(72) マクロな亀裂とミクロな亀裂の相互作用

INTERACTION BETWEEN MACRO-CRACKS AND MICRO-DISTRIBUTED CRACKS

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この論文では、マクロな亀裂とミクロに分散した亀裂の相互作用を均質化理論を用いて説明する。まず、理論的背景について数学的に述べる。ここで、マクロな特性におけるジョイントの寸法効果を考えるために影響関数 $f(\varepsilon)$ を導入する。つぎに、局所問題と大域問題について均質化理論の立場から述べる。亀裂の寸法効果については、影響関数の章で簡潔に述べることとする。最後に、マクロな亀裂がミクロな亀裂に与える影響についていくつかの数値解析例を示す。そうすることで、掘削時のマクロな亀裂の周辺でおこるミクロな挙動を説明することが可能となるであろう。

1. INTRODUCTION

Watching any geological map, whether how large is its scale, one can see that geomaterials are damaged materials and this damage is in a range of scales. Rock mass, strictly speaking, is a structural system. The elements in this system are damaged blocks, geological structural surfaces, underground water and so on. Damaged blocks interact each other through geological structural surface to form a structural system. Each block has its own microstructure such as inclusions, interfaces and micro-cracks. Every blocks are bonded by so called 'geological structural surface' such as faults, distributive micro-cracks. The deformation law of 'geological structural surface' is quite different from that of intact rocks. The deformation of rock masses is the comprehensive behaviors of intact rock and 'geological structural surface' as a structural system. The geological structural surface has its spatial distribution, range and effect zone. Faults are very large in size and behave as a structural element in macro-scale, but distributive micro-cracks are very small in size and distribute in all rock masses except the geological structural surface. Each micro-crack has a little effect on the mechanical properties of rock blocks so it cannot be regarded as a structural element in macro-scale. But a lot of micro-cracks have great effect on the macro-properties of rock blocks, thus on the mechanical properties of rock masses. In order to consider the interactions between macro-cracks and micro-cracks the method proposed by Wang and Ichikawa(1995a) is extended in this paper.

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2. STATEMENT OF THE PROBLEM

2.1 Fundamental equation in an incremental form

Governing equations

Equilibrium equation — Geometrical relation

$$\frac{\partial \Delta \sigma^{\varepsilon}_{ij}}{\partial x_j} + \Delta f^{\varepsilon}_i(\boldsymbol{x};t) = 0 \qquad \Delta \varepsilon_{ij}(\boldsymbol{u}^{\varepsilon}) = \frac{1}{2}\left(\frac{\partial \Delta u^{\varepsilon}_i}{\partial x_j} + \frac{\partial \Delta u^{\varepsilon}_j}{\partial x_i}\right) \quad \text{in} \quad \Omega^{\varepsilon} \times \Delta T \tag{1}$$

Constitutive equation of each material

$$\Delta \sigma^{\varepsilon}_{ij} = E^{\varepsilon}_{ijkl}(\boldsymbol{x}, \varepsilon^0_{rs}; t)\Delta \varepsilon_{kl}(\Delta \boldsymbol{u}^{\varepsilon}) \quad \text{in } \Omega^{\varepsilon} \times \Delta T \quad \text{except interface J} \tag{2}$$

where superscript ε ($\varepsilon = \boldsymbol{x}/\boldsymbol{y}$) denotes Y-periodicity and $0 < \varepsilon \ll 1$. $\boldsymbol{y}$ is the fast spatial variable in a unit cell and $\boldsymbol{x}$ is the slow spatial variable, almost fixed in the unit cell. $E^{\varepsilon}_{ijkl}(\boldsymbol{x}, \varepsilon^0_{rs}; t)$ is the modulus tensor, being oscillatory on fast spatial variable $\boldsymbol{y}$. It satisfies the symmetry condition such that $E^{\varepsilon}_{ijkl}(\boldsymbol{x}, \varepsilon^0_{rs}; t) = E^{\varepsilon}_{jikl}(\boldsymbol{x}, \varepsilon^0_{rs}; t) = E^{\varepsilon}_{ijlk}(\boldsymbol{x}, \varepsilon^0_{rs}; t) = E^{\varepsilon}_{jilk}(\boldsymbol{x}, \varepsilon^0_{rs}; t)$. Deformation modulus is the function of strain / stress history. In fact, Eq.(2) expresses a generalized constitutive law. Note that $\boldsymbol{u}^{\varepsilon}(\boldsymbol{x}, \boldsymbol{y}; t) = \boldsymbol{u}(\boldsymbol{x}, \boldsymbol{y}; t)|_{\boldsymbol{y}=\frac{\boldsymbol{x}}{\varepsilon}}$, $\sigma^{\varepsilon}_{ij} = \sigma_{ij}(\boldsymbol{x}, \boldsymbol{y}; t)|_{\boldsymbol{y}=\frac{\boldsymbol{x}}{\varepsilon}}$, and $\varepsilon^{\varepsilon}_{ij} = \varepsilon_{ij}(\boldsymbol{x}, \boldsymbol{y}; t)|_{\boldsymbol{y}=\frac{\boldsymbol{x}}{\varepsilon}}$ are the displacement vector, stress tensor and strain tensor, respectively. They are all Y-periodicity at any time t. That is, $\varphi(\boldsymbol{x}, \boldsymbol{y}) = \varphi(\boldsymbol{x}, \boldsymbol{y} + \boldsymbol{Y})$ where φ is an abstract Y-periodicity function, and $\boldsymbol{Y}$ the minimum periodicity of microstructures.

2.2 Boundary conditions

Boundary conditions should be divided into external boundary which is the boundary of domain Ω^{ε} and internal boundary or imperfect bonding which describes the properties between sub-domains of Ω^{ε}.

2.2.1 External boundary Γ_F and Γ_0

$$\Delta \sigma^{\varepsilon}_{ij} n_j = \Delta F_i(\boldsymbol{x}; t) \quad \text{on} \quad \Gamma_F \times \Delta T \qquad \Delta u^{\varepsilon}_i = \Delta \bar{u}_i(\boldsymbol{x}; t) \quad \text{on} \quad \Gamma_0 \times \Delta T \tag{3}$$

2.2.2 Imperfect bonding or internal boundary

The micro-slip mechanism at interface is very intricate (Bandis et al. 1983). But generally

a) Traction on an interface J is continuous, which is a basic requirement of static equilibrium:

$$[\![\Delta \sigma_{ij} n_j]\!] = 0 \tag{4}$$

$[\![\bullet]\!]$ denotes the jump of $\bullet$, and n_j is the directional cosines on interface J.

b) Displacement on J may be discontinuous. Slipping obeys the constitutive law of interface generally:

$$f(\varepsilon)[\Delta \sigma]_J = [\bar{D}]_{ep}[\![\Delta \boldsymbol{u}]\!] \qquad \text{in } y_1 - y_2 \text{ coordinates} \tag{5}$$

$$f(\varepsilon)[\Delta \sigma]_J = [D]_{ep}[\![\Delta \boldsymbol{u}]\!]_J \qquad \text{in } n - s \text{ coordinates shown in Fig.1} \tag{6}$$

$f(\varepsilon) = \varepsilon$ case has been studied by Wang and Ichikawa (1995a). Generally $f(\varepsilon)$ can be expanded as

$$f(\varepsilon) = A_0 + A_1\varepsilon + A_2\varepsilon^2 + \cdots \tag{7}$$

$$A_0 = \begin{cases} 1 & \text{When } (x,y) \in \text{ Faults} \\ 0 & \text{When } (x,y) \bar{\in} \text{ Faults} \end{cases} \qquad A_1 = \begin{cases} 1 & \text{When } (x,y) \bar{\in} \text{ Faults} \\ 0 & \text{When } (x,y) \in \text{ Faults} \end{cases} \tag{8}$$

if there is no micro-distributive crack, $A_1 \equiv 0$. Where $[\Delta\sigma]_J$ is the traction increment on J, $[\![\Delta \boldsymbol{u}]\!]_J$ the jump of displacement increment, and $[\bar{D}]_{ep}$ the stiffness matrix of joint materials. They are all expressed in $y_1 - y_2$ coordinates. Subscript J shows that quantities are expressed in interface local coordinates n-s shown in Fig.1. ε is the measure of the relative size of a unit cell compared to that of the global problem. Introduction of effect function $f(\varepsilon)$ in Eq.(5) and (6) indicates that the equation is suitable not only for the local problem (unit cell or REV), but also for the global problem. This expression is different from that thereinbefore(Wang and Ichikawa 1995a). For example, when $\varepsilon \to 0$, that is, the unit cell is very small, the effect of micro-cracks in a unit cell will reduce to zero, too.

A simplified constitutive model of crack or interfaces proposed by Goodman et al.(1968) can be written as if nondilatant joints are considered.

$$f(\varepsilon)\Delta\sigma_n = K_n[\![\Delta u_n]\!] \quad f(\varepsilon)\Delta\sigma_s = K_s[\![\Delta u_s]\!] \tag{9}$$

or $[D]_{ep} = \begin{bmatrix} K_s & 0 \\ 0 & K_n \end{bmatrix}$ but general form is $[D]_{ep} = \begin{bmatrix} K_s & K_{sn} \\ K_{ns} & K_n \end{bmatrix}$ on the $n - s$ local coordinates (Wang and Ichikawa 1995b). Here K_n is the normal stiffness, K_s the shear stiffness, K_{ns} the dilatancy term, and K_{sn} the internal friction term. Note that $\Delta u_n = \Delta u_i n_i$ and $\Delta\sigma_n = \Delta\sigma_{ij}n_i n_j$ are components in normal direction of $\Delta\boldsymbol{u}$ and $\Delta\sigma$, respectively, and similarly $(\Delta u_s)_i = \Delta u_i - \Delta u_n n_i$ and $(\Delta\sigma_s)_i = \Delta\sigma_{ij}n_j - \Delta\sigma_n n_i$ are components in tangential direction.

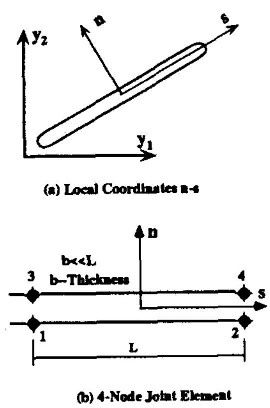

Fig.1 Local Coordinates and Joint Element

3. LOCAL PROBLEM AND GLOBAL PROBLEM

Suppose that the displacement can be expanded as a series of ε:

$$\Delta u_i^\varepsilon(\boldsymbol{x};t) = \Delta u_i^0(\boldsymbol{x},\boldsymbol{y};t) + \varepsilon\Delta u_i^1(\boldsymbol{x},\boldsymbol{y};t) + \varepsilon^2\Delta u_i^2(\boldsymbol{x},\boldsymbol{y};t) + \cdots \tag{10}$$

where $\Delta u_i^\alpha(\boldsymbol{x},\boldsymbol{y};t) = \Delta u_i^\alpha(\boldsymbol{x},\boldsymbol{y}+\boldsymbol{Y};t)$ $(\alpha = 0,1,2,\cdots)$ is Y-periodic.
The chain rule of differentiation suggests

$$\frac{d}{dx_i} = \frac{\partial}{\partial x_i} + \frac{1}{\varepsilon}\frac{\partial}{\partial y_i} \tag{11}$$

Governing equations and boundary conditions are expanded separately by Eq.(10). After some mathematical treatment, a global problem and a local problem are finally obtained as follows[3]

Global problem

[3]The crack or interface, even though how large is its range or size, can be regarded as special internal boundary in the domain Ω^ε. This implies that domain Ω^ε is fissured or non-smooth. Special care should be taken to treat such a problem with Gauss-Green theorem (Okada and Nemat-Nasser 1994).

$$\frac{\partial \langle \Delta\sigma^1_{ij}\rangle}{\partial x_j} + \langle \Delta f_i(\boldsymbol{x}, \boldsymbol{y}; t)\rangle = 0 \qquad \text{in } \Omega \tag{12}$$

$$\Delta\varepsilon^0_{ij} = \frac{1}{2}\left[\frac{\partial \Delta u^0_i}{\partial x_j} + \frac{\partial \Delta u^0_j}{\partial x_i}\right] \qquad \Delta u^0_i(\boldsymbol{x}, \boldsymbol{y}) = \Delta u^0_i(\boldsymbol{x}) \qquad \text{in } \Omega \tag{13}$$

$$\langle \Delta\sigma^1_{ij}\rangle = E^h_{ijkl}\Delta\varepsilon^0_{kl} \qquad \text{in } \Omega \tag{14}$$

$$E^h_{ijkl} = \frac{1}{|Y|}\int_Y E^\varepsilon_{qnpm}\left[\delta_{qi}\delta_{nj} - \frac{\partial \omega^{ij}_q}{\partial y_n}\right]\left[\delta_{pk}\delta_{ml} - \frac{\partial \omega^{kl}_p}{\partial y_m}\right] dy + \frac{1}{|Y|}\int_J [\![\omega^{ij}_p]\!][D_{pq}][\![\omega^{kl}_q]\!] dJ \tag{15}$$

with external boundary condition

$$\langle \Delta\sigma^1_{ij}\rangle n_j = \Delta F_i(\boldsymbol{x}; t) \quad \text{on} \quad \Gamma_F \qquad \Delta u^0_i = \Delta \bar{u}_i(\boldsymbol{x}; t) \quad \text{on} \quad \Gamma_0 \tag{16}$$

and internal boundary

$$[\Delta\sigma^1]_J = [\bar{D}]_{ep}[\![\Delta \boldsymbol{u}^0]\!] \qquad \text{on faults} \tag{17}$$

This is a closed problem if characteristic functions ω^{ij}_p are known.

Cell problem

$$\frac{\partial}{\partial y_l}\{E^\varepsilon_{klij}(\boldsymbol{x}, \varepsilon^0_{rs}; t)[\Delta\varepsilon^0_{ij} + \Delta\varepsilon^{1y}_{ij}]\} = 0 \qquad \text{or} \qquad \frac{\partial\{E^\varepsilon_{ijkl}\Delta\varepsilon^{1y}_{kl}\}}{\partial y_j} = -\frac{\partial E^\varepsilon_{ijkl}}{\partial y_j}\Delta\varepsilon^0_{kl} \qquad \text{in Y} \tag{18}$$

with Y-periodic boundary condition for unknowns $\Delta\varepsilon^1_{kl}$ or $\Delta u^1_i(\boldsymbol{x}, \boldsymbol{y}; t)$. $\Delta\varepsilon^0_{kl}$ is known for cell problem. And internal boundary condition

$$[\Delta\sigma^1]_J = [\bar{D}]_{ep}[\![\Delta \boldsymbol{u}^1]\!] \qquad \text{on J} \tag{19}$$

Where

$$\Delta\varepsilon^{\alpha\beta}_{ij} = \frac{1}{2}\left[\frac{\partial \Delta u^\alpha_i}{\partial \beta_j} + \frac{\partial \Delta u^\alpha_j}{\partial \beta_i}\right] \qquad i, j = 1, 2, 3 \quad \alpha = 0, 1, 2, \cdots \quad \beta = x, \text{ or } y \tag{20}$$

4. SOLUTION FOR LOCAL PROBLEM AND GLOBAL PROBLEM

4.1 Solution for the local problem

The weak form of the local problem is

$$\int_{Y_J} \frac{\partial}{\partial y_j}\left[E^\varepsilon_{ijkl}\frac{\partial \Delta u^1_k}{\partial y_l}\right]\nu_i(\boldsymbol{y})dy = -\int_Y \frac{\partial E^\varepsilon_{ijkl}}{\partial y_j}\Delta\varepsilon^0_{kl}\nu_i(\boldsymbol{y})dy \tag{21}$$

in which $\nu_i(\boldsymbol{y})$ is an arbitrary Y-periodic function with respect to y_i. $\Delta\boldsymbol{u}^1$, which satisfies the internal constraints Eq.(19), is also Y-periodic. Eq.(21) can be rewritten as

$$\int_J [\Delta\sigma^1_p]_J[\nu_p(\boldsymbol{y})]_J dJ + \int_Y E^\varepsilon_{ijkl}\frac{\partial \Delta u^1_k}{\partial y_l}\frac{\partial \nu_i(\boldsymbol{y})}{\partial y_j}dy = -\int_{Y_J} E^\varepsilon_{ijkl}\frac{\partial \nu_i(\boldsymbol{y})}{\partial y_j}dy\ \Delta\varepsilon^0_{kl} \tag{22}$$

Note that this gives the fundamental equation including joint element in a unit cell. Its characteristic function $\boldsymbol{\omega}(\boldsymbol{y})$ is defined as $\Delta u^1_p = -\omega^{kl}_p(\boldsymbol{y})\Delta\varepsilon^0_{kl} + g_p(\boldsymbol{x})$. So the normalized form by $\Delta\varepsilon^0_{kl}$ is

$$\int_J D_{pq} [\![\omega_q^{kl}]\!]_J [\![\nu_p(\boldsymbol{y})]\!]_J dJ + \int_Y E_{ijpq}^{\epsilon} \frac{\partial \omega_p^{kl}}{\partial y_q} \frac{\partial \nu_i(\boldsymbol{y})}{\partial y_j} dy = \int_Y E_{ijkl}^{\epsilon} \frac{\partial \nu_i(\boldsymbol{y})}{\partial y_j} dy \tag{23}$$

4.2 Solution for the global problem

The weak form of global problem is

$$\int_\Omega \langle \Delta f_i \rangle \delta v_i dv + \int_{\Gamma_F} \Delta F_i \delta v_i ds = \int_\Omega \Delta \varepsilon_{ij}^0 E_{ijkl}^h \delta \varepsilon_{kl} dv + \int_{Fault} [\![\Delta u_i^0]\!]_J K_{ij} [\![\Delta v_j]\!]_J dF \tag{24}$$

Where the weight function δv_i is continuous on variable $\boldsymbol{x}$ in Ω^ϵ except faults. $\delta v_i = 0$ on Γ_0.

5. A NUMERICAL EXAMPLE

Fig.2 is the global structure and the microstructure. There is a fault in the global structure and the microstructure has a crack in the inclusion. All are assumed to be linear except the behaviors of cracks. Fig.3 is the relative displacement of two micro-cracks near and far away from the fault.

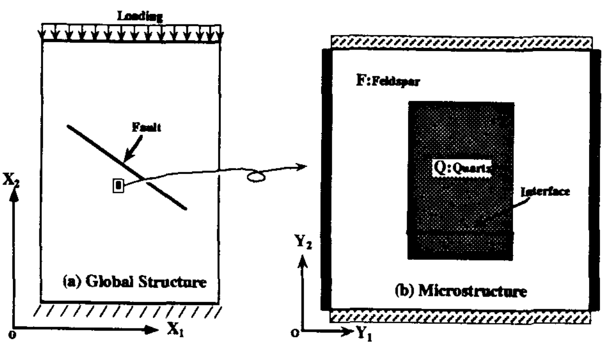

Fig.2 The global structure and the microstructure

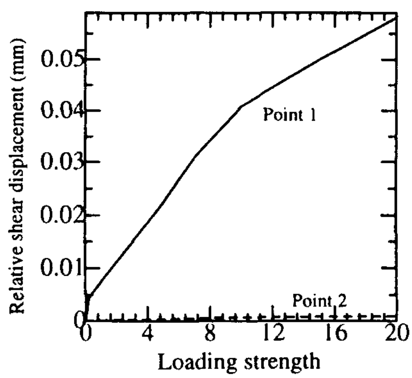

Fig.3 Relative slippage in different points

6. CONCLUSIONS

The proposed method can describe the interaction between macro-cracks and micro-cracks. The behavior of micro-cracks is obviously observed in the simulation.

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