

# NEW PRACTICAL KEY BLOCK ANALYSIS METHOD CONSIDERING FINITE DISCONTINUITY PERSISTENCE IN TUNNELS 有限長さの不連続面を考慮したトンネルのキーブロック解析手法

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Block theory is based on the assumption of infinite discontinuity persistence, therefore cannot handle a complex concave shaped block. This paper proposes a new practical key block analysis method considering finite discontinuity persistence in tunnels. The new analysis method can handle concave and convex shaped blocks. To demonstrate the validity and usefulness of the new practical key block analysis method considering finite discontinuity persistence in tunnels, two analysis example problems for a convex shaped block and a concave shaped block were performed in this study. The comparisons and investigations with the analysis results have confirmed the validity and usefulness of this proposed new practical key block analysis method considering finite discontinuity persistence in tunnels.

Key Words: Key Block Analysis Method, Finite Persistence, Rock Discontinuity, Tunnel, Concave and Convex Shaped Blocks

## 1. Introduction

The properties of rock masses are important factors relevant to the design and construction of tunnels. Rock masses in nature contain numerous discontinuities such as faults, joints, fractures, bedding planes, seams, cracks, schistosity, fissures and cleavages. The behavior of civil structures in hard rocks, therefore, is mainly controlled by numerous discontinuities (Hwang et al, 2002; Ohnishi, 2002; Hwang, 2003; Hwang et al, 2004). The tunnel support design has been mostly empirical. The typical support design pattern is based on the rock/soil types, or based on the rock mass classification. However, both stress and rock structure induced failures should be considered in the design of rock support for tunnel design.

As for the assessment of the rock structure induced failures, the so-called block theory was suggested by Goodman and Shi (1985). Excavations in discontinuous rock masses are frequently affected by key blocks, which are critical blocks of rock bounded by discontinuities and excavation surfaces. Block theory is a geometrically based set of techniques that determine where dangerous blocks can exist in a rock mass intersecting by variously oriented discontinuities in three dimensions. Block theory is a useful method to determine the stability of rock blocks that were created by the intersection of discontinuities. However, the block theory is based on some assumptions which limit its usefulness. In block theory, joint surfaces are assumed to extend entirely through the volume of interest; that is, no discontinuities will terminate within the region of a key block (Ohnishi et al, 1985). The block theory is based on the assumption of infinite persistent discontinuities, and does not consider the effects of finite discontinuity persistence, therefore cannot handle a complex concave shaped block (Fig.1). The rock block

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should be divided into convex sub-blocks. In practice, it is necessary to consider the finite persistence of discontinuities to attempt applying the block theory to successive excavations.

A new practical key block analysis method considering finite discontinuity persistence in tunnels is proposed in this paper. The new analysis method can handle concave and convex shaped blocks. The new practical key block analysis method considering discontinuity persistence consists of the following major steps:

- ① 3D discontinuous rock mass modeling,
- ② Identification of 2D loop,
- ③ Identification of 3D loop,
- ④ Finalization of block shape and volume,
- ⑤ Removability and stability analysis of rock blocks.

In order to demonstrate the effectiveness and usefulness of the new practical key block analysis method considering finite discontinuity persistence in tunnels, two analysis example problems are carried out.

## 2. 3D Discontinuous Rock Mass Modeling

### (1) Discontinuity Disc Model

To build a 3D model considering finite discontinuity persistence, the problem concerning geometric shape and spatial extent of discontinuities must be addressed. Through the analysis of trace data and examination of discontinuity surfaces, several studies (Pollard and Aydin, 1988; Priest, 1993; Ohnishi et al, 1994) have demonstrated that discontinuities are likely to be roughly elliptical or circular. The fundamental feature of the discontinuity disc model is the assumption of circular discontinuity shapes (Pahl, 1981; Ohnishi et al, 1994) as shown in Fig.2.

The size of circular discontinuities is defined completely by a single parameter, the discontinuity radius. Discontinuity radius may be defined deterministically, as a constant for all discontinuities, or stochastically by a distribution of radii. The best and most widely adopted sampling strategy for determining discontinuity size is based on the measurement of the lengths of the traces produced where the discontinuities intersect a planar face (Priest, 1993).

### (2) Analysis Region Modeling

The model region of this study is a closed domain with a certain number of faces. Model region may be of any shape closed by polygons. The excavation faces may be convex or concave polygonal faces. All excavation faces should form a closed domain of target rock mass for the analysis. Fig.3 shows an example of the analysis region modeling for an underground cavern.



Fig.1 An Example of Concave Shaped Blocks in Tunnel

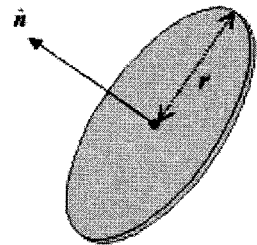


Fig.2 Discontinuity Disc Model

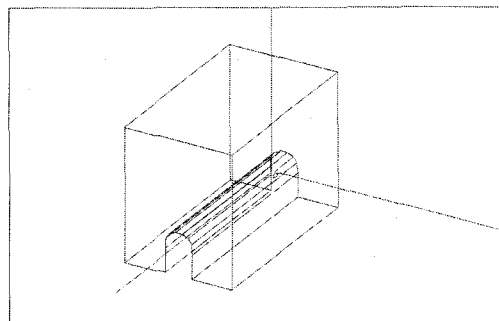


Fig.3 Example of the Analysis Region Modeling for an Underground Cavern

### 3. Identification of 2D Loop

In a discontinuity system, not all discontinuities are connected; some discontinuities do not intersect other discontinuities, and some discontinuities intersect very few discontinuities (Yu, 2000). Before the rock block identification stage, unconnected discontinuities should be identified and eliminated. A discontinuity is referred to as connected only when it plays a part in block formation. A connected discontinuity must feature the two following characteristics: (i) it is connected to at least 3 discontinuities including excavation faces, (ii) within the planar disc of the considered discontinuity, the intersections between the considered discontinuity and other connected discontinuities or excavation faces must form at least one connected loop.

By checking every discontinuity against the two characteristics mentioned above, all unconnected discontinuities can be identified and eliminated. It should be noted that this is an iteration procedure.

### 4. Identification of 3D Loop

After the elimination of unconnected discontinuities, intersections among discontinuities will be calculated, then the discontinuities which form a block will be identified. A face in 3D closed region forms when other discontinuities cut the region. 2D line loop should be established at the surfaces of the three-dimensional region. When this criterion is satisfied at all faces, one or more blocks are formed.

One individual 2D loop may be represented by its boundary nodes arranged in clockwise direction or counter-clockwise direction. The 3D loop may also be represented by its boundary 2D loops arranged in clockwise direction or counter-clockwise direction. Consequently, a block is represented by a number of 3D loops; a 3D loop is represented by a number of 2D loops; a 2D loop is represented by the nodes on its boundaries; and a node is described by 3D coordinates. All the above identification processes will be performed automatically by computer.

### 5. Finalization of Block Shape and Volume

The volume of blocks will be determined by a simplex integration method. Simplex integration method is an accurate solution on n-dimensional domains with any shape. Simplex integration is based on the topology (Shi, 2001). The simplex has the most simple shape in 0, 1, 2, 3,..., n dimensional space as shown in Fig.4. Different from the ordinary integration, the simplex integration has only the simplex as the integral domain. The simplex also has positive and negative orientations. The positive and negative orientations define as positive and negative volumes respectively.

The coordinates of the vertices  $V_0, V_1, V_2, V_3$  on any 3D simplex are supposed as  $(x_0, y_0, z_0), (x_1, y_1, z_1), (x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  respectively. Therefore, the volume of the 3D simplex  $V_0V_1V_2V_3$  is

$$V = \frac{1}{3!} \begin{vmatrix} 1 & x_0 & y_0 & z_0 \\ 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \end{vmatrix} \quad (1)$$

The volume of simplex  $V_1V_0V_2V_3$  is the negative volume of simplex  $V_0V_1V_2V_3$ .

The integration formulas on the domain of 3D simplex  $V_0V_1V_2V_3$  (Fig.4) with non-zero volume can be represented as follows (Shi, 2001). Supposing  $(x_0, y_0, z_0), (x_1, y_1, z_1), (x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  are the coordinates of the vertices  $V_0, V_1, V_2$  and  $V_3$  respectively.

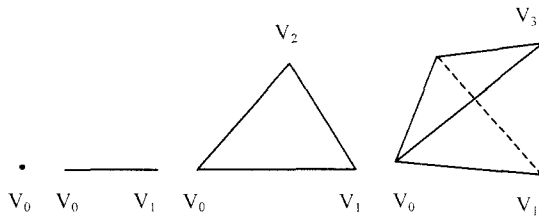


Fig.4 0, 1, 2, 3 Dimensional Simplex

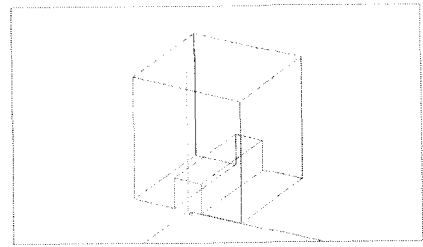


Fig.5 Geometric Model of Square Tunnel

Set

$$J = \begin{vmatrix} 1 & x_0 & y_0 & z_0 \\ 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \end{vmatrix} \quad (2)$$

## 6. Examples of Application to Tunnel

The removability and stability analyses of rock blocks formed by the identification method using simplex integration method are performed. To demonstrate the effectiveness and usefulness of the new practical key block analysis method considering finite discontinuity persistence in tunnels, two analysis example problems were carried out. The example problems for a convex shaped block and a concave shaped block in tunnels are presented in this chapter. Fig.5 shows the simplified three-dimensional geometric model of the square tunnel used in this study. The height and width of the square tunnel are 25m.

### (1) Example of Convex Shaped Block in Tunnel

The first application example is a tunnel problem with a convex shaped block. Table1 shows the dip directions and dips of discontinuity planes for this example problem.

In this study, the key block analysis considered finite discontinuity persistence and the key block analysis based on the assumption of infinite persistent discontinuities were carried out. Then, the analytical results which considered finite discontinuity persistence were compared and investigated with the result based on the assumption of infinite persistent discontinuities. Table2 shows the number of key blocks detected in this example problem. One key block was detected in the tunnel of this example. The number of key blocks which considered finite discontinuity persistence in this study was exactly the same as that based on the assumption of infinite persistent discontinuities as shown in Table2. In this study, the key block which considered finite discontinuity persistence is compared with that based on infinite persistent discontinuities. The key block which considered finite discontinuity persistence was the same as that based on infinite persistent discontinuities. The comparisons and investigations with the analytical results have confirmed the validity and effectiveness and usefulness of this new practical key block analysis method considering finite discontinuity persistence in tunnels.

Table1 Discontinuity Orientation Data for the Example in Tunnel

Discontinuity Plane	Dip Directions (°)	Dips (°)
1	0	45
2	90	45
3	180	45
4	270	45

Table2 Number of Key Blocks

No.	Analysis Based on Infinite Discontinuity Persistence	Analysis Considered Finite Discontinuity Persistence
No. of Key Block	1	1

Table3 Discontinuity Orientation Data for the Example in Tunnel

Discontinuity Plane	Dip Directions (°)	Dips (°)
1	0	45
2	90	60
3	180	45
4	270	30
5	270	60

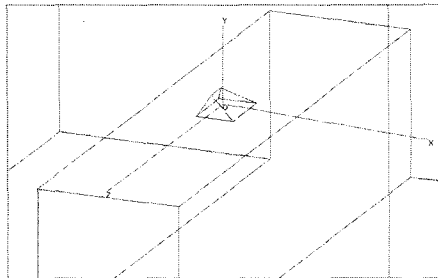


Fig.6 Key Block Analysis  
Based on Infinite Discontinuity Persistence

Table4 Number of Key Blocks

No.	Analysis Based on Infinite Discontinuity Persistence	Analysis Considered Finite Discontinuity Persistence
No. of Key Block	1	0

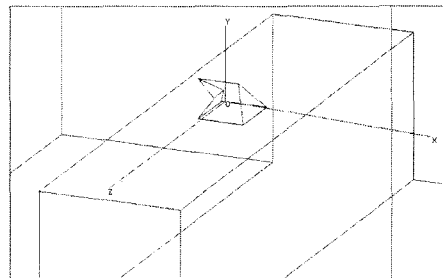


Fig.7 Key Block Analysis  
Considered Finite Discontinuity Persistence

## (2) Example of Concave Shaped Block in Tunnel

The next application example is a tunnel problem with a concave shaped block. Table3 shows the dip directions and dips of discontinuity planes for this example problem.

In this study, the key block analysis considered finite discontinuity persistence and the key block analysis based on the assumption of infinite persistent discontinuities were carried out. Then, the analytical results which considered finite discontinuity persistence were compared and investigated with the result based on the assumption of infinite persistent discontinuities. Table4 shows the number of key blocks detected in this example problem. In the key block analysis based on the assumption of infinite persistent discontinuities, one key block was detected. However, no key block was detected in the key block analysis considered finite discontinuity persistence. The developed new practical key block analysis method considering finite discontinuity persistence provides a practical, economic, efficient and useful support method for determining the stability analyses of rock blocks. In this study, the key block which considered finite discontinuity persistence is compared with that based on infinite persistent discontinuities as shown in Fig.6 and Fig.7. Fig.6 shows the key block analysis based on infinite persistent discontinuities. Fig.7 shows the key block analysis which considered finite discontinuity persistence. The comparisons and investigations with the analytical results have confirmed the effectiveness and usefulness of this proposed practical key block analysis method considering finite discontinuity persistence in tunnels.

## 7. Conclusions

Previous analyses for key blocks have been based on the assumption of infinite persistent discontinuities. This paper has been proposed a new practical key block analysis method considering finite discontinuity persistence in tunnels. The new practical key block analysis method considering discontinuity persistence consists of the following major steps: ① 3D discontinuous rock mass modeling, ② Identification of 2D loop, ③ Identification of 3D loop, ④ Finalization of block shape

and volume, and ⑤ Removability and stability analysis of rock blocks. The new practical key block analysis method considering discontinuity persistence is briefly described below.

Discontinuities are represented by the discontinuity disc model. Unconnected discontinuities are eliminated from the discontinuity system, the connectivity of discontinuities will be calculated, and then the discontinuities that form a block will be identified. The volume of blocks will be determined by a simplex integration method. The removability and stability analyses of rock blocks formed by the three-dimensional rock block identification method are performed.

Using the above-mentioned method, the new practical key block analysis method developed by the authors can handle concave and convex shaped blocks. To demonstrate the validity and usefulness of the new practical key block analysis method considering finite discontinuity persistence in tunnels, two analysis example problems for a convex shaped block and a concave shaped block was carried out. The comparisons and investigations with the analytical results have confirmed the validity and usefulness of this proposed new practical key block analysis method in tunnels.

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