Parameter Determination of Multi-tank Model with Dynamically Dimensioned Search

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Abstract: Based on tank model, multi-tank model is proposed. For the calibration process with many parameters like multi-tank model, it is typically difficult to obtain optimal parameters using the existing methods. A new random optimization approach called modified dynamically dimensioned (DDS) search is introduced for parameters calibration of new model. It is based on heuristic global search and its adjustment is achieved by dynamically and randomly reducing the number of searching dimensions. Multi-tank model with 25 parameters is applied to an actual case, According to the results, good agreement between observed values and calculation results is obtained. It is clarified that this model is a helpful tool in prediction of water table and stability factor of the slope.

Key words: *multi-tank model, modified dynamically dimensioned search, optimization method, groundwater table prediction, slope stability factor*

1. Introduction

The tank model proposed by Sugawara[1,2] and others is structurally simple and useful, since it can represent a non-linear stream flow behavior. Therefore it is widely used for long-term runoff analysis. In real cases there are many kinds of tank distributions as required. Because of many parameters, it is difficult to properly identify those using observed data. In previous literatures some researches tried to find solutions for four-tank model (16 parameters) with many methods: Kobayashi and Maruyama[3] applied Powell's conjugate direction method to the problem. Watanabe[4] suggested using Newton's method. Yasunaga[5] and Hino[6] tried sequential estimation using Kalman filter. Tanakamaru[7] , Suzuki[8] tried to use genetic algorithm (GA) as an efficient search procedure. In this paper, multi- tank model is more complicate, and three series of tanks are introduced, and 25 (or more) parameters are needed to estimate from measurements by use of retrieve functions. For three series of tank distribution, using GA is very time-consuming and also the solutions are not so good. So in order to facilitate calibration process, the development of a new method is needed.

Basically, during the studies of estimating parameters, the calibration function of multi-tank model is replaced with a non-linear optimization function, for example, the most popular way is to minimize the errors between calculations and measurements. In this article some water table measurements are used as criterion function. Using genetic algorithm to find the estimation of tank model parameter, if it is regarded as an inverse problem, it is an ill-posed problem without uniqueness of the solution especially when parameters are over 20. And also it presents a significant computational burden. This is because too many dimensions will lead to distribution order increase with geometric series. In fact though one considers getting results in the limited number of model evaluations, the idea of achieving global optimality becomes unreasonable in most automatic calibration process. So for high dimension optimization problems, a better multipoint random optimization method is necessary, and here a new approach called modified dynamically dimensioned search is provided as one of such good algorithm focused on identifying good calibration results when calculation time is limited.

2. Analytical multi-tank model and its parameter determination.

The tank model is composed of one or several series of tanks with some outlets on the side and bottom in each tank. Outflow through the side outlets represents components of the total discharge due to the immediate or delayed response to the rainfall. Flow through the bottom holes means the portion of infiltrating flow and does not contribute to the surface flow directly.

In this study, in order to monitor the water table of the slope and estimate the slope stability factor during rainfall, three series of multi-tanks are introduced as shown in Fig-1; its 25 parameters generally need to be estimated using the new optimal method.

(1) Schematic figure of tank model

The studied tank model is a three series of in-line six-tank model as illustrated in Fig-1, which is used as surface flow and water table analysis model: R(t) is rain intensity (mm/day), a(i) is the coefficients of runoff from the side hole of the tank; b(i) means coefficients of seepage from the bottom hole; Z(i) represents the height of the runoff on the side of tanks(mm); and Q(i) is seepage runoff volume from the side of the tank. WL0(i) is the initial water level in corresponding tank. Then the lateral flow discharge ($Q_i(t)$) and the vertical seepage volume ($I_i(t)$) at one specific time can be evaluated by equations (1), (2), (3):

$$I(i) = b(i) \times WL_t \quad (1)$$

$$Q(i) = \begin{cases} a(i)\left(WL_t - Z(i)\right) & \text{for } WL_t \geq Z(i) \\ 0 & WL_t \leq Z(i) \end{cases} \quad (2)$$

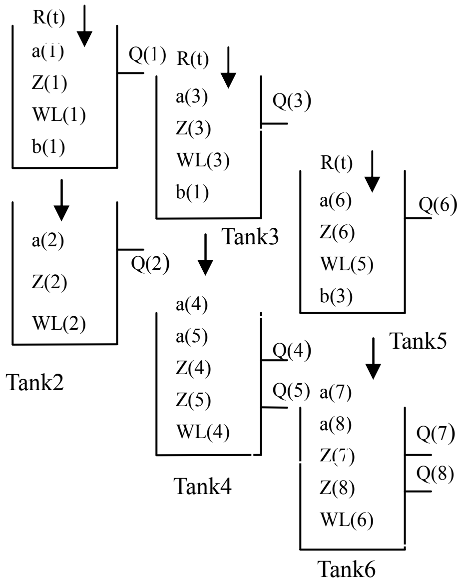

Fig-1 Schematic figure of tank model

$$\begin{aligned} dh_1/dt &= R(t) - Q_1(t) - I_1(t) \\ dh_2/dt &= I_1(t) - Q_2(t) \\ dh_3/dt &= R(t) + Q_1(t) - Q_3(t) - I_2(t) \\ dh_4/dt &= I_2(t) - Q_4(t) - Q_5(t) \\ dh_5/dt &= R(t) + Q_3(t) - Q_6(t) - I_3(t) \\ dh_6/dt &= I_3(t) + Q_4(t) + Q_5(t) - Q_7(t) - Q_8(t) \end{aligned} \quad (4)$$

Here: $WL_i(t)$ and h_i are the water tables of the corresponding tank; it is noted that the water levels of the three lower tanks (h_2, h_4 and h_6) are related to groundwater table. $GWL_i(t)$ at a specific time t is calculated by the following equations:

$$GWL_i(t) = GWL(0) + h_i^{bot}(t)/v \quad (4)$$

Where GWL(0) is the reference groundwater level; v is effective porosity of soil; $h_i^{bot}(t)$ is calculated water levels of three lower tanks (i=2,4 ,6).

(2) Definition of optimization function

In order to find appropriate solutions, the evaluation functions can be used as expressed by the following equation (5).

$$J_{XS} = \frac{1}{M} \sum_{i=1}^{M} \frac{(Q_c(i) - Q_o(i))^2}{Q_c(i)} \quad (5)$$

Here: $Q_o(i)$ is observed measurements of water table of three lower tanks; $Q_c(i)$ is calculated results; M is the number of data measurements.

(3) Definition of modified dynamically dimensioned search

The dynamically dimensioned search algorithm[9]

(DDS) is a novel and simple stochastic single-solution method, and it is based on heuristic global search algorithm that was developed for the purpose of finding good global solutions within the specified maximum function evaluation limit. In short, the algorithm searches globally at the start of the search and becomes more and more local as the number of iterations approaches the maximum allowable number of function evaluations. The adjustment from global to local search is achieved by dynamically and randomly reducing the number of dimensions in the neighborhood. The decision variables in automatic calibration are the model parameters, and the dimension being varied is the number of model parameter, which are changed to generate a new search neighborhood. Candidate solutions are created by perturbing the current solution values randomly selected dimensions only. Its perturbations magnitudes are sampled from a normal distribution N(0,1).

The DDS algorithm is unique relative to current other random optimization approaches because of the way that neighborhood is dynamically adjusted by changing the dimension of the search. But using GA, the final solutions will be different for every calculation process. The only algorithm parameter to set in DDS is the scalar neighborhood size perturbation parameter (r) that defines the random perturbation size as a fraction of the decision variable range. An initial value of the r parameter is set as 0.2, and with the calculation process going on r will reduce step by step, the minimal value of r is 0.05, which is different from Tolson[9]. This initial sampling region size is designed to allow the algorithm to escape regions around poor local minima. In the final stage because the current solution is close to final results, in order to avoid big perturbation, the value of r must decrease. And also in order to accelerate the rate of convergence, its update algorithm is also modified. The complete calculation process of modified DDS algorithm is provided as the follows:

STEP1. Define modified DDS inputs:

- Neighborhood perturbation size parameter: r(the default initial value is 0.2);
- Vectors of lower, $\mathbf{x}^{\mathbf{min}}$, and upper, $\mathbf{x}^{\mathbf{max}}$, and initial solution, $\mathbf{x}^{\mathbf{0}} = [x_1,x_2,\ldots,x_D]$.

STEP2. Set counter to 1, i=1, and evaluate objective function **F** at initial solution, $F(\mathbf{x}^{\mathbf{0}})$:

- $F_{best} = F(\mathbf{x}^{\mathbf{0}})$, and $\mathbf{x}^{\mathbf{best}} = \mathbf{x}^{\mathbf{0}}$

STEP3. Randomly select J of the D decision variables for inclusion in neighborhood, {N}.

STEP4. For j=1,…,J decision variables in {N}, perturb x_j^{best} using a standard normal random variable: N(0,1), reflecting at decision variable bounds if necessary:

STEP5. Evaluate $F(x^{new})$ and update current best solution if necessary:

- If $F(x^{new}) \leqq F^{best}$, update new best solution:
  $F^{best} = F(x^{new})$ and $x^{best} = x^{new}$
- If $F(x^{new}) > F^{best}$ and
  $\exp(-(F^{new} - F^{best})/f(j)) >$ random(Pn)
  $F^{best} = F(x^{new})$ and $x^{best} = x^{new}$

STEP6. Update iteration count, i=i+1, and check stopping criterion:

- If i= Maxiter, STOP, print output (e.g: F^{best} and x^{best})
- Else go to STEP3

The only parameter 'r' is defined as the following lines: P_n decreases with the increase of the number of function evaluations (Maxiter is maximum number of function evaluation; 'i' is the current calculation step):

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Pn=1.0-dlog(dfloat(i))/dlog(dfloat(Maxiter)
if (0.4<Pn.)   r_val=0.20
if(0.3<Pn.and.Pn<0.4)    r_val=0.18
if (0.2<Pn.and.Pn<0.3)    r_val=0.15
if (0.1<Pn.and.Pn<0.2)    r_val=0.08
if (Pn<0.1)   Pn=0.1;     r_val=0.05
```

3. Case studies on the actual slope using multi-tank model

The multi- tank model is applied to the slope along Japanese national road No.12 to simulate fluctuations of groundwater table induced by rainfall.

(1) Outline of the slope

From the boring survey results, it is revealed that in the slope, weathered rock is about 3 to 10 meter thick. With the history of collapses, it was regarded that it is urgent to determine its water table fluctuations and evaluate its stability. As illustrated in Fig-2, it is the configuration of multi-tank model in the slope: Top tank (tank2) is assumed on the top hill(x=325m); middle tank(tank4) is at the center (x=165m) and bottom tank

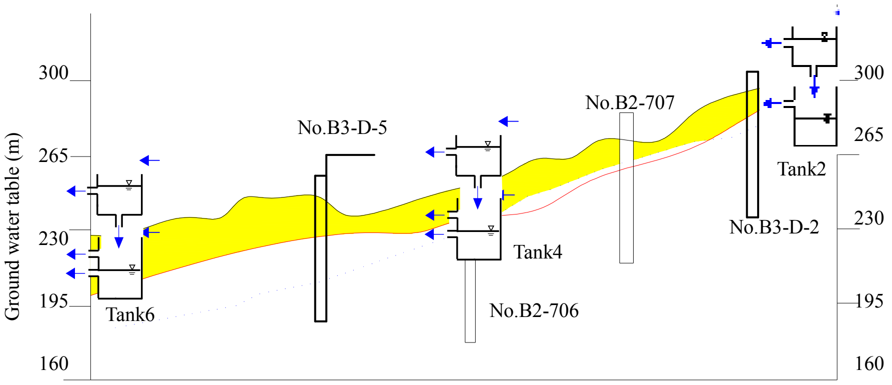

Fig.2 Configuration of multi-tank model in the slope

(tank6) lies on the lowest part of the slope (x=7m), and the porosities near the three parts are 0.09, 0.18 and 0.15 respectively. As aforementioned, in order to evaluate tank parameters, historical data of rainfall and ground water table are required. In this case, there are four observation borings along this cut-slope: boring No.B3-D-2(x=310 m), B2-707(x=225m), B2-706(x=165m), and B3-D-5(x=98m) are drilled to monitor the ground water table at the four locations. Rainfall intensities and ground water tables of 185 days from 1994-5-27 have been recorded, and the initial measurements of 100 days are used to parameters optimization; the rest measurements of 85 days are used to check model's validity.

(2) Analytical conditions

The parameters of multi- tank model can be identified by the reproduction of observed hydrographs assuming that basic watershed characteristics remain unchanged during the observation of events. In the runoff analysis, because the studied slope area is fairly small in size, the travel time is considered to be relatively short. The data used for the analysis are the 185 daily measurements of precipitation (shown as Fig-3 between 1994-5-27 and 1994-11-27) and water tables averaged over catchment of the slope. By using multi-tank model, water tables can be obtained; and using optimization method of modified dynamically dimensioned search algorithm, parameters are retrieved. Here there are totally 25 parameters; their lower and upper bounds of the search for parameters are listed in Table 1. The bounds of search are set based on the result of an application of the three-series tank model. The maximum number of function evaluation (Maxiter) is 4000, and it is considered large enough for practical purposes.

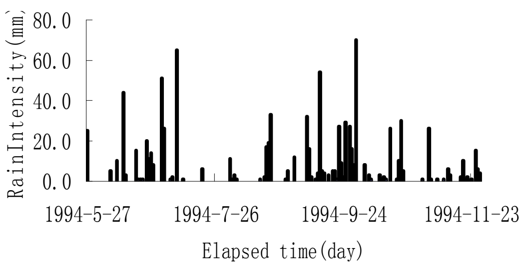

Fig-3. Daily rainfall intensities

Table1. Parameter bounds

0.0<a(1)<0.4	5.0<Z(1)<200	0.0<b(1)<0.4
0.0<a(2)<0.4,	5.0<Z(2)<300	0.0<b(2)<0.4
0.0<a(3)<0.4	5.0<Z(3)<200	0.0<b(3)<0.4
0.0<a(4)<0.4	5.0<Z(4)<300	1< WL0 (1)<80
0.0<a(5)<0.4	5.0<Z(5)<200	10< WL0 (2)<300
0.0<a(6)<0.4	5.0<Z(6)<200	1< WL0 (3)<80
0.0<a(7)<0.4	5.0<Z(7)<300	10< WL0 (4)<300
0.0<a(8)<0.4	5.0<Z(8)<200	1 < WL0 (5)<80
	Unit/ mm	10< WL0 (6)<450

(3) Analytical results

The analysis results of by dynamically dimensioned search are shown in Table 2. The calculation error J_{XS} is represented by 0.1412.

Objective function values plotted against the number of function evaluation is shown in Fig-4 From the figure, it is clear that at the beginning objective function values are over 15000, but with the calculation process going on, they go down quickly, at the same time, the perturbation is very big at the initial stage, and then become smaller and smaller gradually, finally the current solution is close to final results; the objective function values decrease oscillatorily with some perturbations, which shows modified DDS can escape from the poor local minima.

Differences of water table between observed and estimated errors are shown in Fig-5, from which it can be concluded that good agreement between the observed values (Boring B2-706) and the calculation results (Tank4) is obtained, especially during the prediction phase the agreements are very good too. Fig-6 shows measurements of several borings and calibration results of at Tank2; compared with peak value of Tank2 and Tank4, the lagged effect of groundwater table peak of No. B2-707 and B3-D-5 is also reproduced. By use of spline interpolation method, the virtual measured water tables can be estimated quickly. As shown in Fig-6, its interpolation curve of Tank2 is similar to the calculated results there. So it can be concluded that during the rainfall, multi-tank model proposed in this paper can effectively and quickly simulate the transport behavior of ground water table in the slope.

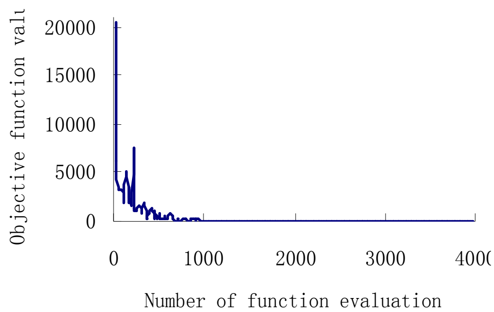

Fig-4. Objective function value plotted against the number of function evaluations

Table2 Optimization solutions

a(1)= 0.341	Z(1)= 63.429	b(1)= 0.399
a(2)= 0.041	Z(2)= 101.88	b(2)= 0.371
a(3)= 0.353	Z(3)= 50.010	b(3)= 0.396
a(4)= 0.387	Z(4)= 297.56	WL0(1)= 1.598
a(5)= 0.046	Z(5)= 185.14	WL0(2)= 118.009
a(6)= 0.365	Z(6)= 75.246	WL0(3)= 14.436
a(7)= 0.035	Z(7)= 297.69	WL0(4)= 245.226
a(8)= 0.035	Z(8)= 53.541	WL0(5)= 7.3401
	Unit/mm	WL0(6)= 134.260

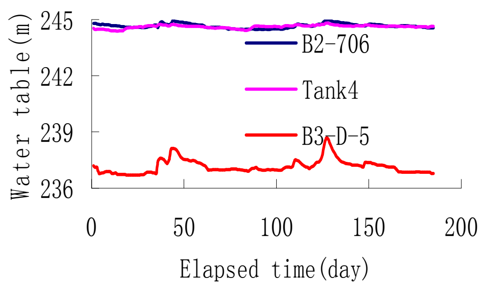

Fig-5. Measurements and calibration results

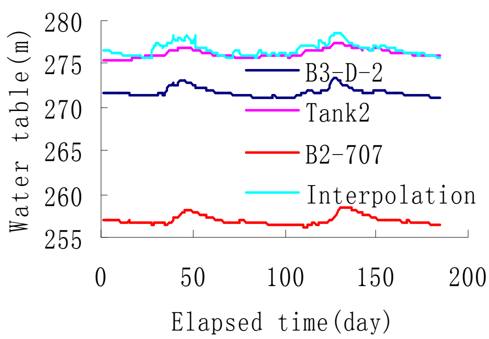

Fig-6. Measurements and calibration results

After getting the water tables of the several locations during the rainfall, the instantaneous groundwater lines can be estimated quickly by spline interpolation method. Then the slope stability factor is calculated with the division methods commonly. In the division method, there are many methods such as Fellenius, Bishop, Janbu and Spencer. Fig-7 is the results of stability analysis of this slope using Bishop method during the analysis period: it is clear that when rain is big, its stability factor decreases sharply, which also demonstrates that rainfall has great influence on slope stability.

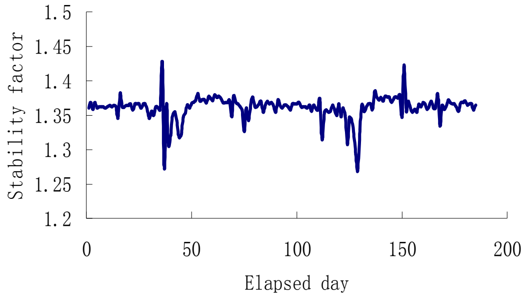

Fig-7. Slope stability factor changes during rainfall

4. Conclusions

The paper focused on the behavior of rainfall infiltration process and aimed to develop a simple and quick analytical tool to evaluate underground water table and slope stability factor. The insights gained through this study are summarized as the following:

According to water balance (tracking flows of water into and out of the particular hydrologic system of interest), a multi-connected tank model that can reproduce the rainwater movement behavior during rainfall infiltration process was developed. A new stochastic single-solution method called modified dynamically dimensioned search was adopted to identify optimal solutions. The new method could find relatively good solutions in a shorter time. Multi-tank model was applied to the actual slope. Its consistency with field data was confirmed, and its practicability was proved.

Although multi-tank model has an advantage of predicting water table fluctuations, it can not give information of infiltration process in the unsaturated zone. The authors are planning to work on the development of unsaturated tank model, so the stability for shallow landslide can be assessed during by rainfall. Combined with a new accurate rain gauge, the methodology will be evolved further into an assessment system for correctly predicting the hazards of rainfall that may lead to slope failure.

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