

Evaluation of the Flow Pump and Constant Head Techniques When Testing Low-Permeability Geotechnical Materials

難透水性岩盤の透水試験に際するフローポンプ法及び定水位法の比較・評価

T. Esaki*, M. Zhang**, M. Takahashi***, and K. Sakai*
江崎 哲朗, 張 銘, 高橋 学, 坂井 健太郎

石油、圧縮空気の貯蔵や各種廃棄物の処理・処分など様々な地下利用において難透水性の岩や土の水理学的特性を評価すべき機会が多くなってきている。本研究は 10^{-10} m/s以下の難透水性の岩や土を対象とする通常の定水位試験とフローポンプ試験について比較検討する。まず、難透水性の場合の定水位法の問題点を指摘した。次にこの2つの試験方法の正確な解析理論を誘導し、実験の開始から試験体内の水頭のみならず動水勾配の経時的変化についてシミュレーションを行った。そして、2つの方法における試験時間、水頭、動水勾配及び透水係数の値の信頼性などについて比較を行った。その結果、新しく開発した解析法を用いるフローポンプ試験法は、計測時間も短く現場に近い低動水勾配状態で低い透水係数を正確に評価できることを明らかにした。また、本論文で確立した両試験法の厳密解析理論は、それぞれの方法を用いた難透水性材料の透水試験の計画・試験結果に対する適切な評価にも適用できると考えられる。

Keywords: flow pump, constant head, laboratory test, comparison, low-permeability, geotechnical materials

1. Introduction

Interest in accurate measurement of low-permeability (lower than 10^{-10} m/s) of intact rocks and fine grained soil materials has increased significantly in recent years. This growing interest is mainly due to the need to predict accurately, for example, the leakage of stored materials from underground facilities and the subsurface migration of hazardous wastes around disposal repositories accompanying the movement of underground water. Although geotechnical materials with such low permeability would be considered impervious in conventional engineering projects, it is very important to obtain accurately the permeability values of the materials, even when they are extremely low, for effectively designing the facilities associated with many kinds of underground utilization coming into vogue in recent years, such as LNG storage, CAES and geological disposal of hazardous wastes, etc.

Conventionally, there are two laboratory testing techniques, the constant head and the falling head methods, which have been used to obtain permeability of geotechnical materials. The constant head method is principally suitable for testing materials with relatively-high permeability (10^{-4} to 10^{-5} m/s) such as sands, while the falling head method is suitable for materials with relatively-low permeability (10^{-5} to 10^{-8} m/s) such as sandstones, clays and silts¹⁾. The two methods, however, are still being used to measure even-lower permeability ($<10^{-8}$ m/s) for the simplicity and low costs of experimental equipment. The permeability of the specimen is evaluated by means of the formula for steady state flow in the constant head test, based on the measurements of volumetric flow rate, even when the falling head method is used for testing extremely-low permeability materials because the head drops during the test are very small. Limited by the volume measurement techniques conventionally used in geotechnical laboratories with the maximum practical resolution of about 10^{-3} ml²⁾, accurate measurements of low-permeability can be achieved only if the tests last for a long period of time, from several tens of hours to several weeks depending on the permeability of materials tested³⁾⁴⁾, or the imposed gradients are very high, up to even more than 1000⁵⁾. Both the prolonged testing period of time and the higher

* Institute of Environmental Systems, Faculty of Engineering, Kyushu University

** Faculty of Engineering, Kyushu University; Now at Geological Survey of Japan

*** Geological Survey of Japan

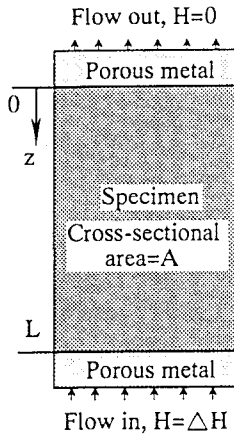
hydraulic gradient will usually cause significant error of permeability measurements. The former may cause errors resulting from temperature fluctuation and bacterial growth, the latter may cause errors from the difference between the hydraulic gradients in situ and those used in the test (correspondingly the laminar flow and turbulent flow) and changes in effective stresses within the specimen. The lower the permeability, the longer the time required for establishing the steady state flow in the specimen. Consequently, the hydraulic gradients are certainly different along the length of specimen and continuously change their values from the start of the test. These important factors have not previously been considered when evaluating very-low permeability with the conventional constant head and falling head permeability test methods because an exact theoretical analysis of the methods interpreting the initial non-steady flow in the specimen had not been available. Therefore, the permeability values of nearly-impermeable materials obtained from the conventional tests assuming the steady flow in the specimen are sometimes questionable. An additional problem for the conventional methods is that they can not provide the value of specific storage of specimen, another important hydraulic parameter associated with the fluid flow in porous materials.

Fundamentally, there are also two techniques, the Transient Pulse Technique⁶⁾ and the Flow Pump Technique⁷⁾, which have been developed to evaluate the low permeability of rocks and soils respectively. The two methods evaluate the permeability from the measurements of transient variation of pressure (or differential pressure), a physical quantity that can be measured with much higher precision and resolution compared with those of volumetric flow rate measurement. However, not only the Transient Pulse Technique but also the Flow Pump Technique have not been widely used because of the relative-high costs of equipments and the unnecessary of evaluating very-low permeability in the engineering projects conventionally encountered, and only in the past few years have the methods received wider attention and used for testing rocks and soils with extremely-low permeability⁸⁾⁹⁾. As a special type of falling head test, the Transient Pulse Technique imposes relatively-high hydraulic head on the specimen end on the side of upstream reservoir at the beginning of an experiment. This may also induce extremely-high hydraulic gradients (correspondingly turbulent flow) near the end where the hydraulic head is imposed. In the flow pump technique, a low flow rate is controlled to infuse into (or withdraw from) one of the specimen ends, and cause gradual increment in hydraulic head (consequently the hydraulic gradient) across the specimen. Turbulent flow in the test can be avoided provided that the flow rate used is appropriately low. When using the flow pump method to measure relatively-high permeability, the permeability values are evaluated by means of the well-known Darcy's law for steady flow in saturated porous materials, as this was done by Aiban and Znidarcic¹⁰⁾. In their study, apparent advantages and disadvantages of the flow pump and constant head methods were also given in detail. When testing low-permeability materials and/or with relatively-high specific storage, the flow-pump method also requires long period of time to reach the steady state⁹⁾. For determining the permeability during early testing time, theoretical analysis of the transient pressure response from a constant flow-rate permeability test has been developed by Morin and Olsen¹¹⁾. The governing equation used for the analysis is equivalent to that used by Terzaghi¹²⁾ for describing one-dimensional consolidation of saturated soils. However, their approach does not consider the storage capacity of the flow pump equipment and is therefore reasonably accurate only when the storage capacity of flow pump system, i.e., the equipment compliance, is negligible compared with that of the specimen.

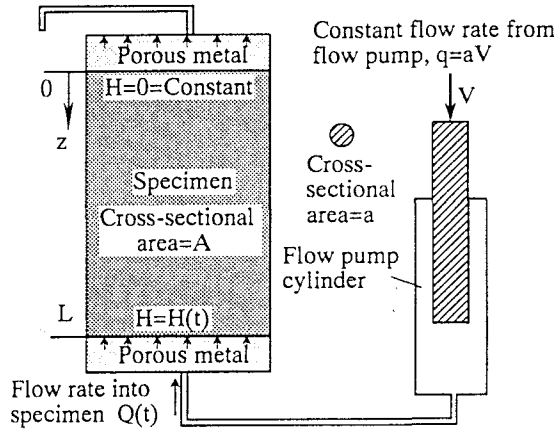
This paper presents exact theoretical analyses of constant head and flow pump permeability tests in which both the permeability and specific storage of specimen are considered. In addition, the storage capacity of flow pump system is considered for the flow pump test. The new analyses are further extended so that they can be used to simulate accurately the changes in hydraulic gradient in the specimen during the test. Through a simulation example, this paper also evaluates the constant head and flow pump permeability test methods through a theoretical comparison. The comparison is emphasized on the testing period of time, hydraulic gradients imposed on or induced across the specimen and the reliability of the permeability obtained. The objective specimens in the theoretical study are those with extremely-low permeability less than 10^{-10} m/s. Geotechnical materials with such low permeability are desirable for many kinds of new uses of underground space as described earlier.

2. Mathematical Models

The schematic diagrams and the boundary conditions associated with the constant-head and flow pump permeability tests are depicted in Fig. 1 a) and b), respectively¹⁾¹³⁾.



a) Constant head permeability test



b) Flow pump permeability test

Fig.1 Schematic diagrams and the boundary conditions associated with the constant head and flow pump permeability tests

The equation of one-dimensional flow of a slightly compressible fluid in a saturated, porous medium is¹⁴⁾

$$\frac{\partial^2 H}{\partial z^2} - \frac{S_s}{K} \cdot \frac{\partial H}{\partial t} = 0 \quad (1)$$

where

H = the hydraulic head (L),

z = vertical distance along the specimen (L),

S_s = specific storage (L^{-1}),

K = hydraulic conductivity (LT^{-1}),

t = time (T).

This equation has also been used to interpret the constant-head and flow pump permeability tests¹¹⁾¹⁵⁾.

For the constant-head permeability test, the initial and boundary conditions are

$$t = 0, \quad H = 0 \quad \text{at} \quad 0 < z \leq L \quad (2)$$

$$t > 0, \quad H = 0 \quad \text{at} \quad z = 0 \quad (3)$$

$$t > 0, \quad H = \Delta H \quad \text{at} \quad z = L \quad (4)$$

For the flow pump permeability test, the initial condition is same as that expressed by Eq 2. The boundary conditions are

$$t > 0 \quad H = 0 \quad \text{at} \quad z = 0 \quad (5)$$

$$t > 0 \quad \frac{\partial H}{\partial z} = \frac{1}{KA} \left(q - C_e \frac{\partial H}{\partial t} \right) \quad \text{at} \quad z = L \quad (6)$$

Where

L = the length of the specimen (L),

ΔH = the constant head (L),

A = cross-sectional area of specimen (L^2),

q = flow rate of flow pump (L^3/T),

C_e = storage capacity of the flow pump system (L^2), i.e., the change in volume of the permeating fluid in the flow pump system per unit change in hydraulic head.

3. Analytical Solutions

Theoretical expression of the constant head permeability test can be obtained from the solution of Eq 1 together with the initial and boundary conditions Eq 2, Eq 3 and Eq 4:

$$H(z, t) = \Delta H \left\{ \frac{z}{L} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} \cdot \sin \frac{n\pi z}{L} \cdot \exp \left(-\frac{K}{S_s} \cdot \frac{n^2 \pi^2}{L^2} t \right) \right\} \quad (7)$$

The hydraulic gradient distribution $i(z,t)$ in the specimen during the constant head permeability test can be obtained by differentiating the above Eq 7 with respect to the variable z :

$$i(z,t) = \Delta H \left\{ \frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} \cos(n\pi) \cdot \cos\left(\frac{n\pi z}{L}\right) \cdot \exp\left(-\frac{K}{S_s} \cdot \frac{n^2 \pi^2}{L^2} t\right) \right\} \quad (8)$$

Correspondingly, the flow rates at $z=0$ and $z=L$ can be obtained as follows:

$$q(0,t) = K \cdot A \cdot \Delta H \left\{ \frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} (-1)^n \cdot \exp\left(-\frac{K}{S_s} \cdot \frac{n^2 \pi^2}{L^2} t\right) \right\} \quad (9)$$

$$q(L,t) = K \cdot A \cdot \Delta H \left\{ \frac{1}{L} + \frac{2}{L} \sum_{n=1}^{\infty} \exp\left(-\frac{K}{S_s} \cdot \frac{n^2 \pi^2}{L^2} t\right) \right\} \quad (10)$$

Theoretical expression of the flow pump permeability test can be obtained from the solution of Eq 1 together with the initial and boundary conditions Eq 2, Eq 5 and Eq 6⁽¹³⁾¹⁶⁾:

$$H(z,t) = \frac{qL}{AK} \left\{ \frac{z}{L} - 2 \sum_{n=0}^{\infty} \frac{\exp\left(-\frac{K}{S_s} \beta_n^2 t\right) \sin(\beta_n z)}{L \delta \beta_n \cos(\beta_n L) \left[L \left(\beta_n^2 + \frac{1}{\delta^2} \right) + \frac{1}{\delta} \right]} \right\} \quad (11)$$

in which $\delta = C_e / (AS_s)$ and, β_n are the roots of following equation

$$\tan(\beta L) = 1 / (\beta \delta) \quad (12)$$

Similarly, the hydraulic gradient distribution in the specimen during the flow pump permeability test can be obtained by differentiating Eq 11 with respect to the variable z :

$$i(z,t) = \frac{qL}{AK} \left\{ \frac{1}{L} - 2 \sum_{n=0}^{\infty} \frac{\exp\left(-\frac{K}{S_s} \beta_n^2 t\right) \cos(\beta_n z)}{L \delta \cdot \cos(\beta_n L) \left[L \left(\beta_n^2 + \frac{1}{\delta^2} \right) + \frac{1}{\delta} \right]} \right\} \quad (13)$$

4. Simulations and Discussion

A $\phi 5 \text{ cm} \times 8 \text{ cm}$ cylindrical specimen is assumed for the theoretical simulations. Correspondingly, we have A and L equal 19.63 cm^2 and 8 cm respectively. The hydraulic conductivity K and specific storage S_s of the specimen used here are assumed to be $5 \times 10^{-11} \text{ m/s}$ and 3×10^{-3} respectively. Besides, the small and constant flow rate q of flow pump, the storage capacity of flow pump system C_e and the hydraulic head imposed on the specimen in a constant head test are assumed to be $1.47 \times 10^{-6} \text{ ml/s}$, $3 \times 10^{-6} \text{ m}^3/\text{MPa}$ and 11.77 kPa ($120 \text{ cmH}_2\text{O}$, correspondingly, $i=15$) respectively. The values of parameter are assumed with reference to the experimental results previously obtained by the first two authors⁽¹³⁾. The small flow rate can be generated by either commercially available syringe pumps (e.g., Harvard Apparatus, Model 909 with minimum flow rate of $7.9 \times 10^{-5} \text{ ml/min}$) or a self-designed flow pump⁽⁹⁾.

Using the exact solutions of the constant head and flow pump permeability tests developed in the previous section together with the above assumed parameters, theoretical simulations for the two tests are performed. The hydraulic head versus time curves induced in the specimen for constant head and flow pump tests are illustrated in Fig. 2.

The testing period of time required for either test to reach the steady state will be long up to several hundreds of hours if a specimen and flow pump equipment have the parameter values on the same orders of those assumed in the simulation. Also, one can easily find from Eq 7 and Eq 11 that the lower the permeability K and/or the higher the specific storage S_s , the longer the time required to reach the steady state in the two tests. The flow pump test requires even longer period of time than the constant head test to reach the steady state because the flow pump system has the storage capacity of itself. However, this long time required for establishing the steady state in a flow pump test can be shortened to some extent by using flow pumps and permeating fluids with less compressibility and consequently with less storage capacity. In addition, the steady state in a flow pump permeability test can be easily monitored through the differential head across the entire length of the specimen because the hydraulic heads in the specimen reach their steady values at the same time and the differential head across the entire specimen can be monitored precisely by means of either a differential pressure transducer or a high-sensitivity pressure gauge. An approach of evaluating not only the permeability

and specific storage of specimen but also the storage capacity of flow pump system from the early time non-steady measurements in a flow pump permeability test has also been developed. The approach permits of obtaining the hydraulic parameters with relatively-short testing time and with relatively-low hydraulic gradients as close as possible to those in situ without the necessity of waiting for the steady state¹³⁾.

Simulated results of flow-in and flow-out rates for the constant head permeability test are plotted in Fig. 3. The flow-in rate is much greater than flow-out rate in the early phase of the test. The difference between the flow-in and flow-out rates are the volume of permeating fluid absorbed in the specimen due to the specific storage of specimen related to the compressibility of both specimen and permeating fluid. Evaluation of the permeability using either flow-in or flow-out rate measured in the early phase of a test is not feasible. The flow-in and flow-out rates reach a same value when the steady state is established at an extended testing period of time. Note that the flow rates are very small. Therefore, it is extremely difficult to measure them accurately even though the steady state is reached. As reviewed earlier in this paper, the maximum practical resolution of volumetric flow rate measurement is about 10^{-3} ml, thousands of times larger than the value of the steady flow rate. Therefore, an early time evaluation of the parameters for the constant head permeability test is impractical because accurate measurement of such small flow rates is impractical. The only way to obtain the low permeability from a constant head test is to wait until the accumulated volumes of flow-in and flow-out rates can be monitored with relatively-high precision. The specific storage of specimen, however, can not be evaluated from the steady state measurements.

Transient variation of hydraulic gradient in the specimen for the constant head and flow pump tests are provided in Fig. 4 a) and b), respectively. In the early period of constant head test, the hydraulic gradients induced in the region near to the end of specimen where the constant head is externally imposed are significantly greater than that established in the steady state. Cautions should be excised when interpreting the results of a constant head test. Evaluation of the permeability using an apparent hydraulic gradient, i. e., the differential head across the entire specimen divided by the full specimen length, may sometimes cause significant errors unless the steady state is certainly established. For example, using non-steady measurements of flow-in rate and the apparent hydraulic gradient to evaluate permeability by means of the well known Darcy's Law will obtain greater values of the permeability because the hydraulic gradient used for the calculation is smaller than the actual values. In the flow pump test, however, the hydraulic gradients along the entire length of specimen increase gradually from 0 to the values of steady state. Therefore, the hydraulic gradients in a flow pump permeability test can be well controlled to be less than a specified value. Lower hydraulic gradients can be obtained easily by using lower flow rates and increasing the ratio of A/L .

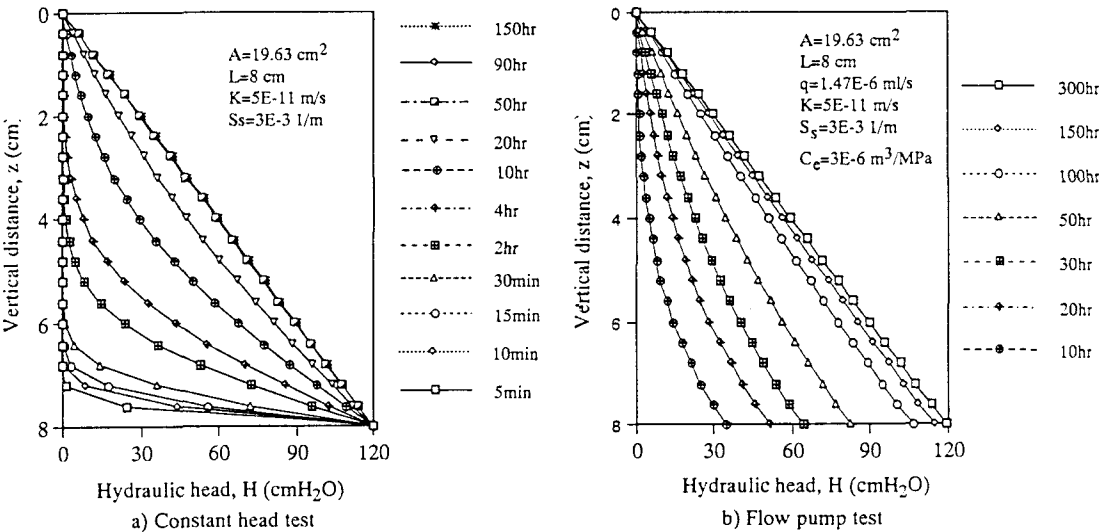


Fig.2 The hydraulic head versus time curves simulated for the constant head and flow pump tests

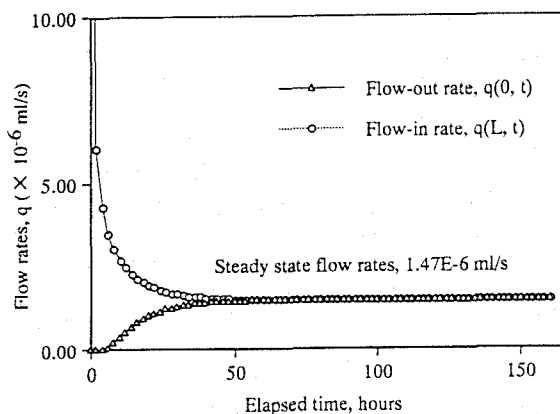


Fig.3 Simulated flow-in and flow-out rates for the constant head test

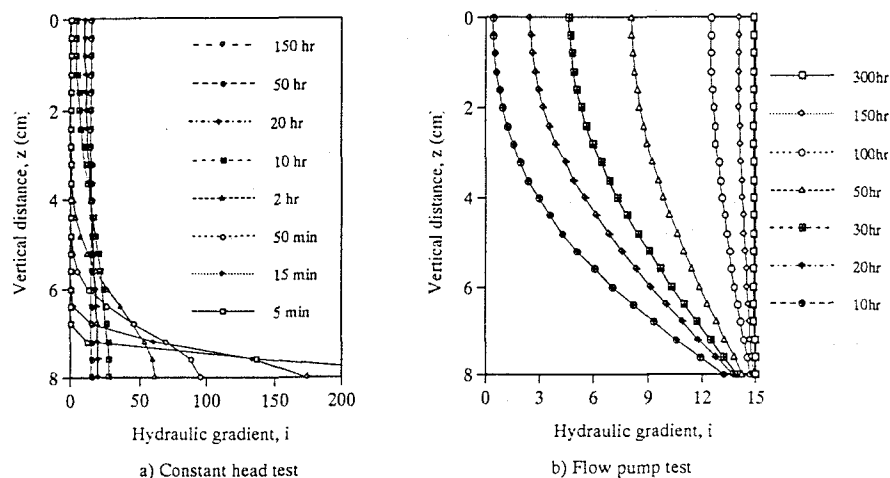


Fig.4 Transient variations of hydraulic gradients in the specimens in the constant head and flow pump tests

5. Conclusions

Exact theoretical analyses of the constant head and flow pump permeability tests are derived and extended to those permit of simulating accurately the changes in hydraulic gradient from the start of test in specimen. Through a simulation example, the new analyses are also applied to evaluate the two methods when testing low-permeability materials. Major conclusions drawn from this study are as follows:

1. When testing rocks or soils with relatively-low permeability and/or relatively-high specific storage, both constant-head and flow pump techniques require relatively-long response times to reach the steady state. However, the flow pump method permits to obtain not only the extremely-low permeability but also the specific storage of specimen much more rapidly than the constant head technique. This is because it is much easier to control small flow rates precisely than to measure them accurately, and the early time evaluation approach has already been developed. Early time evaluation from the non-steady flow pump measurements also permits to obtain the hydraulic parameters of specimen as rapidly as possible under the conditions of even lower hydraulic gradients close to those in situ. Therefore, errors and management problems associate with the long-term permeability tests can thus be decreased to the minimum if the flow pump technique is adopted for testing extremely-low permeability materials.

2. The flow-in and flow-out rates are time-dependent and very small in the constant head test on low-permeable materials. Evaluation of the permeability from a constant head test is possible only if the testing time is extended to be sufficiently long so that accumulated flow in and flow-out rates can be measured and judged to be the same. However, evaluation of the specific storage of low-permeability materials from a constant head test is impossible because the time-dependent small flow rates can not be accurately measured.

3. The hydraulic heads in the specimen during a constant head permeability test increase much more steeply than those do in a flow pump permeability test from the start of the experiment. The maximum hydraulic gradient in a flow pump test can be artificially controlled to be lower than a specified value while the hydraulic gradients induced in one side of specimen in the early period of a constant head permeability test is much greater than that established in the steady state. Interpreting constant head permeability measurements using an apparent hydraulic gradient across the full length of the specimen is feasible only if the steady state flow is reached. However, a definitive judgement of the steady state in a constant head permeability test is much more difficult and time-consuming than that in a flow pump permeability test due to the reason that pressure can be monitored with much higher resolution than flow rate to be done. Using non-steady measurements of flow-in rate and the apparent hydraulic gradient to evaluate permeability from the constant head test by means of the well known Darcy's Law will obtain greater values of the permeability because the hydraulic gradient used for the calculation is smaller than the actual values during non-steady state.

The exact theoretical analyses of the two methods developed in this paper may also be helpful to interpret appropriately the test results and to make optimally the plan of permeability tests on nearly-impermeable materials.

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