Structural healing and decay of sedimentary soft rock under slide-hold-slide process and their constitutive description

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In order to investigate the long-term behavior of rock masses, slide-hold-slide triaxial tests have been conducted on a sedimentary rock under several patterns of effective confining stress conditions. The strength recovery was observed in the re-sliding process after the holding period especially when the confining pressure is relatively low. A model incorporating the healing and decay of the rock structure is proposed to describe the strength recovery in the re-sliding process. The proposed model is based on a critical state soil model incorporating the subloading surface concept. The structural healing and decay are modeled by the movements of the normally consolidation line and the critical state line in the plane of mean effective stress and void ratio. Triaxial slide-hold-slide tests are simulated by the proposed model and applicability of the model is discussed.

Keywords: Sedimentary rock, Slide-hold-slide, Triaxial test, Structural healing and decay, Rate- and state-dependent friction

1. Background

Earthquakes have been considered as mechanical consequences of stick-slip frictional instabilities of faults (Brace and Byerlee, 1966), where the slip is the earthquake and the stick corresponds to the interseismic period. Thus, laboratory tests on the rock friction have been extensively conducted for more than half a century and several friction laws derived from the experimental evidences have been proposed in order to investigate the mechanisms of earthquake and to discuss the seismic cycles. The rate- and state-dependent friction (RSF) (Dieterich, 1979, 1981; Ruina, 1983) have been widely applied to describe the stick-slip instabilities of faults and have successfully simulated several aspects of earthquake such as seismic cycles including preseismic slip, earthquake nucleation, dynamic slip instabilities, healing of fault, afterslip and aftershocks (Tse and Rice, 1986; Stuart, 1988; Stuart and Tullis, 1995; Marone, 1998). Hydrothermal influences on the frictional behavior of faults have also been investigated through several experimental studies and have been incorporated with the RSF (Chester, 1994; Blanpied et al., 1998). The mechanism of the time-dependent strengthening of faults have been discussed by several researchers in the context of the pressure solution phenomena (Rutter, 1983; Yasuhara et al., 2005).

Fault exhibits volumetric dilation when it is sheared. Although Beeler and Tullis (1997) insisted that volumetric behavior of faults will not contribute to the frictional resistance and concluded that no correlation needs to be considered in the modeling, authors believe that the dilatancy will significantly affect the frictional behavior of the fault as: negative dilation of saturated rock masses under undrained condition will increase the pore water pressure and will consequently decrease the mean effective stress; positive dilation under drained condition will increase the porosity and this will lead to the decreases in the stiffness and the peak strength even if the effective confining stress remains constant.

In this research work, drained triaxial tests on a tuff with slide-hold-slide (SHS) processes have been conducted to explore the structural healing and decaying behaviors of rock masses and their description through a continuum elastoplastic constitutive model.

2. Slide-Hold-Slide Triaxial Tests

Drained triaxial tests with SHS processes have been conducted on a saturated, pumice lapilli tuff, a kind of volcanic sedimentary rock. The isotropic consolidation path is firstly applied until the predetermined effective confining stress σ_r', namely 0.3, 0.5, 0.7, 5.0 and 7.0 MPa. The specimen is sheared with a constant axial strain rate of 0.01 %/min under constant confining stress to the post-peak phase where stress approaches the residual state. The holding process is then applied

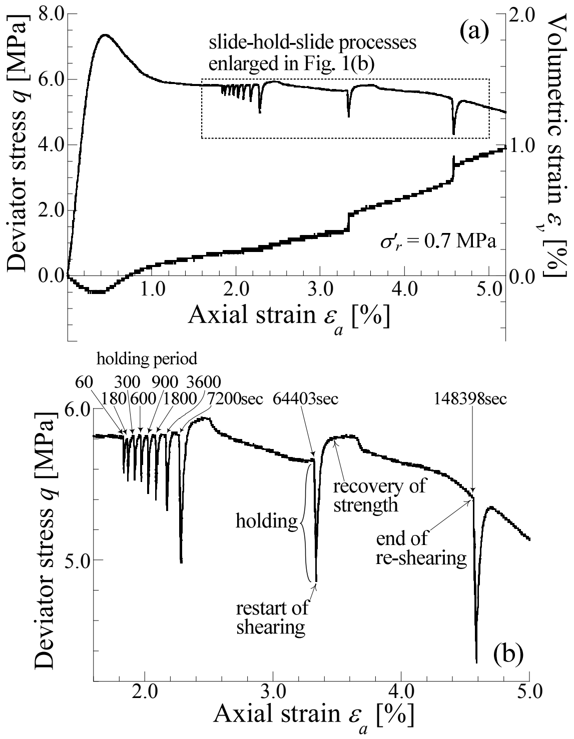

Fig. 1 An example of the stress-strain relationship in the drained triaxial slide-hold-slide test (T = 20°C, σ_r = 0.7MPa) (a) Overall view (b) Enlarged view.

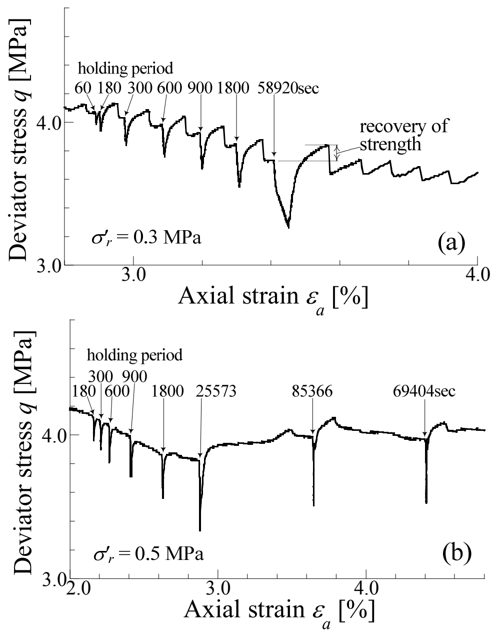

Fig. 2 Stress-strain relationship during slide-hold-slide processes (T = 20°C) (a) σ_r = 0.3MPa (b) σ_r = 0.5MPa.

viz. keeping constant axial strain under various holding time period, i.e. 60, 180, 300, 600, 900, 1800, 3600, 7200 seconds and more. The re-shearing process is again applied. The tests are conducted under several temperatures T as 20, 60, 75, and 90 °C.

An example of the results of the SHS triaxial tests is shown in **Fig. 1**(a), where σ_r = 0.7 MPa, T = 20 °C. It is indicated that the specimen exhibits typical stress-strain behavior of soft sedimentary rocks and the result shown here seems to be consistent with the past experimental observations (e.g. Adachi and Ogawa, 1980).

In the holding process in which the axial strain is kept constant, stress relaxation with a reduction of the deviator stress can be observed. In the following re-sliding process, the deviator stress increases with a relatively high stiffness, reaches to a peak value and then returns again to the residual value. The magnitude of the strength recovery is dependent on the duration of holding as the higher peak strength is particularly seen after a longer time of the holding period.

Stress-strain relations in the SHS processes under relatively lower confining pressures of 0.3 and 0.5 MPa is similar to higher confining pressure as shown in **Fig. 2**. It is however noted that the strength recovery seems to be more significant under lower stress levels.

3. Elastoplastic constitutive model considering the healing and decay of the rock structure.

Novello (1995) compared behaviors of soils, soft rocks and hard rocks in triaxial tests and insisted that the brittle-ductile transition in rocks is similar to the brittle-ductile transition in rocks is similar to the change in overconsolidated to normally consolidated behavior in soils. In this regard, the critical state framework, which was originally developed for clay, has a broader range of application to various kinds of geomaterials and several extended versions of the critical state models have been actually proposed to describe the behaviors of structured geomaterials such as naturally deposited soils or sedimentary soft rocks (e.g. Adachi and Oka, 1995; Rouainia and Muir Wood, 2000). In this study, an elastoplastic model for soft rocks which can describe the time-dependent healing effect and deformation-dependent decay of rock structure is developed based on a critical state model.

It is known that the normally consolidation line (NCL) and the critical state line (CSL) are usually utilized in the formation of critical state soil model. Specific volume v of normally consolidated soil at the current stress (p, η) is uniquely given considering the combined effects of consolidation and dilation as:

$$v = N - \lambda \ln \frac{p}{p_a} + (\Gamma - N)\zeta(\eta) \tag{1}$$

where p is mean effective stress, η (= q/p) is stress ratio, q is deviator stress, p_a (= 98Pa) is atmospheric pressure, $\zeta(\eta)$ is a monotonic increasing function satisfying $\zeta(0)$ = 0 and $\zeta(M)$ = 1. N and Γ represent specific volumes on NCL and CSL at $p = p_a$, respectively.

A plane containing NCL and CSL given by eq. (1) is shown in **Fig. 3**. The plane is a linear representation of the state boundary surface in the ln$p$-$\zeta(\eta)$-v space and specific volume of normally consolidated soil always stays on this plane. In the current model, we employ eq. (2) as a function of $\zeta(\eta)$ in accordance with the modified Cam clay (Roscoe and Burland, 1968).

$$\zeta(\eta) = \ln\left\{1 + \left(\frac{\eta}{M}\right)^2\right\} / \ln 2 \tag{2}$$

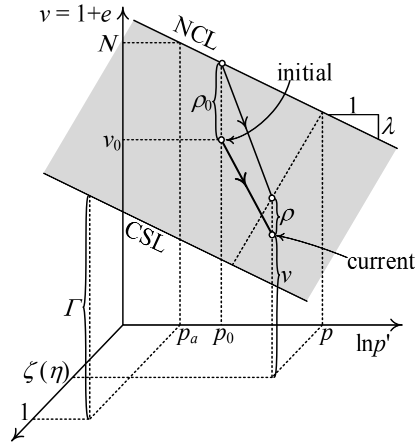

Fig. 3 Modeling of volumetric behavior of overconsolidated soils by the subloading concept.

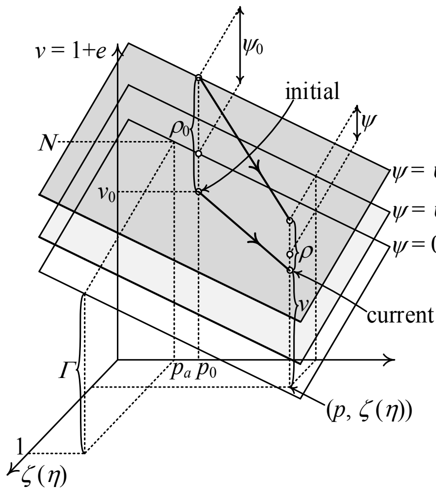

Fig. 4 Modeling of volumetric behavior of soft rocks considering the structural change via state parameter ψ.

The concept of subloading surface (Hashiguchi and Ueno, 1977) is consequently employed to consider the effect of density or overconsolidation ratio *OCR*. However, instead of using their original state parameter *R*, reciprocal of *OCR*, we select a state parameter ρ which is given by the combination of void ratio and stress to describe the changing strength and stiffness of soils. As all states of soil locate below the state boundary surface in **Fig. 3**, the state boundary surface defines the loosest, upper limit of specific volume of soils and a state parameter ρ is thus defined as the specific volume difference between the current state and the loosest state under the same stress (p, η) on the state boundary surface as shown in **Fig. 3**.

$$\rho = N - \lambda \ln \frac{p}{p_a} + (\Gamma - N)\zeta(\eta) - v \tag{3}$$

Though Been and Jefferies (1985) proposed a state parameter for sand as volumetric distance of the sand from the reference state on the steady state line and Nakai and Hinokio (2004) used similar state parameter defined as a void ratio difference from the normally consolidation line, our state parameter ρ always refers to the volumetric distance from the loosest state of soil under the current stress condition. In the similar form as eq. (1), variation in the specific volume of soil from initial state (p_0, η = 0, v_0) to current state (p, η, v) indicated in **Fig. 4** is given by equation (4).

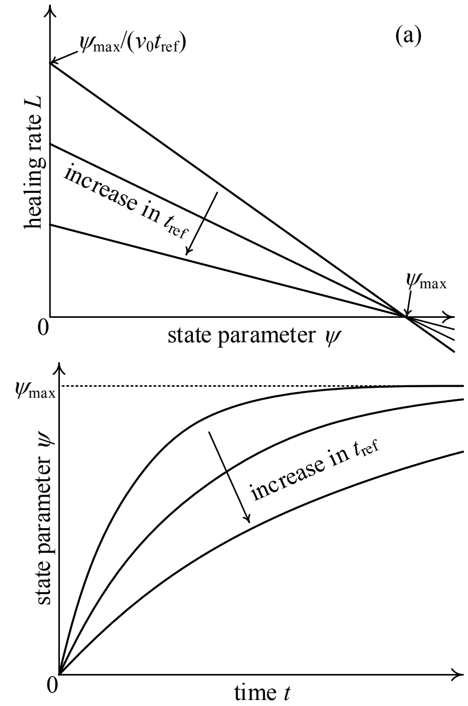

Fig. 5 Modeling of the time-dependent healing of the rock structure via state parameter ψ (a) rate of the healing (b) image of structural healing.

$$(v + \rho) = (v_0 + \rho_0) - \lambda \ln \frac{p}{p_0} + (\Gamma - N)\zeta(\eta) \tag{4}$$

The rock structure is assumed to have a role of shifting the state boundary surface upward in the specific volume direction to increase the loosest specific volume. We propose to introduce a new state variable ψ to represent the upward shift of the state boundary surface in the p-$\zeta(\eta)$-v space as indicated in **Fig. 4**, and the state parameter ρ is defined as the volumetric distance from the shifting state boundary surface. The volume change of structures soft rocks exhibiting structural change shown in **Fig. 4** is thus given in a similar way as eq. (5).

$$(v + \rho - \psi) = (v_0 + \rho_0 - \psi_0) - \lambda \ln \frac{p}{p_0} + (\Gamma - N)\zeta(\eta) \tag{5}$$

With the swelling index κ, the elastic volumetric strain is assumed to follow a usual relationship as:

$$\varepsilon_v^e = \frac{\kappa}{v_0} \ln \frac{p}{p_0} \tag{6}$$

From equations (5) and (6), the plastic volumetric strain ε_v^p is obtained as follow.

$$\begin{aligned} \varepsilon_v^p &= \varepsilon_v - \varepsilon_v^e = \frac{v_0 - v}{v_0} - \varepsilon_v^e \\ &= \frac{\lambda - \kappa}{v_0} \ln \frac{p}{p_0} + \frac{N - \Gamma}{v_0} \zeta(\eta) + \frac{\rho - \rho_0}{v_0} \end{aligned} \tag{7}$$

A yield function f is introduced from eq. (7) as eq. (8).

$$f = \frac{\lambda - \kappa}{v_0} \ln \frac{p}{p_0} + \frac{N - \Gamma}{v_0} \zeta(\eta) + \frac{\rho - \rho_0}{v_0} - \varepsilon_v^p \tag{8}$$

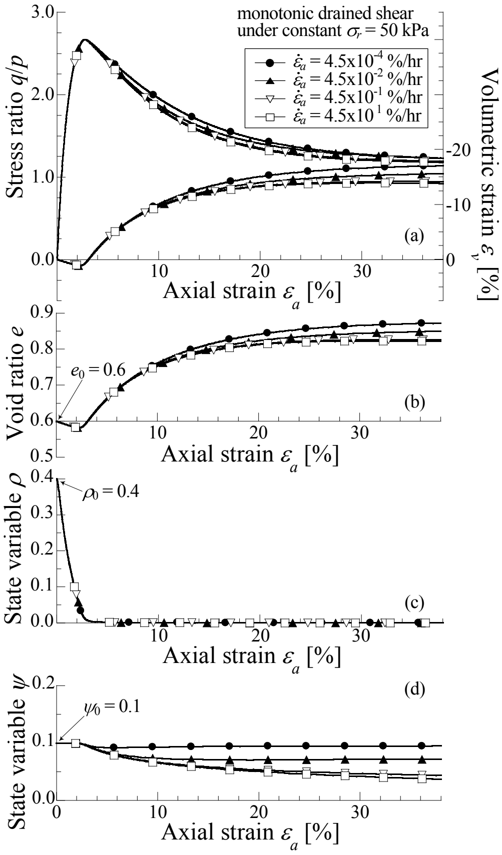

Fig. 6 Simulation results of the effect of parameter b on decaying rate in monotonic shear test under triaxial consolidated drained condition

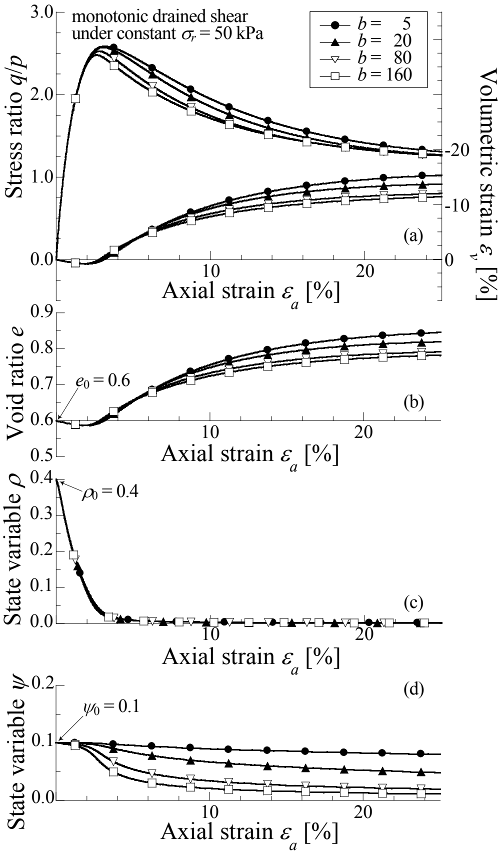

Fig. 7 Simulation results of the effect of strain rates on decaying rate in monotonic shear test under triaxial consolidated drained condition

Associated flow is assumed in the model. As soil exhibits unlimited distortional strain at critical state without any change in stress or volume, $\partial f/\partial\sigma_{ii}$ becomes zero when η is equal to M. $(N-\Gamma)$ is thus equal to $(\lambda-\kappa)/\ln 2$ in case eq. (2) is applied. For consistency condition of f, the evolution law of ρ is needed. The essential characteristic of ρ is that it monotonically decreases in accord with the plastic deformation and finally converges to zero when the soil approaches normally consolidated plane. Evolution law satisfying this is given as:

$$\frac{d\rho}{v_0} = -a\rho^2\sqrt{d\varepsilon_{ji}^p d\varepsilon_{ji}^p} \tag{9}$$

where a is a parameter controlling the effect of density that describes the convergence rate to 0 of ρ.

Considering the time-dependent healing and the deformation-dependent decay of rock structure, the evolution law of ψ considering the both mechanisms can be given as eq. (10):

$$\frac{d\psi}{v_0} = -b\psi^2\sqrt{d\varepsilon_{ji}^p d\varepsilon_{ji}^p} + Q(\psi)dt \tag{10}$$

The first term on the right hand side of eq. (10) represents the deformation-dependent decay of the structure. State parameter ψ decreases towards zero with the plastic strain development, where b is a constitutive parameter controlling the rate of the decay. The second term describes the time-dependent healing and $Q(\psi)$ is a function defining the healing rate. Though this function would be dependent on temperature, confining pressure and some other factors, a tentative expression is given as:

$$Q(t) = \frac{1}{v_0}\frac{\psi_{\max}-\psi}{t_{ref}} \tag{11}$$

where $\psi_{\max}$ is a parameter defining the maximum value of ψ and t_{ref} is a parameter having a dimension of time which describes the convergence rate of ψ from 0 to $\psi_{\max}$. If the rock initially has no structure $(t, \psi) = (0, 0)$ and if no plastic deformation is exhibited, differential equations (10) and (11) can be directly solved and we obtain equation (12).

$$\psi = \psi_{\max}\left\{1-\exp\left(-\frac{t}{t_{ref}}\right)\right\} \tag{12}$$

The meaning of eqs. (11) and (12) is that ψ will increase as time increases and gradually ψ will reach its maximum value $\psi_{\max}$ as indicated in **Fig. 5** (b).

4. Results and discussions

(1) Monotonic Shear under Drained Condition

Numerical simulations results for a monotonic shear test in triaxial consolidated drained condition with constant confined pressure σ_r = 50 kPa are shown in

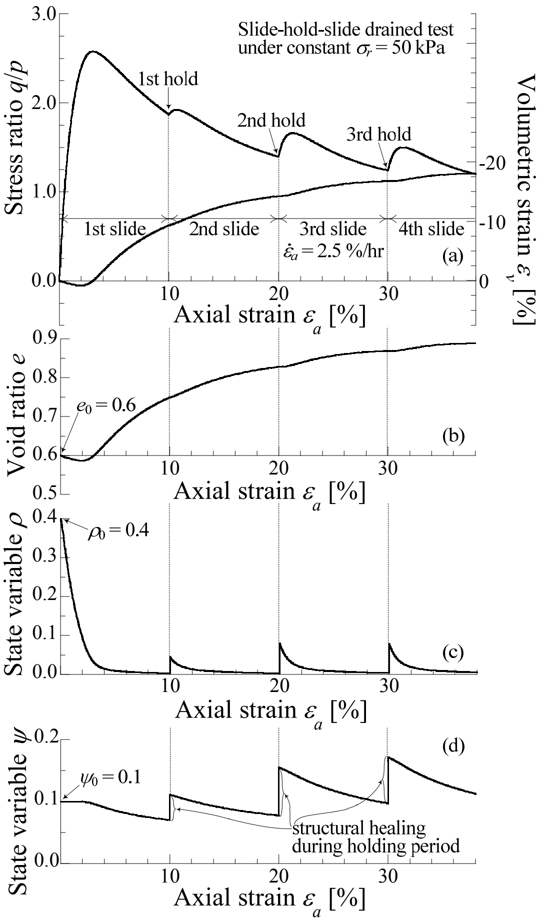

Fig. 8 Simulation results of shear-hold-shear test (variations in relation to axial strain).

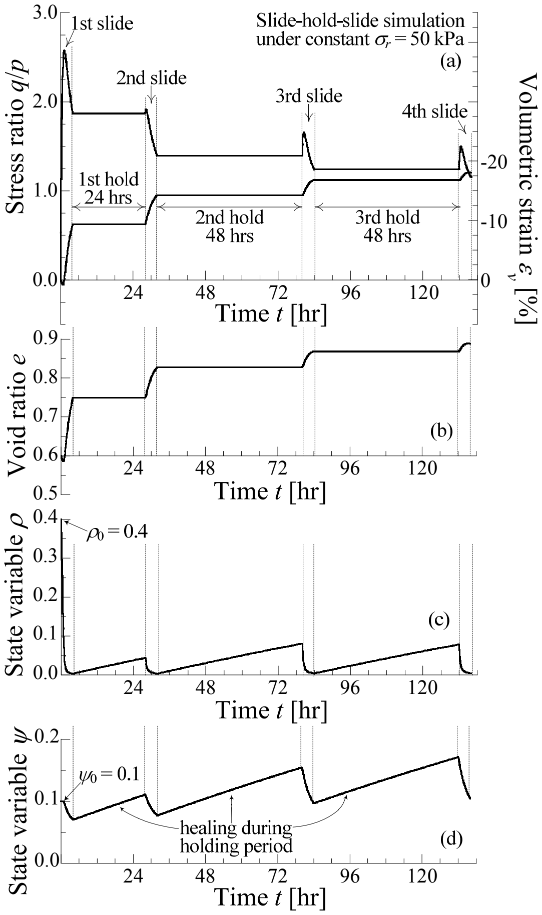

Fig. 9 Simulation results of shear-hold-shear test (variations in relation to time).

Fig. 6 and **Fig. 7**. The assumed constitutive material parameters are: λ = 0.104, κ = 0.01, $M = (q/p)_{cs}$ = 1.2, a = 5000, t_{ref} = 300; ψ_{max} = 0.6, N = 1.83 and the initial conditions are: e_0 = 0.6 and ψ_0 = 0.1. The analysis of the effect of the parameter b accounting for decaying rate with different values of b ranges from 5 to 160 and the effect of the strain rates from 4.5x10^{-4} to 4.5x10^{1}%/h were performed.

Comparing with the empirical equation of rock friction proposed by Dieterich (1979) and Ruina (1983), our proposed model has an advantage in terms of considering dilatancy which is important factor in rock behavior under great depth of the Earth.Different decaying rates of rock friction for different kinds of rock can be controlled by the material parameter b as well as strain rate value as shown in **Fig. 6**(a), **Fig. 7**(a). The larger the value of b or strain rate is, the faster the decaying of rock friction and a lower negative dilatancy during softening process are observed. As observed in **Fig.6** (a), axial strain reaches a critical value, the rock material will reach its critical state in which $(q/p)_{cs} = M$. The state variable ρ which is the distance from current void ratio to the ratio on the normal consolidation plane at the same mean stress as noted in the model section, starts from an initial value and gradually decreases to 0. The rate of structural decaying can be studied by observing the variation of state variable ψ in **Fig. 6**(d) and **Fig. 7**(d) when b or strain rate are changed. **Fig. 7**(d) shows a tendency to reach a critical state of ψ in which $d\psi$ is zero, after a critical time has been approached. In this critical state, healing and decaying given by equation (5) are likely to have the same amount of effect on rock friction.

(2) Slide-Hold-Slide Test in triaxial consolidated drained condition

Figs. 8 and **9** show the simulation results of a SHS test in triaxial consolidated drained condition with assumed constitutive material parameters as follows: λ = 0.104, κ = 0.01, $M = (q/p)_{cs}$ = 1.2, a = 5000, b = 20, t_{ref} = 300, ψ_{max} = 0.6 and N = 1.83. Initial conditions are: e_0 = 0.6 and ψ_0 = 0.1. Initial conditions are: e_0 = 0.6 and ψ_0 = 0.1. The simulation was performed at 2.5 %/hr axial strain rate, with 3 holding time periods, 24, 48 and 48h, respectively.

During the holding process in which the axial strain is kept constant, the time-dependent healing which may arise from the increase in contact area between sliding surfaces by indentation creep of asperities (e.g. Scholz and Engelder, 1976; Dieterich, 1978) can be observed by the variation of state variables ψ and ρ in (**Fig. 9**(c), **9**(d)). First, ψ increases (**Fig. 9**(d)) thanks to the effect of holding time in the evolution law of state variable in (9) and the increase of ρ is due to the upper movement of the state boundary surface as ψ increases. The longer the holding time is, the larger ψ will be come. After a very long holding time, ψ will gradually reach its maximum value ψ_{max} which is assumed the stiffest state of the rock. The decaying phenomenon is observed in the re-sliding process. First, ψ decreases (**Fig. 9**(d)) because of the larger effect of plastic strain over the effect of holding time in the evolution law of state variable in eq. (9). The decrease of ψ moves the Normal Consolidation Plane down which means the material has less stiffness. Therefore, the value of state variable ρ, will decrease, too (**Fig. 9**(c)). In addition, the deviator stress increases with a rather high stiffness, reaches a peak value and then returns again to the residual value. The magnitude of the strength recovery depends on the duration of holding process as the higher peak strength is particularly seen after a longer time period of holding process. This is consistent with Dieterich's experimental discovery of the increase of friction coefficient as a function of logarithm of time. During the later sliding process after 1st one, negative dilatancy is always observed through ε_v-ε_a relationship in (**Fig. 8**(a)). Different healing and decaying rates for various kinds of rocks can be controlled through material parameters t_{ref} and b respectively.

5. Conclusion

Our proposed model is an innovative alternative method to study earthquake's mechanism. However, our model clearly has some limitations including the difficulty of finding correct constitutive material parameters for rock and the lack of healing effect of strain rate on rock friction. Nevertheless, the proposed model may prove to be more useful than empirical constitutive equations by Dieterich and Ruina in terms of dilatancy. Our future research plan is to develop a model cover both healing and decaying effect of strain rate to rock friction as well as to estimate constitutive rock parameters based on experimental tests.

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