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# (60) Modeling of Dissolved Oxygen in Lakes

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**Abstract;** The accurate prediction of dissolved oxygen(DO) in lakes specially in eutrophic lakes is of paramount importance in view of the water quality. The temporal and spatial variations of DO in the lake Calhoun is studied with the aid of a numerical model. When the water quality is found to be highly degraded, the DO depletion may cause fish kills and anoxic conditions for other living organisms in the ecosystem. The main factors that affect the DO budget are temperature and organic matters present. Computational values and measured values of DO are compared. The model predicts that at the surface, reaeration is predominant whereas photosynthesis and algal respiration contribute considerably.For the management point of view, it is useful to predict water quality in terms of DO, thereby, if necessary, remedial measures such as artificial aeration can be adopted to improve the water quality. The model prediction also provides an idea about the period during which an improvement is necessary without extensive costs.

**Keywords;** Photosynthesis,reaeration,biological oxygen demand,phytoplankton and zooplankton respiration,sediment oxygen demand.

## 1. Introduction;

With the seasonal change it is evident that the water quality changes causing severe problems and detrimental effects to flora and fauna in lakes. One of the parameters that predicts the quality of the lake water is DO level. The accurate prediction of DO, therefore, provides useful guidance to analyze the degree of pollution in the lake. The model provides useful insights into the causes of severe oxygen depletion, phytoplankton dynamics and sediment oxygen demand(SOD). In the formulation of the DO budget, carbonacious biological oxygen demand(BOD), nitrogenous BOD, reaeration with the atmosphere, photosynthesis, zooplankton and fish respiration and SOD are considered to be the major contributing processes.

## 2. Model Formulation;

The rate of change of DO is primarily dependent on the consumption and production parameters. The structure of the mathematical model shows the interrelationships among physical and biological processes with the DO level (Fig. 1). In the model formulation, nutrients and sediment carbon are considered together whereas all other variables are taken into account separately.

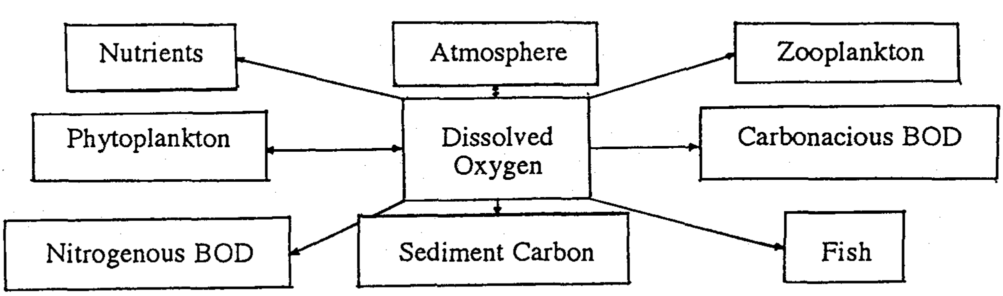


Fig.1. Structure of the model depicting the contributing variables to the DO budget.

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## 3. The Governing Equation;

$$\frac{\partial C}{\partial t}+V_s\frac{\partial C}{\partial z}=-k_1L-4.57k_NN+k_2(C_s-C)+a_p\mu P-a_prP-SOD-k_F(\frac{F}{H})-a_zk_zZ \quad (1)$$

where C is DO concentration (mg/L); $V_s$ is settling velocity(m/day); z is the depth in question(m); P is phytoplankton concentration (μg/L Chl-a); N is ammonia nitrogen (mg N/L); L is first order ultimate carbonacious oxygen demand (mg/L); $C_s$ is saturated dissolved oxygen concentration (mg/L); SOD is sediment oxygen demand (mg/l day); $k_1$ is biochemical deoxygenation rate (day$^{-1}$); $k_N$ is nitrification rate (day$^{-1}$); $k_2$ is reaeration coefficient (day$^{-1}$) $a_P$ is the ratio of oxygen production to uptake per algal mass ($O_2$/mass of phyto.); r is phytoplankton respiration rate (day$^{-1}$); $k_F$ is fish respiration rate (day$^{-1}$); H is mean depth of the lake (m); $a_Z$ is the ratio of oxygen production to uptake per unit mass of zooplankton carbon (mg $O_2$/mg C); $k_Z$ is zooplankton death rate (day$^{-1}$); Z is zooplankton concentration (mg/L) and F is fish stocking density.

### 3.1 Carbonacious and Nitrogenous BOD;

The ultimate BOD is calculated as follows.

$$(BOD_5)=(BOD_U)(1-e^{-5k_1}) \quad (2)$$

$$L=(BOD_U)e^{-k_1t} \quad (3)$$

$$k_1(T)=k_{20}\theta^{T-20} \quad (4)$$

$$k_N(T)=k_{N20}\theta^{T-20} \quad (5)$$

where t is time(days) and T is temperature in $^0$C.

Carbonacious and nitrogenous wastes are contributed by mainly organic matters and higher order species excreta. The DO in the lake is therefore, taken up by the carbonacious wastes giving rise to carbonacious BOD as well as the oxidation of ammonia to nitrate nitrogen through nitrite during nitrification process resulting nitrogenous BOD. However, in the literature, deoxygenation rate at 20 degrees and arrhenious constant for carbonacious BOD are equal to 0.23day$^{-1}$ and 1.047 (Ambrose et al. 1988) whereas nitrification rate and arrhenious constant for nitrogenous BOD are equal to 0. day$^{-1}$ and 1.08 (Ambrose et al. 1988).

### 3.2 Reaeration;

When the lake water is below saturation level, oxygen from the atmosphere can diffuse in to the lake by the process of wind induced turbulent mixing. Conversely, when the water is supersaturated oxygen is released to the atmosphere. The reaeration is, therefore, proportional to the oxygen deficit ($C_s$-C).The reaeration coefficient and saturated oxygen concentration are determined as follows (Thomann and Mueller 1987).

$$k_2(20)=\frac{0.728W^{0.5}-0.317W+0.0372W^2}{H} \quad (6)$$

$$k_2(T)=k_{20}\theta^{T-20} \quad (7)$$

$$C_s = 1.43[(10.291 - 0.2809T + 0.006009T^2 - 0.0000632T^3)] \quad (8)$$

where W is wind speed (m/s) and H is mean depth (m). The arrhenious constant is taken as 1.024.

### 3.3 Photosynthesis and Algal Respiration;

To predict the phytoplankton growth rate all species of phytoplankton are assumed to be represented by Chlorophyll-a. The growth rate is also assumed to be mainly dependent on nutrients,light intensity and temperature. In the presence of sunlight, phytoplankton absorbs solar radiation to synthesize plant matter from dissolved carbon dioxide and water to produce oxygen. The phytoplankton growth rate can be described as

$$\mu = \mu_{max} \exp(aT + b) \frac{DL}{24} \frac{N}{N + N_k} \frac{I}{I_s} \exp(1 - \frac{I}{I_s}) \quad (9)$$

where $\mu_{max}$ is non nutrient limited growth rate at 20 $^0$C=2.0 day$^{-1}$ (Eppley 1972); a=0.0632($^0$C$^{-1}$); b=-1.26; T is water temperature ($^0$C); DL is daily solar hours for given day; N is nutrient concentration (mg/L); $N_k$ is the half saturation constant=15μg N/L (Thomman and Mueller 1987); I is solar radiation in the layer for which the computation is done (W/m$^2$); $I_s$ is optimum light intensity = 0.2 langleys/min (Lasen et al. 1974 and Megard et al. 1972).

Solar radiation is found from the expressions given below.

$$I = I_0 \exp(-kz) \quad (10)$$

$$k = k_w + k_c P \quad (11)$$

where $I_0$ is solar radiation intensity at the water surface; z is the median depth for which the computation is done; k is light attenuation coefficient(m$^{-1}$); $k_w$ is light attenuation coefficient due to water color and to dissolved and nonplanktonic suspended solids = 0.6 m$^{-1}$; $k_c$ is light attenuation coefficient due to phytoplankton=30(mg/m$^3$ Chl-a)$^{-1}$.

The energy for phytoplankton is absorbed by the Chl-a in the form of solar radiation. The photosynthetically active radiation(PAR) is reported to be less than the incoming solar radiation and covers only the range of 0.4-0.7μ wavelength. The available solar radiation for phytoplankton is therefore, assumed as a fraction of incoming solar radiation.

The direct downward component of solar radiation at the surface with the assumption of constant atmospheric transmissivity is computed as (Hoffert and Storch 1979 and Kaufmann and Weatherred 1982)

$$S = \frac{S_0}{r^2} \tau Cos Z_e (1 + \eta)(1 - 0.65n) \quad (12)$$

where S is solar radiation(W/m$^2$); $S_0$ is solar constant=1360(W/m$^2$); τ is the atmospheric transmissivity=0.6; η is the proportion of diffuse-to-direct solar radiation; n is the degree of sky cloudiness; $r^2$ is the correction factor to account for the elliptical orbit of the earth

around the sun; $Z_e$ is solar zenith angle(degrees).
In this model, the degree of sky cloudiness is assumed as zero.

The correction factor $r^2$ is determined as

$$r^2 = 0.9998 + 0.0014\delta_0 \tag{13}$$

where $\delta_0$ is the declination angle of the sun.
The declination angle of the sun in degrees is given by

$$\delta_0 = 23.45 Sin[0.986(284 + Julian.Day)] \tag{14}$$

and solar zenith angle is given by

$$Z_e = Cos^{-1}(Sin\phi_0 Sin\delta_0 + Cos\phi_0 Cos\delta_0 Cos\varepsilon_0) \tag{15}$$

where $\phi_0$ is the latitude of the position; $\delta_0$ is the declination angle of the sun and $\varepsilon_0$ is the hour angle from the solar noon.

The proportion of the diffused solar radiation to the direct solar radiation can be computed as

$$\eta = 0.05 + 0.10(1 - CosZ_e) \tag{16}$$

where $Z_e$ is the solar zenith angle.

The algal respiration rate is typically in the range of 0.05-0.15 $day^{-1}$( Bowie et al. 1985). In this model it is assumed as 0.05 $day^{-1}$. Arrhenious constant for algal respiration is taken as 1.08 (Thomann and Mueller 1987).

The algal respiration can be expressed as

$$r = k_r \theta^{T-20} \tag{17}$$

The ratio of oxygen production to uptake per algal mass is expresses as

$$a_P = 2.65CCHL \tag{18}$$

where CCHL is carbon to Chlorophyll ratio.

In lakes CCHL varies seasonally depending on the growth rate of phytoplankton. But, in this model, after calibration a constant value of 30 is adopted at the surface and found to predict fairly accurate results.

### 3.4 Sediment Oxygen Demand;

SOD in lakes involves the transport of oxygen from overlying water through the sediment-water interface and pore water to the place where bacteria and macroinvertebrates consume it. Turbulent or laminar flow may control transport in the overlying water while molecular diffusion may control transport in the sediments. SOD mainly comprises of two parts such as biological and chemical .
SOD can be expressed as

$$SOD = \mu_\beta \frac{C}{k_{O_2} + C} + k_c C \tag{19}$$

where $\mu_\beta$ and $k_c$ are constants to be determined for each lake; $k_{O_2}$ is half saturation constant=1.4mg/L (Robert et al. 1986).

### 3.5 Zooplankton and Fish Respiration;

The fish stocking density varies from lake to lake. This depends on the market, season and availability of nutrients. Oxygen uptake per unit mass of zooplankton carbon is taken as 2.67;zooplankton death rate at 20 degrees is in the range of 0.2-0.25(day$^{-1}$) and arrhenious constant for zooplankton death is 1.045( Lee et al. 1991)

Zooplankton death coefficient is expresses as

$$k_Z = k_{Z20}\theta^{T-20} \tag{20}$$

In the lake Calhoun, zooplankton and fish respiration are found to be negligible and therefore, not considered in the computation.

## 4. Results and Discussions;

The Crank Nicolson finite difference scheme is used to calculate the temporal variation of DO. Fig.2. shows the comparison of computational and measured values of DO at the surface.Fig. 3. shows each component separately. The model is built on well established concepts as well as in situ measurements.Fig.4. and fig.5. show the DO variations at depths of 2.5m and 5.0m respectively. For these two figures, the temporal variation is taken from the months of June to August. For the entire model, settling velocity is taken as zero at the surface whereas in all other depths it is taken as 1.5m/day.

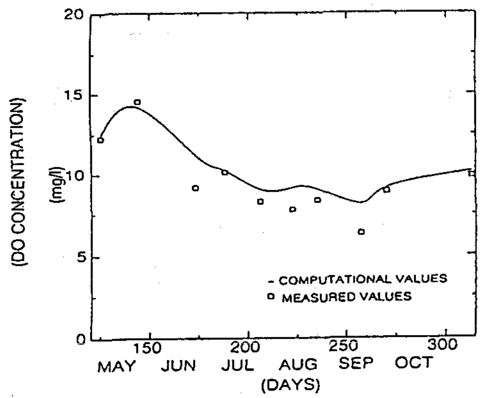


Fig 2. Comparison of computational and measured DO at the surface.

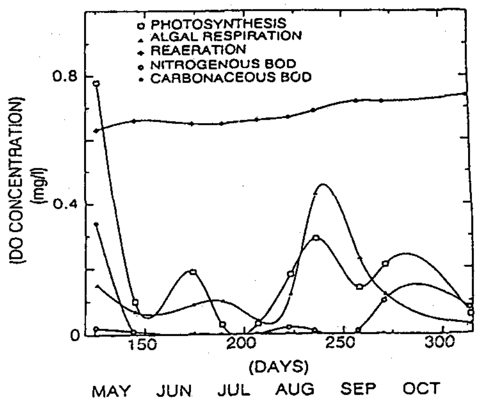


Fig 3. Contribution of physical and biological processes separately.

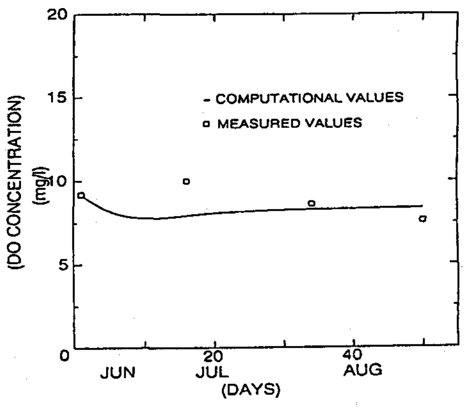


Fig4. Comparison of computational and measured DO at 2.5m depth.

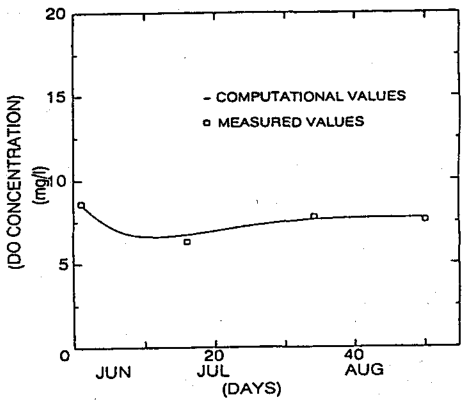


Fig 5. comparison of computational and measured DO at 5.0m depth.

Time step of 1 day is adopted in the computation. In general, various kinds of nutrients are needed for the growth of phytoplankton. In this model, only nutrients which limit the growth rate are considered. Usually, in lakes, either nitrogen or phosphorus becomes limited nutrient depending on the seasonal variation. In the model limited nutrient is determined from the measured values.The initial values of DO are also taken from the measured values and assigned to the first nodal points of the time scale. The necessary biological and hydrological data are obtained from Shapiro and Pfannkuch(1973) whereas meteorological data are obtained from a meteorological station located at Minneapolis St. Paul International Airport. The model simulation agrees favorably with observations and manifests that the reaeration is predominant at the surface whereas photosynthesis and algal respiration contribute fairly well.Further improvements of the model can be made with the consideration of advection and bubble plume hydrodynamics.

## 5. Conclusions;

Eutrophication of lakes has now been an acute environmental problem in many parts of the world. Hence for the management point of view, it is useful to predict water quality in terms of DO, thereby ,if necessary, remedial measures such as artificial aeration can be adopted to improve the water quality.The model prediction also provides an idea about the period during which an improvement is necessary without extensive costs.

## References;

1. Ambrose, R.B. et, al. (1988).WASP4, A hydrodynamic and water quality model. Rep. EPA600/3-87/039,U.S. Environmental Protection Agency, Washington,D.C.
2. Bowie, G.L.et , al.(1985). Rates, constants and kinetics formulations in surface water quality modeling. Rep. EPA 600/3-85/040, U.S. Environmental Protection Agency, Washington, D.C.
3. Eppley, R.W.(1972). Temperature and phytoplankton growth in the sea, Fisheries Bulletin, Vol.70, p.1063.
4. Hoffert, M.I. and Storch, J.(1979). Scheme for computing surface fluxes from mean flow observations, Boundary layer Metoroi., 17, p429-442.
5. Kaufmann, M.R. and Weatherred, J.D. (1982). Determination of potential direct beam solar irradiance, U.S.Dep. of Agr. For. Serv., Fort Collins Co., Res. Pap. MM-242.
6. Larsen, D.P., Mercier, H.T. and Malveg, K.W. (1974). Modeling algal growth dynamics in Shawaga Lake, Minnesota,With comments concerning projected restoration of the lake, E.J.Middlebrooks, D.H. Falkenborg, and T.E. Maloney, eds., Modeling the eutrophication process, Ann Arbor Science, Ann Arbor, Mich.,
7. Lee, J.H., Wu, R.S.S. and Cheung, Y.K. (1991). Forecasting of dissolved oxygen in marine fish culture zone, J. of Env. Eng., Vol.117, No6.