PERFORMANCE OF A NEW EDDY VISCOSITY MODEL FOR SPILLING BREAKERS

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本研究では，平面２次元場の砕波帯における波高減衰と、浮遊砂輸送の両方の計算への利用を目的とした新しい渦動粘性モデルを提案する．本研究では，RANS-VOF (Reynolds Average Navier-Stokes with Volume of Fluid)の出力に近似した渦動粘性モデルを提案し、波高減衰を水平拡散で表現する人工渦動粘性への換算方法を導出する。砕波帯内では、鉛直拡散によるエネルギー逸散が支配的であるが、Boussinesq方程式を使った平面２次元計算では、それが評価できずKennedyが水平拡散を使って近似し、その中に現れるのが人工渦動粘性である。Spilling breaker wave typeの波高減衰について、Boussinesq方程式に組み込んで、本モデルとKennedyモデルを比較したところ、砕波点付近では両者はほぼ同一で、汀線付近では本モデルがRANS-VOFに近い結果となった。また、砕波帯内における既存の波高の実験値とモデルに本モデルを用いた場合の計算値を比較したところ，Kennedyモデルと比べて良好に再現可能であった．

***Key Words:** Eddy viscosity, spilling breaker, surf zone, wave decay*

1. INTRODUCTION

Estimation of wave propagation in coastal region is essential for researchers. Ability to estimate this behavior affects the design of coastal structures as well as our ability to forecast beach profile evolution.

Spilling breakers receive much less attention from casual observers of the ocean surface than their more dramatic and powerful plunging counterparts. However, spilling breakers probably occur more frequently than plunging breakers and are important contributors to turbulence, spray, and bubble generation at the water surface.

Numerical models have been developed for simulating wave breaking in surf zone. In the past decade, Boussinesq-type equations (BTE) have been attractively developed by researches. In this model, breaking phenomena is predicted in two main approaches; surface roller and eddy viscosity. The first approach is developed by Svendsen[1)], Schaffer et al.[2)] and Madsen et al.[3)]. The second approach is a popular method[4)] and related with this study.

The basic idea behind the eddy viscosity model is the addition of a momentum diffusion term to the BTE with the diffusivity strongly localized on the front face of a broken wave. Zelt[5)] proposed an artificial eddy viscosity to produce the dissipation term due to turbulence generated by wave breaking and bore propagation. It was treated by a diffusion term in the momentum conservation equation. Kennedy et al.[6)] used a momentum-conserving eddy viscosity technique to model breaking. This is somewhat like the eddy viscosity formulation by Zelt[5)], but with extensions to provide a more realistic description of the initiation and cessation of wave breaking. Roeber et al.[4)] introduced two parameters defining the onset and termination of the wave-breaking process.

These models have not been validated yet in term of eddy viscosity. On the other hand, it has also been recognized that the eddy viscosity is an important variable to determine the suspended sediment distribution. In this study, an eddy viscosity model in breaking term will be developed which can be used solving the problem.

In this paper, we first present the mathematical framework for the model, and then compare the spilling wave breaker type to show the performance of the new model. Spilling breaker condition can be defined according to surf similarity parameter[7].

$$\zeta = \frac{m}{(H_0 / L_0)^{0.5}} \quad (1)$$

where H_0 and L_0 are wave height and wavelength in the offshore, m is beach slope.

2. GOVERNING EQUATIONS

Numerical models based on Boussinesq-type equations have become an important tool in coastal engineering, especially in applications where reflection and diffraction as well as nonlinear wave-wave interactions are important. The Boussinesq-type equations of Nwogu[8] for conservation of mass may be written as:

$$\eta_t + \nabla \cdot M = 0 \quad (2)$$

where η is the free surface elevation, the subscript t denotes partial derivative with respect to time, and:

$$M = D\left\{\tilde{u} + \left(Ah - \frac{D}{2}\right)(2\nabla h F_{22} + \nabla F_{21}) + \left(Bh^2 - \frac{D^2}{3}\right)\nabla F_{22}\right\}$$

where h is the still water depth and $D = h + \eta$. The associated momentum equation is:

$$U_t = -g\nabla\eta - \frac{\delta}{2}\nabla\left(|\tilde{u}|^2\right) + \Gamma_1 + \Gamma_2 \quad (3)$$

U, Γ_1 and Γ_2 are given by:

$$U = \tilde{u} + \left[(A-1)h(2\nabla h F_{22} + \nabla F_{21}) + (B-1)h^2\nabla F_{22}\right]$$

$$\Gamma_1 = \nabla\left[\eta F_{21t} + (2h\eta + \eta^2)F_{22t}\right]$$

$$\Gamma_2 = -\nabla\left\{\tilde{u} \cdot \left[(Ah - D)(\nabla F_{21} + 2\nabla h F_{22}) + (Bh^2 - D^2)\nabla F_{22}\right] + \frac{1}{2}(F_{21} + 2DF_{22})^2\right\}$$

where:

$$A = \frac{1}{h}\left[\beta(h + z_a) + (1-\beta)(h + z_b)\right]$$

$$B = \frac{1}{h^2}\left[\beta(h + z_a)^2 + (1-\beta)(h + z_b)^2\right]$$

$$z_a = \left[\frac{1}{9} - \left\{\frac{8\beta}{567(1-\beta)}\right\}^{1/2} + \left\{\frac{8\beta}{567(1-\beta)}\right\}^{1/2}\right]^{1/2} - 1$$

$$z_b = \left[\frac{1}{9} - \left\{\frac{8\beta}{567(1-\beta)}\right\}^{1/2}\right]^{1/2} - 1$$

$$F_{21} = G\nabla h \cdot \tilde{u}$$

$$F_{22} = \frac{1}{2}G\nabla \cdot \tilde{u}$$

$$G = \frac{1}{1 + |\nabla h|^2}$$

The above equations are only valid for nonbreaking waves, so some additional approximation must be made to model wave breaking. Kennedy et al.[6] used a simple eddy viscosity-type formulation to model the turbulent mixing and dissipation caused by breaking. The mass conservation in Eq.(2) remains unchanged, while, with the additional of breaking terms (R_b), the equation for momentum conservation, Eq.(3), becomes:

$$U_t = -g\nabla\eta - \frac{\delta}{2}\nabla\left(|\tilde{u}|^2\right) + \Gamma_1 + \Gamma_2 + R_b \quad (4)$$

where:

$$R_b = \frac{1}{h+\eta}\left(\nu_K\left((h+\eta)u_a\right)_x\right)_x \quad (5)$$

The artificial eddy viscosity (ν_K) is formulated as:

$$\nu_K = B\delta_b^2(h+\eta)\eta_t \quad (6)$$

where δ_b is a mixing length coefficient. This artificial eddy viscosity is used to produce horizontal diffusion term. The quantity B varies smoothly from 0 to 1 so as to avoid an impulsive start of breaking and the resulting instability.

3. DEVELOPMENT OF EDDY VISCOSITY MODEL

According to Eq.(A.12) in Appendix, the Kennedy's model of artificial eddy viscosity (ν_K) in Eq.(6) is equal to a real eddy viscosity multiplied by a factor square of wavelength and water depth ratio. The breaking term in Eq.(5) is modified by introducing a new real eddy viscosity (ν_t) model as follow:

$$R_b = \frac{1}{h+\eta}\left(\left(\frac{L}{h}\right)^2 \nu_t\left((h+\eta)u_a\right)_x\right)_x \quad (7)$$

This eddy viscosity is not only used to produce horizontal diffusion term, but also can be used to determine vertical velocity profile as well as suspended sediment distribution. The factor square of wavelength and water depth ratio in this equation indicates that to satisfy the momentum equation in Eq.(4) and also to produce the real eddy viscosity, the new model should produce eddy viscosity much smaller than Kennedy's model.

In this study, ν_t is derived empirically and applied in 1D-FUNWAVE (Fully Nonlinear Boussinesq WAVE) developed by Kirby et al.[9]. Eddy viscosity model in Eq.(6) is replaced with:

$$\nu_t = C\sqrt{\left[(d - d_{br})u\right]^2 + \left[dw\right]^2} \quad (8)$$

where C is a coefficient (0.001), d is total local water depth, d_{br} is total water depth at breaking, u and w are horizontal and vertical velocity, respectively. Vertical velocity is calculated as follow[9]:

$$w(x,z,t) = -\mu^2(-F_{21} + 2\xi F_{22}) \tag{9}$$

where ξ is the distance from bottom and dispersion (μ) is obtained as follow:

$$\mu = k_0 h_0 \tag{10}$$

In contrast to the Kennedy's model that only acts locally on the front face of a broken wave, the proposed model of eddy viscosity may extend along the surf zone.

The numerical simulations was set according to the definition given in the experimental work of Ting · Kirby[10),11)] as that used by Zhao et al.[12)]. Spilling breaker wave with surf similarity 0.2 was generated in a constant depth section. The incident wave height and wave period are 0.125 m and 2.0 s, respectively. Fig. 1 shows a schematic diagram of the numerical computation. We consider a computational domain of 40 m with a 0.025 m grid. An internal source at 6 m in front of the toe of bottom slope generates the incident waves and a sponge layer absorbs all waves propagating to the left of the source.

4. RESULT AND DISCUSSION

(1) Simulation of Ting · Kirby's[10),11)] experiments

Eddy viscosity cannot be observed directly in laboratory, the validation is conducted comparing the output result of RANS-VOF developed by Ontowirjo[13)]. Fig. 2 shows the eddy viscosity produced by new model, Kennedy's model and RANS-VOF model.

This figure indicates that the new model can produce eddy viscosity better than the Kennedy's model. However the magnitudes of eddy viscosity are underestimated near the shoreline. This figure proves the previous explanation in Section 3 that the real eddy viscosity (ν_t) should be much smaller than the artificial eddy viscosity (ν_K).

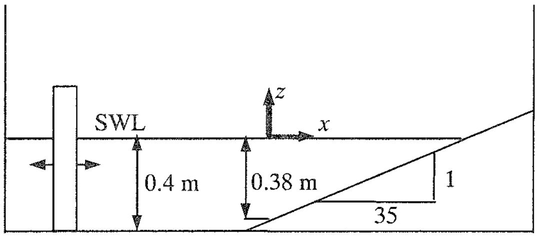

Fig. 1 Numerical computation arrangement.

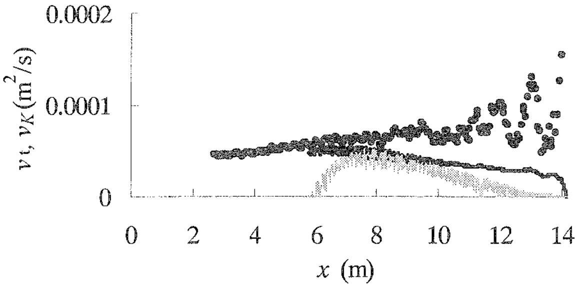

Fig. 2 Time-averaged of eddy viscosity. Black line: new model; gray line: Kennedy's model (with multiplier 0.01); dots: RANS-VOF model.

The numerical model is validated against the laboratory observations[10),11]], specifically wave decay, free-surface elevations, horizontal and vertical velocity. Fig. 3 shows the comparison of the model results with the measurement of cross-shore variations of wave height. Although the new model predicted breaking point earlier than laboratory experiment data which was observed at x = 6.4 m, the wave height was reasonable accepted in this point. Both eddy viscosity models over predicted the wave height in the surf zone. This discrepancy can be explained by considering the eddy viscosity magnitude in Fig. 2, which is under predicted compared to RANS-VOF model. This small eddy viscosity produces small dissipation in surf zone. However, the agreement between Kennedy's model and new model is fairly good.

Phase-averaged free surface elevation from new model results are compared to Kennedy's model results and laboratory data, as depicted in Fig. 4. Near the breaking point, as shown in Fig. 4.a, these two models of eddy viscosity produce surface elevation close to experiment data. As the wave enters the surf zone, both models show deviations between the computed wave height and laboratory data. The over predicting of surface elevation corresponds to the over predicting of wave height, as explained previously.

The corresponding phase-averaged horizontal and vertical velocities are presented in Fig. 5. Horizontal velocity can be produced well, although vertical velocity was overestimated at x = 9.11 m.

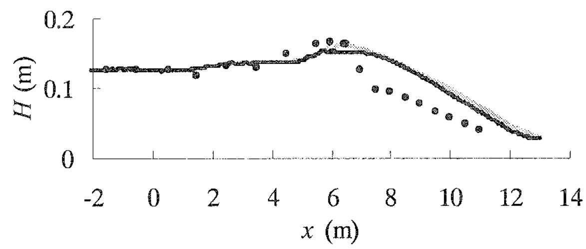

Fig. 3 Comparison of modeled and measured wave height. Black line: new model; gray line: Kennedy's model; dots: experimental data from Ting and Kirby[10)].

For these comparisons, it can be concluded that the performance of the new model and Kennedy's model is the same in simulating wave breaking. The advantage of using the new model is the real eddy viscosity to be calculated; meanwhile the Kennedy's model only calculated the artificial eddy viscosity.

(2) Simulation of Hansen · Svendsen's[14] Experiments

Data for the wave height of shoaling waves was obtained from the results presented by Hansen · Svendsen[14] as cited by Kennedy et al.[6]. In Table 1, the wave parameters at wave maker for each experiment are tabulated.

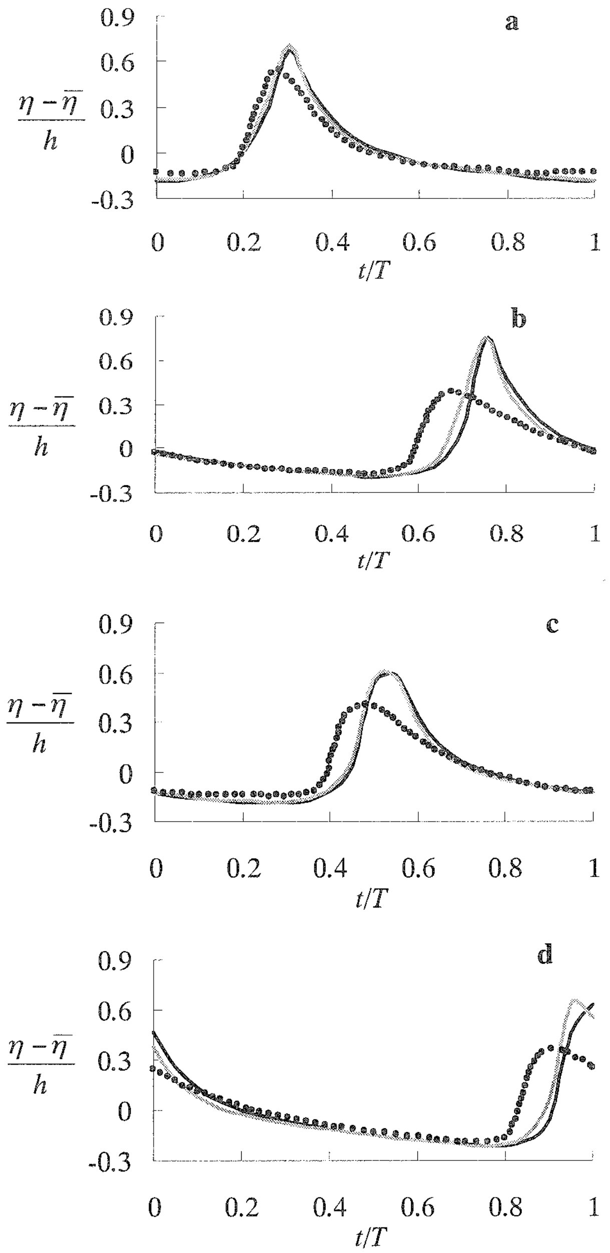

Fig. 4 Comparison of modeled and measured phase-averaged surface elevations. (a) x = 7.275 m, (b) x = 7.885 m, (c) x = 8.495 m and (d) x = 9.11 m (d). Black line: new model; gray line: Kennedy's model; dots: experimental data[12].

The surf similarities (ζ) are calculated by Eq.(1). Waves were generated on a horizontal bottom at a depth of 0.36 m, shoaled, and broke on a 1:34.26 planar slope.

The numerical results of the wave height are presented in Fig. 6. New model give a good description of wave shoaling and breaking, although wave heights are slightly underpredicted as the wave shoals.

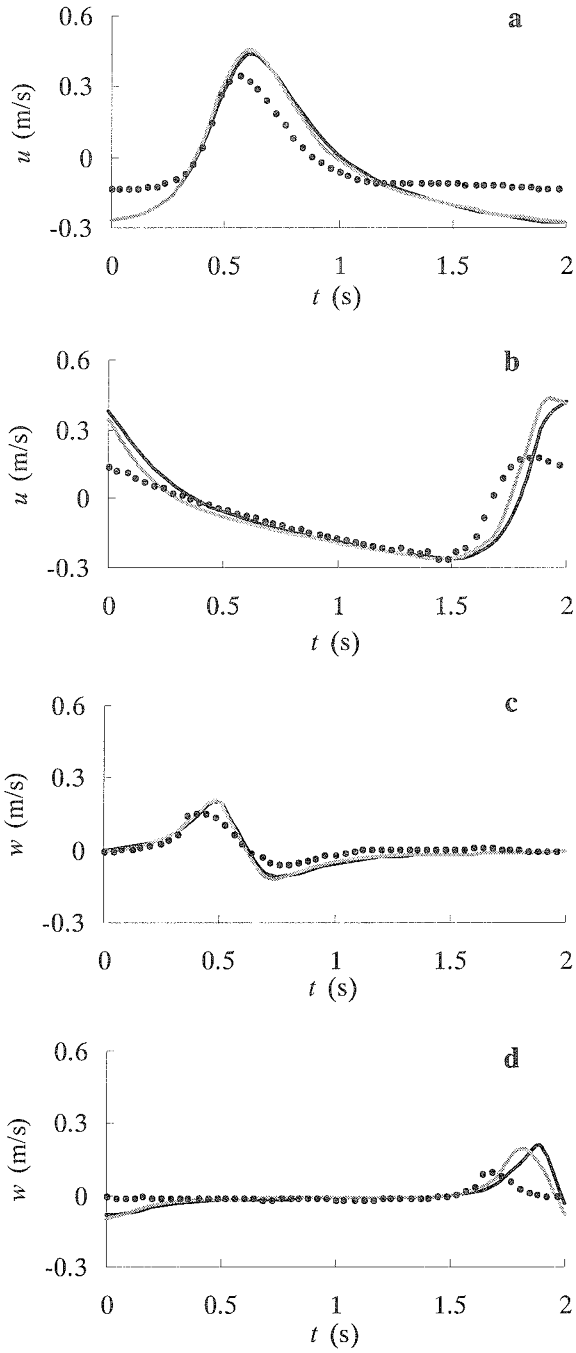

Fig. 5 Comparison of modeled and measured phase-averaged horizontal velocity: (a) x = 7.275 m, (b) x = 9.11 m and vertical velocity: (c) x = 7.275 m, (d) x = 9.11 m. Black line: new model; gray line: Kennedy's model; dots: experimental data[12].

Table 1 Wave parameters at source generation[6].

Case	T(s)	H (m)	ξ
051041	2.0	0.036	0.384
061071	1.67	0.067	0.235
A10112	1.0	0.067	0.141

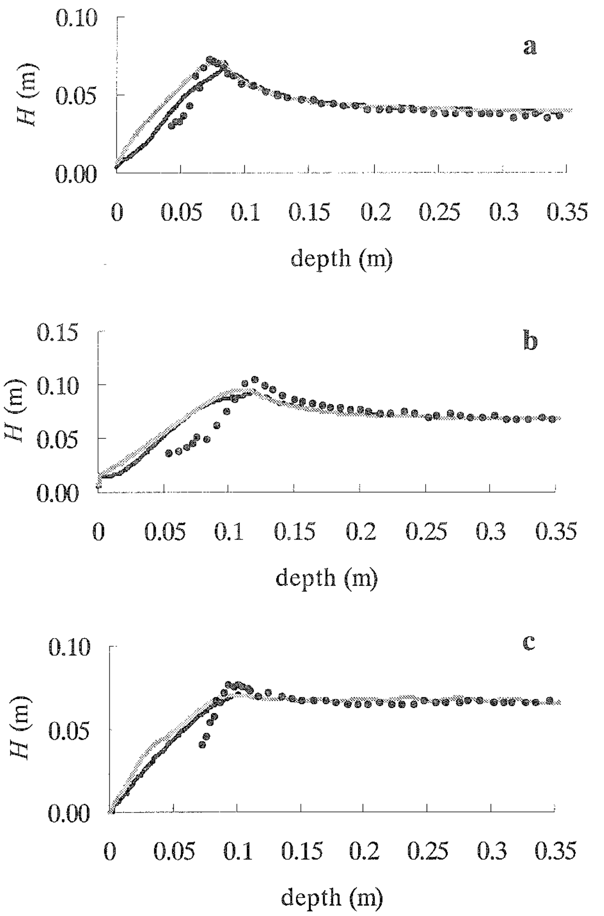

Fig. 6 Computed and measured wave height and setup: (a) case 051041, (b) case 061071, (c) case A10112. Black line: new model, gray line: Kennedy's model, dots: measured data[3].

Comparing to the Kennedy's model, the new model predicted wave height more accurately in surf zone, especially for case 051041. Fig. 3 (ξ = 0.201) and Fig. 6 show that as the surf similarity is smaller, both model produce wave height in surf zone closer.

5. CONCLUSIONS

This study has introduced a new model of eddy viscosity for wave propagation in surf zone based on the Boussinesq-type equation. Eddy viscosity was calibrated against RANS-VOF model. Performance of this model has been validated for spilling breaker case in a plane bottom slope in terms of wave decay, surface elevation and wave-induced current. Small eddy viscosity produces a deviation of wave height compared to laboratory data in surf zone. The new model simulated wave height better than Kennedy's model for ξ = 0.384. Future work is required to apply the present model to numerical simulation of plunging breaker case and suspended sediment distribution in surf zone as well.

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APPENDIX

The purpose of this derivation is to find the relationship between the real eddy viscosity (ν_t) and artificial eddy viscosity (ν_K).

Reynolds stress equations

The derivation begins with the momentum equation for the instantaneous fluctuating velocity as:

$$\frac{\partial u_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = g_i - \frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j^2} - \frac{\partial \overline{u_i' u_j'}}{\partial x_j} \qquad \text{(A.1)}$$

By introducing eddy viscosity, the turbulence shear stress or Reynolds stress is expressed as follows:

$$\tau_{t_{ij}} \cong -\rho \overline{u_i' u_j'} = \rho \nu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \qquad \text{(A.2)}$$

in similarity with:

$$\tau_{l_{ij}} = \rho \nu \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \qquad \text{(A.3)}$$

Then the total shear stress is given by:

$$\tau_{ij} = \tau_{l_{ij}+\tau_{t_{ij}}} = \rho(\nu + \nu_t)\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)$$

$$= \rho \nu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \qquad \text{(A.4)}$$

(if $Re >> Re_c$, Re_c is the critical Reynolds number), then:

$$R.S. = \frac{\partial}{\partial x_j}\left[\nu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \right] \qquad \text{(A.5)}$$

If we confine, i = 1 with three-dimensional space, then:

$$R.S. = \frac{\partial}{\partial x}\nu_t\left(\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\nu_t\left(\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\nu_t\left(\frac{\partial u}{\partial z}\right)$$

$$R.S._{(1)} \cong \frac{\partial}{\partial z}\left(\nu_t \frac{\partial u}{\partial z}\right) \qquad \text{(A.6)}$$

$$R.S._{(II)} \cong \frac{\partial}{\partial x}\left(\nu_K \frac{\partial u}{\partial x}\right) \qquad (A.7)$$

The first approximation (Eq.A.6) is derived from the major term, while the second approximation (Eq.A.7) is based on Kennedy's model.

Then the relationship between the two eddy viscosities can be obtained by the following order estimation. We introduce the following normalization:

$$x^* = \frac{x}{L_0},\ z^* = \frac{z}{h_0},\ u^* = \frac{u}{\sqrt{gh}} \qquad (A.8)$$

where L_0 is typical wavelength, h_0 is typical water depth and T is wave period, then:

$$R.S._{(I)} \cong \frac{\partial}{\partial z}\left(\nu_t \frac{\partial u}{\partial z}\right) = \frac{\sqrt{gh_0}}{h_0^{\ 2}}\frac{\partial}{\partial z^*}\left(\nu_t \frac{\partial u^*}{\partial z^*}\right) \qquad (A.9)$$

$$R.S._{(II)} \cong \frac{\partial}{\partial x}\left(\nu_K \frac{\partial u}{\partial x}\right) = \frac{\sqrt{gh_0}}{L_0^{\ 2}}\frac{\partial}{\partial x^*}\left(\nu_K \frac{\partial u^*}{\partial x^*}\right) \qquad (A.10)$$

The Boussinesq model cannot evaluate the vertical diffusion term of eddy viscosity, then we need to make an approximation as follow:

$$R.S._{(II)} = R.S._{(I)} \qquad (A.11)$$

$$\frac{\sqrt{gh_0}}{L_0^{\ 2}}\frac{\partial}{\partial x^*}\left(\nu_K \frac{\partial u^*}{\partial x^*}\right) = \frac{\sqrt{gh_0}}{h_0^{\ 2}}\frac{\partial}{\partial z^*}\left(\nu_t \frac{\partial u^*}{\partial z^*}\right)$$

$$\nu_K = \left(\frac{L_0}{h_0}\right)^2 \nu_t \qquad (A.12)$$

where, ν_K and ν_t are artificial eddy viscosity and real eddy viscosity, respectively. We find the relationship between two eddy viscosities.

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