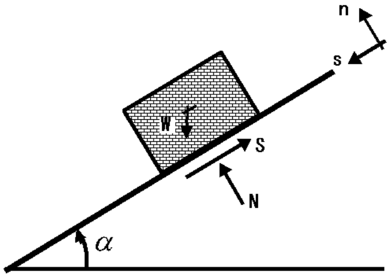


Figure 1 Mechanical model for tilting experiments

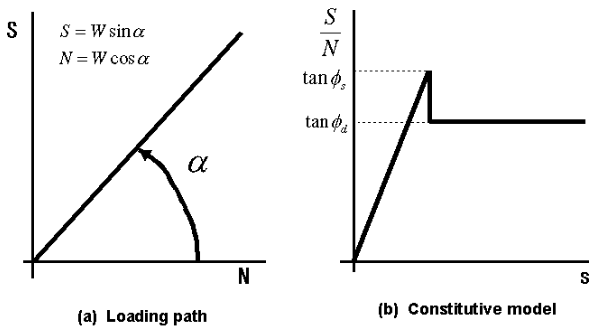


Figure 2 Loading path in tilting experiments and constitutive relation

During motion

$$\frac{S}{N} = \tan\phi_d \tag{4}$$

At the initiation of sliding, the inertia terms are zero so that the following relation is obtained:

$$\tan\alpha = \tan\phi_s \tag{5}$$

The above relation implies that the angle of inclination (rotation) at the initiation of sliding should correspond to the static friction angle of the discontinuity.

If the normal inertia term is negligible during the motion and the frictional resistance is reduced to dynamic friction instantenously, one can easily obtain the following relations for the motion of the block

$$\frac{d^2s}{dt^2} = A \tag{6}$$

where $A = g(\sin\alpha - \cos\alpha\tan\phi_d)$.

The integration of differential equation (6) will yield the following

$$s = A\frac{t^2}{2} + C_1t + C_2 \tag{7}$$

Since the followings hold at the initiation of sliding

$$s = 0 \text{ and } v = 0 \text{ at } t = T_s \tag{8}$$

Equation (7) takes the following form

$$s = \frac{A}{2}(t - T_s)^2 \tag{9}$$

Coefficient A can be obtained either from a given displacement s_n at a given time t_n with the condition, that is,

$$t_n > T_s$$

$$A = 2\frac{s_n}{(t_n - T_s)^2} \tag{10}$$

or from the application of the least square technique to measured displacement response as follows

$$A = 2\frac{\sum_{i=1}^{n} s_i(t_i - T_s)^2}{\sum_{i=1}^{n}(t_i - T_s)^4} \tag{11}$$

Once constant A is determined, the dynamic friction angle is obtained from the following relation

$$\phi_d = \tan^{-1}\left(\tan\alpha - \frac{1}{\cos\alpha}\frac{A}{g}\right) \tag{12}$$

3.2 Tilting Device and Set-up

An experimental device consists of a tilting device operated manually. During experiments, the displacement of the block and rotation of the base is measured through laser displacement transducers produced by KEYENCE while the acceleration responses parallel and perpendicular to the shear movement are measured by a three component accelerometer (TOKYO SOKKI) attached to the upper block and WE7000 (YOKOGAWA) data acquisation system. The measured displacement and accelerations are recorded onto lap-top computers. The weight of the accelerometer is about 0.96 N Figure 3 shows the experimental set-up.

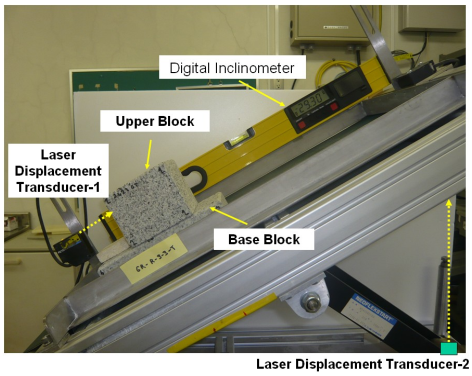


Figure 3 A conceptional illustration of experimental set-up

4. THEORY AND DEVICE OF STICK-SLIP TESTS

4.1 Theory of Stick-Slip Tests

In this model, the basal plate is assumed to be moving with a constant velocity v_m, and overriding block is assumed to be elastically supported by the surrounding medium as illustrated in Figure 4. The basic concept of modelling assumes that the relative motion between the basal plate and overriding block is divergent. Let as assume that the motion of the plate during can be modeled as a stick-slip phenomenon[14,15]. The governing equation of the motion of the overriding block may be written

During the stick phase, the following holds

$$\dot{x} = v_s, \qquad F_s = k \cdot x \tag{13}$$

where v_s is belt velocity and k is the stiffness of the system. The initation of slip is given as (Figure 5)

$$F_y = \mu_s N \tag{14}$$

where μ_s is static friction coefficient, N is normal force. For block shown in Figure 4, it is equal to block weight W and it is related to the mass m and gravitational acceleration g through mg. During slip phase, the force equilibirum yields:

$$-kx + \mu_k W = m\frac{d^2x}{dt^2} \tag{15}$$

where μ_k is dynamic friction angle. The solution of above equation can be obtained as.

$$x = A_1 \cos\Omega t + A_2 \sin\Omega t + \mu_k \frac{W}{k} \tag{16}$$

If initial conditions ($t = t_s$, $x = x_s$ and $\dot{x} = v_s$) are introduced in Eq. (16), the integration constants are obatined as follow.

$$x = \frac{W}{k}(\mu_s - \mu_k)\cos\Omega(t - t_s) + \frac{v_s}{\Omega}\sin\Omega(t - t_s) + \mu_k\frac{W}{k}$$

$$\dot{x} = -\frac{W}{k}(\mu_s - \mu_k)\Omega\sin\Omega(t - t_s) + v_s\cos\Omega(t - t_s) \qquad (17$$

$$\ddot{x} = -\frac{W}{k}(\mu_s - \mu_k)\Omega^2\cos\Omega(t - t_s) - v_s\Omega\sin\Omega(t - t_s)$$

where $\Omega = \sqrt{k/m}$ and $x_s = \mu_s \frac{W}{k}$.

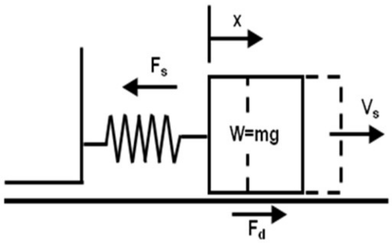


Figure 4 Mechanical modelling of stick-slip phenomenon

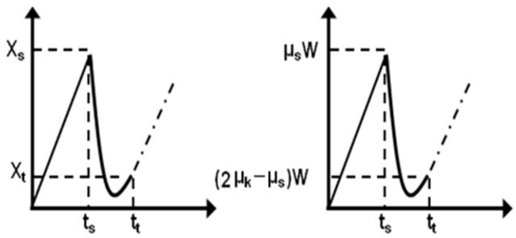


Figure 5 Frictional forces during a stick-slip cycle

At $t = t_t$,velocity becomes equal to belt velocity, which is given as $\dot{x} = v_s$. This yield the slipe period as:

$$t_t = \frac{2}{\Omega}\left(\pi - \tan^{-1}\left(\frac{(\mu_s - \mu_k)W\Omega}{k \cdot v_s}\right)\right) + t_s \tag{18}$$

where $x_s = v_s \cdot t_s$. The rise time, which is slip period is given by

$$t_r = t_t - t_s \tag{19}$$

Rise time can be specifically obtained from Eqs. 19 and 18 as

$$t_r = \frac{2}{\Omega}\left(\pi - \tan^{-1}\left(\frac{(\mu_s - \mu_k)W\Omega}{k \cdot v_s}\right)\right) \tag{20}$$

If belt velocity is neglible, that is, $v_s \approx 0$, the rise time reduce (t_p) to the following form

$$t_r = \pi\sqrt{\frac{m}{k}} \tag{21}$$

The amount of slip is obtained as

$$x_r = |x_t - x_s| = 2\frac{W}{k}(\mu_s - \mu_k) \tag{22}$$

The force drop during slip is given by

$$F_d = 2(\mu_s - \mu_k)W \tag{23}$$

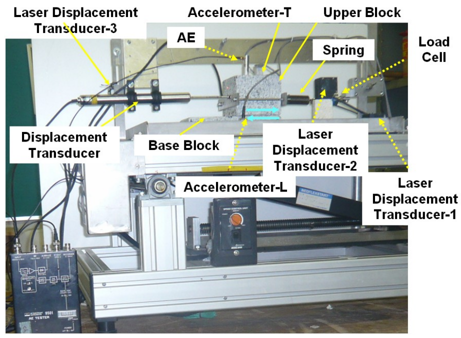
It should be noted that the formulation given above does not consider the damping associated with slip velocity. If the damping resistance is linear, the governing equation (15) will take the following form

$$-kx - \eta\dot{x} + \mu_k W = m\frac{d^2x}{dt^2} \tag{24}$$

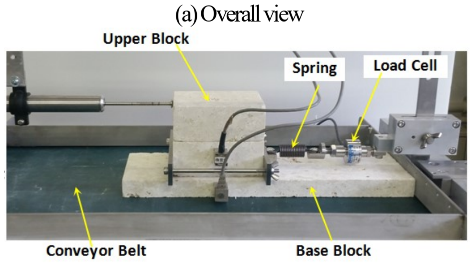
4.2 Device of Stick-slip Tests

Figure 6 shows a view of the experimental device. The experimental device consists of an endless conveyor belt and a fixed frame. The inclination of the conveyor belt can be varied so that tangential and normal forces can be easily imposed on the sample as desired. To study the actual frictional resistance of interfaces of rock blocks, the lower block is stuck to a rubber belt while the upper block is attached to the fixed frame through a spring as illustrated in Fig. 1. We conducted the experiment using the rock samples of granite with planes having different surface morphologies. The base blocks were 200-400mm long, 100-150mm wide and 40-100mm thick. The upper block was 100-200mm long, 100mm wide and 150-00 mm high.

When the upper block moves together on the base block with at a constant velocity (stick phase), the spring is stretched at a constant velocity. The shear force increases to some critical value and then a sudden slip occurs with an associated spring force drop. Because the instability sliding of the upper block occurs periodically, the upper block slips violently over the base block. Normal loads can also be easily increased in experiments.



(a) Overall view



(b) Detailed view

Figure 6 Stick-slip experimental setup

To measure the frictional force acting on upper block, the load cell (KYOWA LUR-A-200NSA1) is installed between spring and fixed frame. During experiments, the displacement of the block is measured through a laser displacement transducer produced by KEYENCE and a contact type displacement transducer with a measuring range of 70 mm, while the acceleration responses parallel and perpendicular to the belt movement are measured by a three component accelerometer (TOKYO SOKKI) attached to the upper block. The measured displacement, acceleration and force are recorded onto laptop computers.

5. DISCONTINUITIES

Discontinuities used in tilting and stick-slip experiments are listed in Table 1. Rocks involve igneous, metamorphic and sedimentary rocks. The discontinuities involve natural cooling joints, schistosity planes and saw-cut surfaces.

Table 1. A list of discontinuities tested

Rock	Location	Type	Unit Weight (kN/m^3)
Andesite	Mt. Fuji	Saw-cut	26.1
Andesite	Mt. Aso	Cooling	26.6
Basalt	Mt. Fuji	Saw-cut	24.1
Quartzite	Bayındır	Schistosity	25.1
Quartzite	Kumamoto	Schistosity	27.7
Diorite	Ishigaki	Saw-cut	26.5
Gabro	Unkown	Saw-cut-P	29.8
Limestone	Ryukyu	Saw-cut	22.9
Limestone	Motobu	Saw-cut	25.3
Dolomite	Kita-Daitojima	Saw-cut-P	26.2
Granite	Inada	Rough	25.3
Granite	Inada	Saw-cut	25.3
Granite	Salang	Saw-cut	25.9
Marl-BP	Babadag	Bedding	19.0
Marl-BP	Babadag	Bedding	19.0

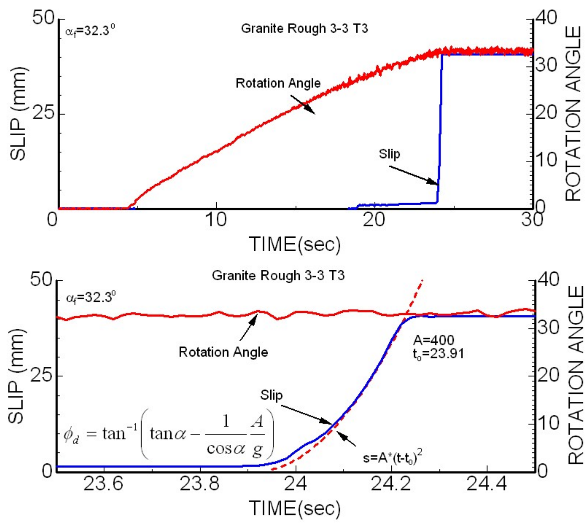


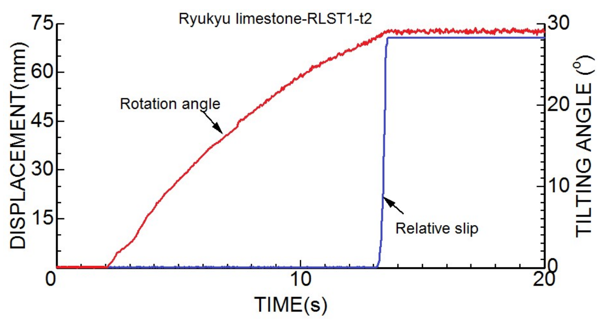
Figure 7. Responses of rough discontinuity of granite during a tilting test.

6. TILTING TESTS

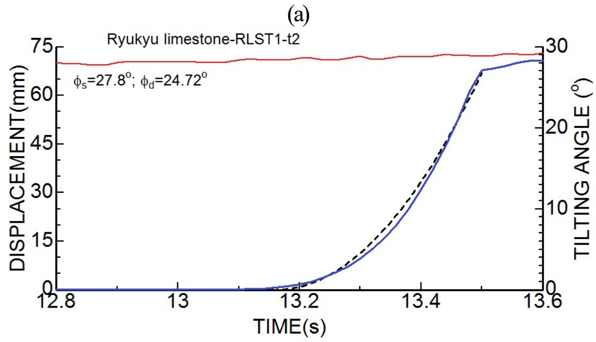
A series of tilting tests are carried out on the discontinuities listed in Table 1. Responses of some of these experimets are reported herein as examples. The measured responses during a tilting test on a rough discontinuity plane is shown in Figure 7 as an example. Figure 8 shows views of the tilting test on roudh discontinuity plane of granite. As noted from the responses of rotation angle, relative displacement and acceleration shown in Figure 8, fairly consistent results are observed. The static and dynamic friction coefficients of the interface were calculated from measured displacement response and weight of upper block using the tilting testing equipment shown in Fig. 3. The static and dynamic friction coefficients were estimated at 32.3-37.6° and 30.3-35.6° respectively.

Similarly experimental results on sawcut discontinuity planes of Ryukyu limestone samples are shown in Figure 9 , which shows responses measured during a tilting experiment on a saw-cut plane of Ryukyu-limestone. The static and dynamic friction coefficients of the interface were calculated from measured displacement response explained in previous section and they were estimated at 28.8-29.6° and 24.3-29.2° respectively.

Figure 8. Views of tilting experiment on rough discontinuity plane of granite



(a)



(b)

Figure 9 Responses of sawcut discontinuity planes of Ryukyu limestone samples during a tilting test.

7. STICK-SLIP EXPERIMENTS

A series of stick-slip experiments are carried out on the discontinuities listed in Table 1 using the stick-slip experiment shown in Figure 6. Responses of some of these experiments are reported herein to provide actual experimental data for the readers. The duration of stick-slip experiments is much longer than the tilting experiments. As the relative slip history involves many cycles of slip, the surface conditions are likely to be changing in relation to the slip history. Therefore, the evaluations are limited to the first two cycles of stick-slip phases.

The peak (static) friction angle can be evaluated from the T/N response while the residual (kinetic) friction angle is obtained from the theoretical relation (23). Figures 10 and 11 show the stick-slip responses of discontinuity planes shown in Figures 8 and 9, respectively. Peak (static) friction angle for both discontinuity plane obtained from tilting tests and stick-slip experiments are very close to each other. The residual or kinetic friction angle for rough discontinyuity plane of granite are also very close to each other. Similarly, the residual (kinetic) friction angle of saw-cut discontinuity plane of Ryukyu limestone obtained from stick-slip experiments are very close to those obtained from tilting experiments. Nevertheless, the kinetic or residual friction angle is generally lower than those obtained from the tilting experiments.

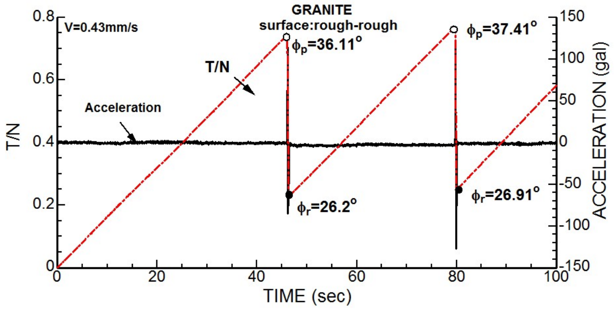


Figure 10 Stick-slip response of rough discontinuity plane of granite

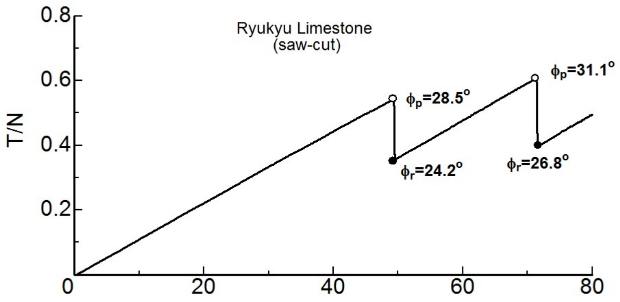


Figure 11 Stick-slip response of saw-cut plane of Ryukyu limestone

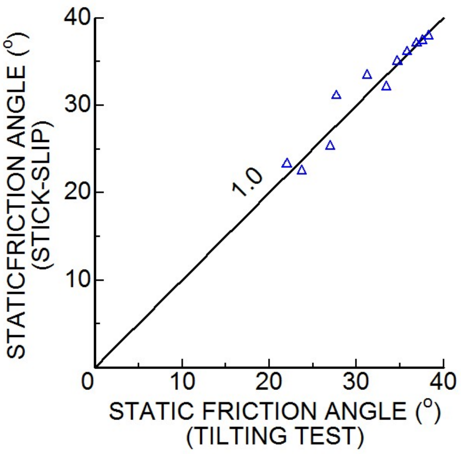


Figure 12. Comparison of static friction angles obtained from titling and stick-slip experiments.

8. COMPARISONS

The static (peak) and residual (kinetic) friction angles obtained from tilring tests and stick-slip experiments are compared herein. Table 2 compares the static and kinetic friction angles obtained from tilting and stick-slip experiments on discontinuities listed in Table 1.

Table 2. A list of discontinuities tested

Rock	Location	Tilting		Stick-slip	
		Static	Kinetic	Static	Kinetic
Andesite	Mt. Fuji	24.9	23.3	24.1	21.3
Andesite	Mt. Aso	36.6	34.6	34.6	32.4
Basalt	Mt. Fuji	23.0	21.9	22.5	20.3
Quartzite	Bayındır	38.3	32.0	37.9	26.5
Quartzite	Kumamoto	36.9	34.1	37.1	30.4
Diorite	Ishigaki	33.4	28.9	32.1	28.7
Gabro	Unkown	23.7	21.7	22.5	19.5
Limestone	Ryukyu	27.7	24.7	31.1	26.8
Limestone	Motobu	31.2	28.7	33.4	28.3
Dolomite	Kita-Daitojima	22.0	19.8	23.3	15.6
Granite	Inada	37.6	35.9	37.4	26.1
Granite	Inada	35.8	33.6	36.1	26.2
Granite	Inada	27.0	24.9	25.3	20.0
Granite	Salang	43.2	37.2	41.6	35.8
Marl-BP	Babadag	40.5	38.7	40.3	31.9
Marl-BP	Babadag	34.7	31.9	35.0	27.3

As stated previously, peak (static) friction angle for discontinuity planes obtained from tilting tests and stick-slip experiments are very close to each other as seen in Figure 12. The residual or kinetic friction angle of discontinuity planes obtained from stick-slip experiments are very close to those obtained from the tilting experiments as seen in Figure 13. Nevertheless, the kinetic or residual friction angle is generally lower than those obtained from the tilting experiments and the relation between kinetic friction angle obtained from stick-slip experiments is about 0.9 times those obtained from tilting experiments as seen in Figure 13.

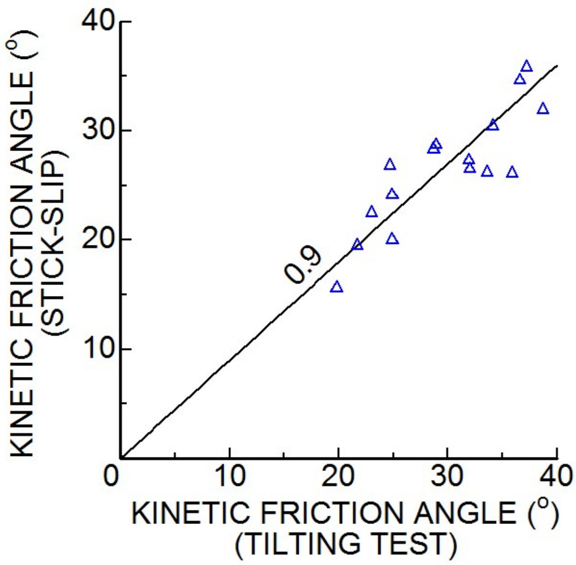


Figure 13. Comparison of kinetic friction angles obtained from titling and stick-slip experiments.

The authors performed some tilting tests on natural and saw-cut surfaces previously by Aydan et al.[12]. The previous results together with those results on static and kinetic friction angles presented in this paper are plotted in Figure 14. Most of experimental indicate that the kinetic friction angle of natural and saw-cut discontinuities is about 0.87 times that of the static friction angle. The previously reported results are quite similar to those obtained in this study. It is expected that these results would be quite usefull for simulating the post-failure motions of failed bodies in rock slope engineering, underground openings and projectile intrusions during impacts.

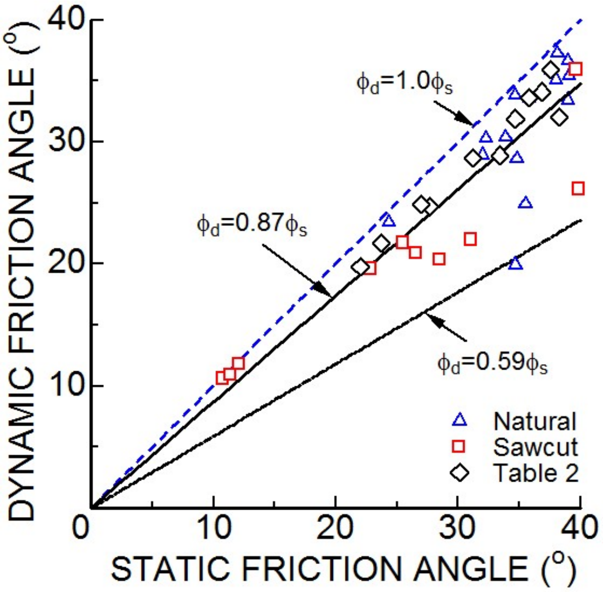


Figure 14. Comparison of static and kinetic friction angles obtained from tilting experiments on various rock discontinuities.

9. CONCLUSIONS

In this study, the authors described ax experimental study on static and kinetic friction angles of various natural rock discontinuities and saw-cut planes utilizing tilting and stick-slip experiments on to determine their static and dynamic frictional properties. In addition the theoretical background of tilting and stick-slip experiments are presented. Natural rock discontinuities involve schistosity planes in quartzite, green-schist, cooling planes in andesite, granite, saw-cut planes of Ryukyu limestone, Motobu limestone, andesite and basalt from Mt. Fuji, dolomite from Kita-Daitojima, grano-diorite from Ishigaki and Inada granite. Experimental results indicated that peak (static) friction angle for both discontinuity planes obtained from tilting tests and stick-slip experiments are very close to each other. Furthermore, the residual or kinetic friction angle for rough discontinuity plane of granite and saw-cut discontinuity plane of Ryukyu limestone obtained from stick-slip experiments are very close to those obtained from the tilting experiments. Nevertheless, the kinetic or residual friction angle is generally lower than those obtained from the tilting experiments.

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