(101) Effect of Dilatancy on Pore Water Pressure in Porous Medium

Jian-Guo WANG Nagoya University
°Takahiro ITO Nagoya University
Yasuaki ICHIKAWA Nagoya University

多孔質体における間隙水圧のダイレイタンシー効果

名古屋大学大学院 学生員 王 建国
名古屋大学大学院 伊藤貴宏
名古屋大学工学部 正会員 市川康明

Abstract

本論文では、Skempton の間隙圧方程式についてダイレイタンシー効果を考慮してつぎの一般式を導入した。

$$du = dp + a_1 dq + a_2 q d\theta \quad \text{(微分形)}$$
$$\Delta u = \Delta p + a_1^* \Delta q + a_2^* q \Delta\theta \quad \text{(有限増分形)}$$

つぎに間隙圧定数 a_1 の閉じた解がいくつか存在し、その予測はＣＵ試験により確かめられることを示した。さらに一般化構成式と結びついた Biot の圧密理論によって、過剰間隙水圧が消散する過程におけるダイレイタンシー効果を考え、過剰間隙水圧が生成、あるいは消散するとき、特に Mandel-Cryer 効果が起こったときでさえダイレイタンシーは顕著に観察されることを示した。

1. Introduction

Dilatancy is a very important property in frictional geomaterials. Some researchers consider it as one of two fundamentals of soil mechanical properties (the other is compression-hardening)[1]. Compression-hardening has well been studied from its deformation to strength. But the dilatancy is more complex phenomenon than the compression-hardening. It is not clear which mechanism makes this phenomenon happen and what is its evolution rule in general. Comparative study has shown that the difference of volumetric strain of constitutive models are great[3,4]. A pore water equation, firstly proposed by Skempton[8] and improved by many researchers[5], is a useful index to express the dilatancy effect on pore water pressure. But the Skempton's coefficients are not directly connected to constitutive models, especially under complex loading.

2. Pore water equation and its coefficients

2.1 Generalized pore-pressure equation

As we know, Skempton's formulus[7] of pore water pressure is given by

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)] \tag{1}$$

B is a coefficient associated with the saturation of soil. If the soil is saturated (saturation $S_r = 100\%$), $B = 1$. Here only $B = 1$ case will be studied. A is the coefficient associated with the dilatancy of soils. There are some experimental data on the variation of coefficient A with respect to deformation and soil type. As we know, undrainage means that volumetric strain keeps unchangeable, i.e.

$$d\varepsilon_v = 0 \tag{2}$$

$$d\varepsilon_v = d\varepsilon_v^e + d\varepsilon_v^p = (d\varepsilon_v^e + d\varepsilon_{vp}^p) + d\varepsilon_{vd}^p, \qquad (d\varepsilon_v^e + d\varepsilon_{vp}^p) = \frac{dp'}{K_*}, \qquad d\varepsilon_{vd}^p = C_1 dq + C_2 q d\theta$$

From Terzaghi's effective stress principle $dp = dp' + du$, a pore-pressure equation can be written as

$$du = dp + K_*(C_1 dq + C_2 q d\theta) \tag{3}$$

or we suggest following pore-pressure equation for isotropic materials:

$$\begin{cases} du = dp + a_1 dq + a_2 q d\theta & \text{:the differential form} \\ \Delta u = \Delta p + a_1^* \Delta q + a_2^* q \Delta\theta & \text{:the finite incremental form} \end{cases} \tag{4}$$

where $a_1^* = \frac{\int_H a_1 dq}{\Delta q}$ $a_2^* = \frac{\int_H a_2 q d\theta}{q\Delta\theta}$ are the functions of strain/stress history (denoted by $'H'$). p: total hydrostatic stress; $d\varepsilon_v^e$: increment of elastic volumetric strain; $d\varepsilon_{vp}^p$: increment of plastic volumetric strain induced by effective hydrostatic p'; $d\varepsilon_{vd}^p$: increment of plastic volumetric strain induced by shear stress (q, θ); K_*: volumetric modulus, which is different in loading and unloading/reloading; σ_1: maximum principal stress; σ_3: minimum principal stress; q: generalized shear stress; θ: stress Lode's angle.

Under conventional triaxial condition we have

$$\Delta\sigma_2 = \Delta\sigma_3, \quad \Delta\theta \equiv 0 \implies \Delta q = \Delta\sigma_1 - \Delta\sigma_3, \quad \Delta p = \frac{1}{3}(\Delta\sigma_1 + 2\Delta\sigma_3) \implies A = a_1^* + \frac{1}{3} \tag{5}$$

For elastic materials, whether it is linear or nonlinear or not, we have

$$a_1 = a_2 \equiv 0 \longrightarrow A = \frac{1}{3} \tag{6}$$

But for elastoplastic materials, the general matrix of materials can be expressed as

$$d\varepsilon_{ij} = C_{ijkl} d\sigma'_{kl} \tag{7}$$

where C_{ijkl} is the inversion of matrix of materials D_{ijkl} which is derived generally as

$$\{d\sigma\} = [D]_{ep}\{d\varepsilon\}, \qquad [D]_{ep} = [D] - \frac{[D]\{\frac{\partial g}{\partial \sigma}\}\{\frac{\partial f}{\partial \sigma}\}^T[D]}{\bar{A} + \{\frac{\partial f}{\partial \sigma}\}^T[D]\{\frac{\partial g}{\partial \sigma}\}} = [D] - \frac{G[X]}{\frac{\bar{A}}{G} + \Phi} \tag{8}$$

Here $[D]$ is elastic matrix; G: shear elastic modulus; $\bar{A}$: plastic hardening modulus; g: plastic potential function; f: yielding function; $\Phi = \{\frac{\partial f}{\partial \sigma}\}^T[D]\{\frac{\partial g}{\partial \sigma}\}/G$. From Eqn(7) we can get the expression of $d\varepsilon_{vd}^p$ for any constitutive models.

2.2 Closed form of a_i $(i = 1, 2)$ for some special constitutive models

- Modified Cambridge model[6]

$$a_1 = \frac{2\eta(\lambda - k)}{\lambda(M^2 - \eta^2) + 2\eta^2 k} \qquad a_2 = 0 \tag{9}$$

It is interesting to note that

$$\bar{\varepsilon} = f_1(\eta) \qquad \text{so that} \qquad a_1 = f_2(\bar{\varepsilon})$$

This equation has been verified by a lot of experimental data of normally consolidated clay. Here λ, k, M are all material parameters. $\eta = \frac{q}{p'}$ is stress ratio.

- Tsinghua elastoplastic model[3]

$$a_1 = -\frac{B_1}{(B_1 + 1)y + m_3} \qquad a_2 = 0 \quad for \quad F_2(\theta) \equiv 1 \tag{10}$$

where

$$B_1 = K_0 m_5 \frac{(A_* - \frac{B^2}{C^2}\eta^2)(\varepsilon_v^p - m_3\bar{\varepsilon}^p + m_6)}{A_*(1 + \frac{B^2}{C^2}\eta^2)}, \qquad y = \frac{\eta^2 - \frac{C^2}{B}(1 + \frac{B^2-1}{C^2}\eta^2)^{\frac{1}{2}}}{\eta[B(1 + \frac{B^2-1}{C^2}\eta^2)^{\frac{1}{2}} + 1]}$$

$$A_* = \{(B^2 - 1)(1 + \frac{B^2}{C^2}\eta^2) + 1\}^{\frac{1}{2}}$$

Here B, C, m_3, m_5, m_6, K_0 are material constants.

- Normalized plastic work model[8,9]

$$d\varepsilon_{vd}^p = \mu F_2(\theta)d\xi - \frac{1}{p'}S_{ij}de_{ij}^p \quad \text{and} \quad du = dp + K_*[\mu F_2(\theta)d\xi - \frac{1}{p'}S_{ij}de_{ij}^p] \tag{11}$$

If S_{ij}, de_{ij}^p are assumed to be coaxial, we have

$$du = dp + K_*[\mu F_2(\theta) - \frac{\| S_{ij} \|}{p'}]d\xi \tag{12}$$

where $d\xi = \| de_{ij}^p \|$. $F_2(\theta)$ is the yielding or failure shape function on π-plane. μ: frictional coefficient; S_{ij}: derivatoric stress; de_{ij}^p: increment of derivatoric plastic strain.

2.3 Experimental verification

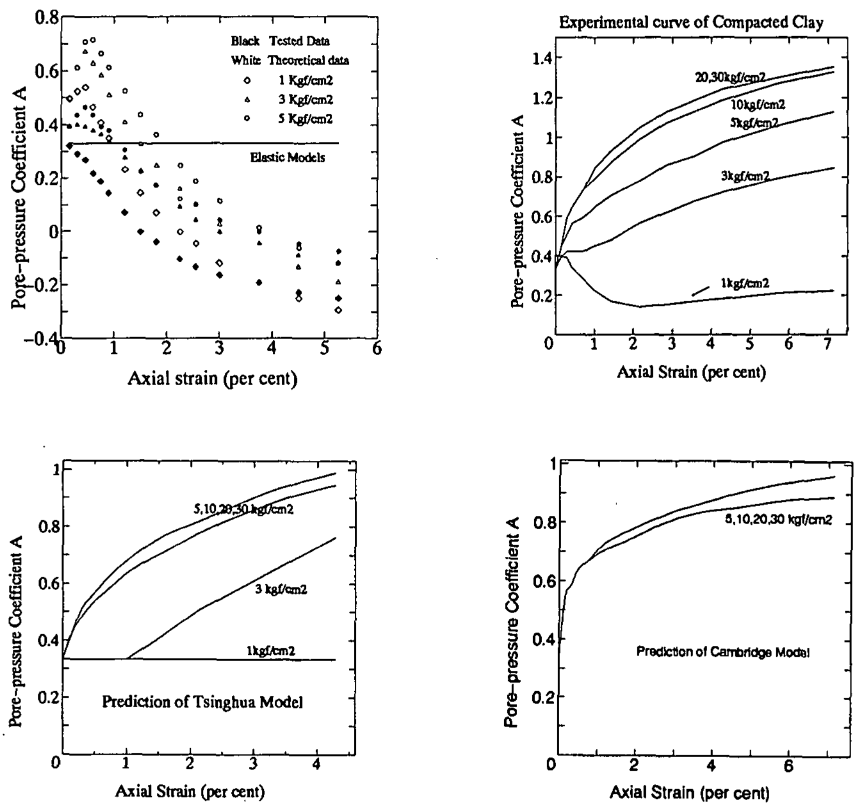


Fig.1 shows the change of A with deformation and confining pressure.

3. Effect of dilatancy on dissipation of pore water pressure in foundations

3.1 Components of Biot's theory incorporating with general constitutive models

Following six concepts form generalized Biot's theory[5]:

- Equilibrium equation

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0$$

- Geometrical equation

$$\Delta\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial \Delta u_i}{\partial x_j} + \frac{\partial \Delta u_j}{\partial x_i}\right)$$

- Constitutive laws of solid skeleton

$$d\sigma'_{ij} = D_{ijkl} d\varepsilon_{kl}$$

- Terzaghi's effective stress principle

$$\sigma_{ij} = \sigma'_{ij} + u\delta_{ij}$$

- Darcy's seepage law

$$q_i = K_{ij}\frac{\partial \psi}{\partial x_j}$$

- Incompressiblility of solid-water mixture

$$\frac{\partial \varepsilon_v}{\partial t} = \frac{\partial q_i}{\partial x_i}$$

3.2 Weak form and FEM discretization

3.2.1 Weak form

The body forces applied on the skeleton are two components: 1) The effective weight b_i per unit volume. 2) Seepage force induced by $(-u_{,i})$. So that the weak form for error-self-corrector mode is:

$$\begin{aligned}
&\int_V \{\delta(\Delta\varepsilon)\}^T[D]_{ep}\{\Delta\varepsilon\}dv - \int_V\{\delta(\frac{\partial \Delta \bar{u}_i}{\partial x_i})\}^T\{u\}^{t+\Delta t}dv - \int_V\{\delta(\Delta\bar{u})\}^T\{\Delta b\}dv \\
&+ \int_{S_\sigma}\{\delta(\Delta\bar{u})\}^T\{n\}u^{t+\Delta t}ds - \int_{S_\sigma}\{\delta(\Delta\bar{u})\}^T\{\Delta\bar{T}\}ds \\
&= -\int_V\{\delta(\Delta\varepsilon)\}^T\{\sigma\}dv + \int_{S_\sigma}\{\delta(\Delta\bar{u})\}^T\{\bar{T}\}ds - \int_V\{\delta(\Delta\bar{u}\}^T\{b\}dv
\end{aligned} \tag{13}$$

$$-\int_V\{\delta u\}^T\{\frac{\partial \Delta \bar{u}_i}{\partial x_i}\}dv = \frac{1}{\tau_w}\int_t^{t+\Delta t}[\int_{S_q}\{\delta u\}^T\{Ku\}ds]dt - \frac{1}{\tau_w}\int_t^{t+\Delta t}[\int_V\{\frac{\partial \delta u}{\partial x_i}\}^T\{K_i\frac{\partial u}{\partial x_i}\}dv]dt \tag{14}$$

3.2.2 Element matrix of stiffness and load vector[2,10]

For plane strain problems ($i=1 \Rightarrow x$; $i=2 \Rightarrow y$; element domain V_m), the displacement $\bar{u}$ and excess pore pressure u are taken to be the same shape function. 4-nodal isoparametric element is used. Its element stiffness is

$$[K]_m = [K_{ij}]_{4\times 4} \quad K_{ij} = K_{ji}^T \quad K_{ij} = \int_{V_m} K_{ij}^0 dv \tag{15}$$

$$K_{ij}^0 = \begin{bmatrix} A_{11} & A_{12} & -N_j\frac{\partial N_i}{\partial x} \\ A_{21} & A_{22} & -N_j\frac{\partial N_i}{\partial y} \\ -N_j\frac{\partial N_i}{\partial x} & -N_i\frac{\partial N_j}{\partial y} & -(1-\theta)\frac{\Delta t}{\tau_w}(K_x G_1 + K_y G_4) \end{bmatrix} \tag{16}$$

Converted loading vector from pre-timing states at time t

$$\int_{V_m}\begin{bmatrix} \frac{\partial N_i}{\partial x}\sigma'_x + \frac{\partial N_i}{\partial y}\tau_{xy} \\ \frac{\partial N_i}{\partial y}\sigma'_y + \frac{\partial N_i}{\partial x}\tau_{xy} \\ \theta\frac{\Delta t}{\tau_w}[K_x\frac{\partial N_i}{\partial x}\sum_r\frac{\partial N_r}{\partial x}u_r^t + K_y\frac{\partial N_i}{\partial y}\sum_r\frac{\partial N_r}{\partial y}u_r^t] \end{bmatrix} dv \tag{17}$$

in which

$$A_{11} = P_1G_1 + P_3G_2 + P_3G_3 + P_9G_4 \qquad A_{12} = P_1G_1 + P_9G_2 + P_2G_3 + P_5G_4$$

$$A_{21} = P_3G_1 + P_2G_2 + P_9G_3 + P_5G_4 \qquad A_{22} = P_9G_1 + P_5G_2 + P_5G_3 + P_4G_4$$

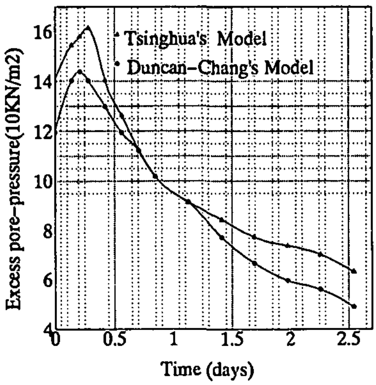
$$B(I) = \frac{\partial N_i}{\partial x} \qquad C(I) = \frac{\partial N_i}{\partial y} \qquad [D]_{ep} = \begin{bmatrix} P_1 & P_2 & P_3 \\ P_2 & P_4 & P_5 \\ P_3 & P_5 & P_9 \end{bmatrix}$$

$$G_1 = B(I) * B(J) \qquad G_2 = C(I) * B(J) \qquad G_3 = B(I) * C(J) \qquad G_4 = C(I) * C(J)$$

All others are the same as usual one. Crank-Nicolson mode ($\theta = \frac{1}{2}$) is adopted for time domain.

3.3 Case study

Some results see Fig.2.



4. Conclusions

- The effect of dilatancy is introduced into the pore pressure equation and a general pore-pressure equation is suggested.
- Dilatancy has a vital effect on the dissipation of excess pore water pressure.

References

1. Jian-Guo Wang and Bin Chen(1993), Consideration on characteristics and recognition theories of soils, Journal of Chongqing University, 16(3):25-29
2. 市川康明 (1990), 地盤力学における有限要素法入門, 日科技連出版社
3. Guangxin Li(1985), *A study of three-dimensional constitutive relationship of soils and an examination of various models*, PhD Dissertation in Tsinghua University, Beijing, China
4. Proceedings of the Workshop on Limit Equilibrium Plasticity and Generalized Stress-Strain in Geotechnical Engineering, McGill University, May 28-30, 1980
5. Jian-Guo Wang(1987), A study on the usage of various constitutive models in pore pressure generation and dissipation, Master Thesis, Tsinghua University
6. Roscoe, K.H., Burland, J.B.(1968), On the generalized stress-strain behavior of 'wet clay', Engineering Plasticity, Ed. J. Heyman and F.A. Leckie, Cambridge University Press
7. Skempton A.W.(1954), The pore-pressure coefficient A and B, Geotechnique, 4(4):143-
8. Jian-Guo Wang(1992), Constitutive properties of granular materials under complex stress/strain paths, Report to National Natural Foundations of Science of China, Institute of Engrg Mech., Chongqing University
9. N. Moroto(1976), A new parameter to measure degree of shear deformation of granular material in triaxial *compression test, Soils and Foundations, 16(4)*:1-11
10. Instructions on COND2.FORT(2-DIM), Ichikawa's Lab., Dept. of Geotechnical and Environmental Engineering, Nagoya University, January 1994