A NEW METHOD FOR WAVE RUN UP SIMULATION BY COUPLING k-ω MODEL WITH SHALLOW WATER EQUATION MODEL

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津波の遡上現象に関する検討を行うために，斜面を遡上する孤立波に関する研究が水理実験・数値計算の両面において多くなされてきた．数値計算の分野では，長波の基礎方程式を用い，抵抗則としてはマニングの式に代表される定常流の知見を援用することが一般的であった．ただし，この手法では底面せん断力が単純に速度の二乗に比例することとなり，複雑な非定常底面境界層の特性が反映されていない．
本研究においては，浅水流方程式とk-ωモデルを組み合わせることにより，孤立波の遡上過程を計算するための新しい手法を構築した．また，本モデルの数値計算結果をSynolakis (1987)による室内実験結果と比較することにより，マニングの式を用いた場合に比べて精度が向上することを示した．また，土砂移動量と密接な関係を有する一周期平均のせん断力などの値を求め，定常流抵抗則を用いた時との差異を定量的に評価した．

Key Words : *Wave run up, numerical simulation, shallow water equation, boundary layer, bed stress*

1. INTRODUCTION

Tsunami wave is a natural phenomenon which has been widely studied. The wave effects to coastal area can be devastating as the wave propagates with massive force. One of the main problems caused by the tsunami wave is the coastal morphology changes due to sediment transport. Detail and thorough study of this phenomenon requires a better approach in the bed stress approximation under the wave. In the near shore region, the bottom boundary layer characteristic of the wave will play an important role in the sediment transport process. The boundary layer in this area is usually very thin and therefore, difficult to observe in nature. For this reasons, experimental and numerical works are preferred to study this phenomenon.

Tsunami wave itself is commonly represented as long wave. Experimental work conducted by Synolakis[1)] has been widely used for long wave study. The canonical problem sets are often used as benchmark for numerical model. Shallow water equation is commonly used for numerical modeling. The equation is considered to be efficient with relatively good accuracy[2)].

Conventional Manning method is usually used in assessing bed stress term. The method estimates the bed stress under the assumption that the bed stress is in phase with the free-stream velocity. Thus, the bed stress can be estimated directly from the free stream velocity. However, this assumption is not always accurate. It is very difficult to accurately predict the sediment fluxes due to the complexity of wave hydrodynamics.

Tanaka[3)] estimated the bottom shear stress under non-linear wave by modified stream function theory and proposed formula to predict bed load transport except near the surf zone in which the acceleration effect plays an important role. Furthermore, Tanaka and Thu[4)] had shown the importance of friction factor and phase difference between velocity and bed stress under the wave where Manning method fails to explain this phenomenon. In general, the bed stress formulations may incorporate both velocity and acceleration relater terms, or may include time varying friction factor, and phase lag (e.g. Nielsen[5)], Kabiling and Sato[6)]).

The bed stress behavior will be influenced by the boundary layer properties beneath the wave. Elfrink and Fredsøe[7)] has shown that the effect of boundary layer to the wave run up process is important. Furthermore, study on boundary layer under solitary wave by Suntoyo and Tanaka[8] has shown the good accuracy of bed stress approximation from the boundary layer using numerical model.

Boundary layer has been studied widely. Two equation models are often used to assess the boundary layer properties with k-ε and k-ω being the most common. Suntoyo[9] had made comparison between the models. k-ω model is considered to be more accurate than the k-ε model in assessing the boundary layer properties. Adityawan et al.[10] used 2D k-ω model to investigate boundary layer properties under the wave motion with good accuracy.

Bed stress estimation is very important in sediment transport modeling. Therefore, it is crucial to have an accurate estimation of bed stress. Adityawan et al.[11] estimated bed stress from the boundary layer using k-ω model. The calculated bed stress were used to assess the friction factor value and applied to the shallow water equation. The results showed improvement as compare to the conventional Manning. However, the method requires two times calculation and therefore, inefficient.

This study objective is to develop a new method for long wave run up simulation which can improve the accuracy of existing shallow water equation (SWE) model by considering the simplicity of the model for practical application in coastal area. Furthermore, bed stress calculation method is crucial in the model. Improvement is obtained by replacing the conventional Manning method with direct assessment of bed stress from the boundary layer. New calculation method is proposed. The SWE model is coupled with k-ω model. Thus, bed stress can be approximated directly from the boundary layer using the k-ω model. The new method is used to simulate canonical problems.

2. MODEL DEVELOPMENT

The governing equations are shallow water equation (SWE) and k-ω equation. The models are calculated separately at each time steps however their results are intertwine, allowing simultaneous calculation.

(1) Shallow Water Equation

The SWE consists of the continuity equation and the momentum equation as follows.

$$\frac{\partial h}{\partial t}+\frac{\partial (Uh)}{\partial x}=0 \qquad (1a)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+gh\frac{\partial (h+z)}{\partial x}=-ghS_f \qquad (1b)$$

where h is the water depth, U is depth averaged velocity, t is time, g is gravity, z is the bed elevation, S_f is bed stress. Manning equation is commonly used to assess this parameter. The bed stress relation in the conventional manning method is assumed linear to the square of free-stream velocity as shown bellow.

$$\frac{\tau_0}{\rho}=gn^2/R_h^{1/3}\times U\times|U| \qquad (2)$$

where τ_o is the bed stress, ρ is density, R_h is the hydraulic radius or can be considered as water depth for a very wide channel, and n is the Manning roughness.

The governing equation above is solved using McCormack predictor corrector finite difference scheme. Forward difference scheme is used in the predictor step and backward difference scheme is used in the corrector step. The new value of h, U the next time step is obtained from the initial time step and the corrector time step. This scheme has been known for its good performance in obtaining numerical solution for the Shallow Water Equation.

Wet dry moving boundary condition is applied in the model to allow run up simulation. A threshold depth is selected. If the calculated water depth is lower than the threshold, then the water depth and velocity in the corresponding grid is given zero value (dry cell). The model also adapts numerical filter for better stability in calculation. The filter acts as an artificial dissipation. A numerical filter[12] which acted as an artificial dissipation is used for each time step at each node. The value of depth and velocities are updated with the following equation.

$$F(i,j)=C\,F(i,j)+0.25\,(1-C)\,\{F(i,j)+F(i+1,j)+F(i,j-1)+F(i,j+1)\} \qquad (3)$$

The C value is set to 0.99 and F corresponds to the filtered parameters which are velocity and depth.

(2) k-ω Equation

The governing equation for the k-ω model is based on the Reynolds-averaged equations of continuity and momentum, follows:

$$\frac{\partial u_i}{\partial x}=0 \qquad (4a)$$

$$\rho\frac{\partial u_i}{\partial t}+\rho u_j\frac{\partial u_i}{\partial x_j}=\frac{\partial P}{\partial x_i}+(2\mu S_{ij}-\rho\overline{u_i'u_j'}) \qquad (4b)$$

where u_i and x_i denotes the velocity in the boundary layer and location in the grid, u_i' is the fluctuating velocity in the x (i = 1) and y (i = 2) directions, P is the static pressure, ν is the kinematics viscosity, ρ is the density of the fluid, $\rho\overline{u_i' u_j'}$ is the Reynolds stress tensor, and S_{ij} is the strain-rate tensor from the following equation.

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{5}$$

The Reynolds stress tensor is given through eddy viscosity by Boussinesq approximation:

$$-\overline{u_i' u_j'} = \nu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}k\delta_{ij} \tag{6}$$

with k is the turbulent kinetic energy and δ_{ij} is the Kronecker delta. The k–ω model equation is given as follows:

$$\frac{\partial k}{\partial t} + u_j\frac{\partial k}{\partial x_j} = \tau_{ij}\frac{\partial u_i}{\partial x_j} - \beta^* k\omega + \frac{\partial}{\partial x_j}\left[\left(\nu + \sigma^*\nu_t\right)\frac{\partial k}{\partial x_j}\right] \tag{7a}$$

$$\frac{\partial \omega}{\partial t} + u_j\frac{\partial \omega}{\partial x_j} = \alpha\frac{\omega}{k}\tau_{ij}\frac{\partial u_i}{\partial x_j} - \beta\omega^2 + \frac{\partial}{\partial x_j}\left[\left(\nu + \sigma\nu_t\right)\frac{\partial \omega}{\partial x_j}\right] \tag{7b}$$

The eddy viscosity is given by:

$$\nu_t = \frac{k}{\omega} \tag{8}$$

The values of the closure coefficients are given by Wilcox[13] as $\beta = 3/40$, $\beta^* = 0.09$, $\alpha = 5/9$, and $\sigma = \sigma^* = 0.5$. Finite difference central scheme is applied to solve the governing equations in time and space. The boundary condition at the bottom is no slip boundary. At the free stream, it is assumed that the velocity gradient, turbulent kinetic energy gradient and the dissipation rate gradient are zero.

(3) Coupling method

The basic idea for the calculation is to upgrade the SWE model by replacing the Manning method with a more accurate method to assess the bed stress term within the momentum equation. The commonly used Manning approach will be replaced by direct approach of bed stress in the near bed region using a k-ω model.

Calculation begins with an initial condition of the parameters. Initial value of friction coefficient is stated for bed stress calculation in SWE model. The velocity obtained from the SWE model is applied as the free stream velocity boundary condition in the k-ω model. Furthermore, the bed stress obtained from the k-ω model is applied in the momentum equation of SWE model. The process continues until the end of simulation time as shown in **Fig. 1**.

A new grid system is developed to allow both models to be coupled simultaneously. The grid system for the method does not require a full horizontal and vertical grid system such as in the full turbulent model. The vertical grid is only required in the near bottom area to assess the boundary layer for bed stress calculation. The water depth becomes very thin at the wave front. Therefore, the boundary layer is not accessible anymore. At this location, the bed stress is calculated from the momentum equation in the SWE. The model domain definition and treatments is shown in **Fig. 2**.

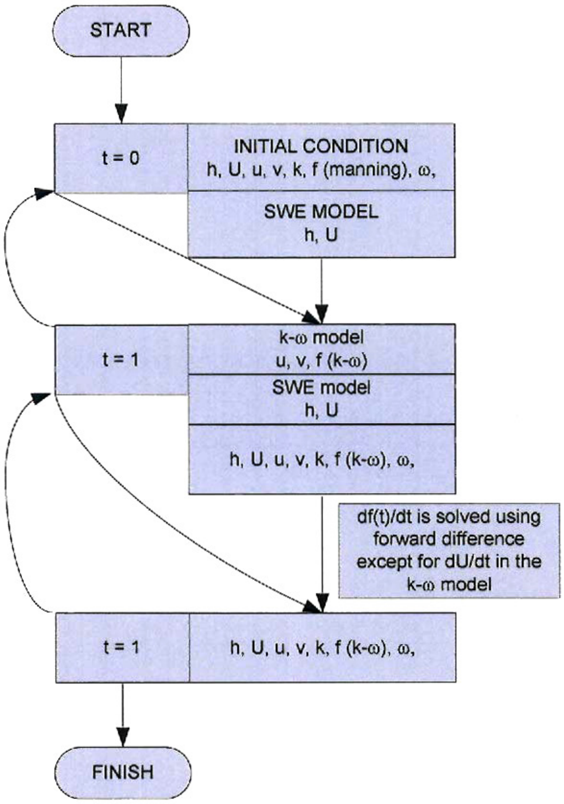


Fig.1 Calculation flowchart.

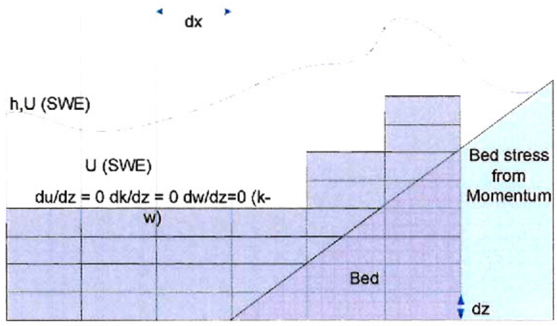


Fig.2 Domain system.

3. WAVE RUN UP SIMULATION

(1) Scenario

The developed method is verified with the case of run up of non breaking wave from previous study by Synolakis[1]. The run up occurs due to a solitary wave on a sloping beach or commonly known as canonical problem. Two type of model are simulated for this scenario. The first one is the Shallow Water Equation model using the conventional Manning method. The second one is the upgraded Shallow Water Equation model using the new method proposed in this study. The model setup for the case is shown in **Fig. 3**.

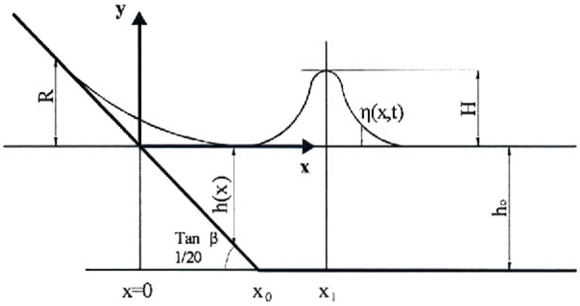


Fig.3 Model sketch setup for benchmark, Synolakis[1].

Non-dimensional variables are introduced

$$x^* = x/h_o \tag{9a}$$

$$h^* = h/h_o \tag{9b}$$

$$\eta^* = \eta/h_o \tag{9c}$$

$$t^* = t(g/h_o)^{0.5} \tag{9d}$$

where h_o is the initial water depth (normal depth), η is the water elevation, x is the coordinate according to the model sketch with asterisk notate the corresponding parameters in non-dimensional. In the experiment, the ratio of initial wave height to the depth is 0.019 with beach slope 1:20.

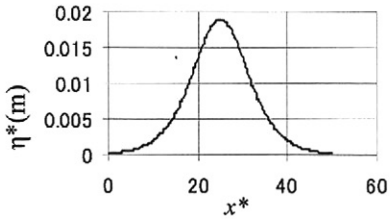


Fig.4 Initial wave profile.

The solitary wave initial profile and velocity is applied for the model initial condition according to the following equation:

$$\eta(x,0) = \frac{H}{h_o}\mathrm{sech}^2\left(\sqrt{\frac{3H}{4h_o}}(x - X_1)\right) \tag{10a}$$

$$U(x,0) = \frac{c\eta}{1+\eta} \tag{10b}$$

$$c = \sqrt{g(H+h_0)} \tag{10c}$$

The wave profile is given by Eq.(10a) with the initial velocity as given from Eq.(10b) and Eq.(10c). The location of this initial wave peak is at X_1 as shown in **Fig. 3**.

X_1 is situated at half of the initial wave length (L/2) from the initial slope (X_0). The wave length (L) can be calculated according Eq. (11). The initial solitary wave profile is given in the **Fig. 4**.

$$L = \frac{2}{\sqrt{3H/4h_o}}\left[\mathrm{arccosh}\left(\sqrt{\frac{1}{0.05}}\right)\right] \tag{11}$$

(2) Results and Discussion

Wave profile comparison between the experimental data, Manning Method, and New Method is shown in **Fig. 5**. The new method is shown to have a better comparison to the experimental data than the Manning method. The model was later verified by simulating the same wave with different slope configuration. The run up height can be calculated using Eq.(12) as given by Synolakis[1].

$$\frac{R}{h_o} = 2.831(\cot\beta)^{1/2} \times \left(\frac{H}{h_o}\right)^{5/4} \tag{12}$$

where R is the estimated run up height and β is the bed slope as shown in **Fig. 3**. The simulated run up heights is compared with various data sets from experiment and the run up law. Run up height prediction using the new method is shown to have a better match with data set and run up law as shown in **Fig. 6** and **Table 1**.

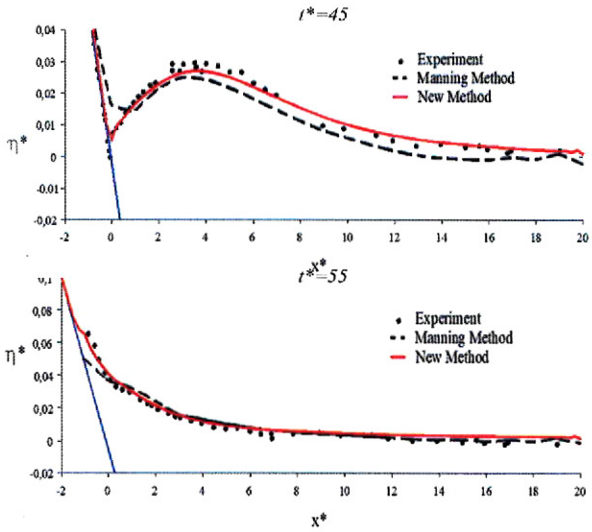


Fig.5 Free surface comparison.

Further analyses regarding bed stress is conducted for both models. The Manning method estimates the bed stress as a function of square of velocity per depth as shown in Eq.(2). This relation often leads to computational error for a very low water depth. The method tends to estimate lower magnitude in the deep area and higher magnitude in the shallow area as shown in **Fig. 7**. It is also observed that the Manning method does not able to explain the shifting of bed stress peak to velocity and overshooting in the deceleration.

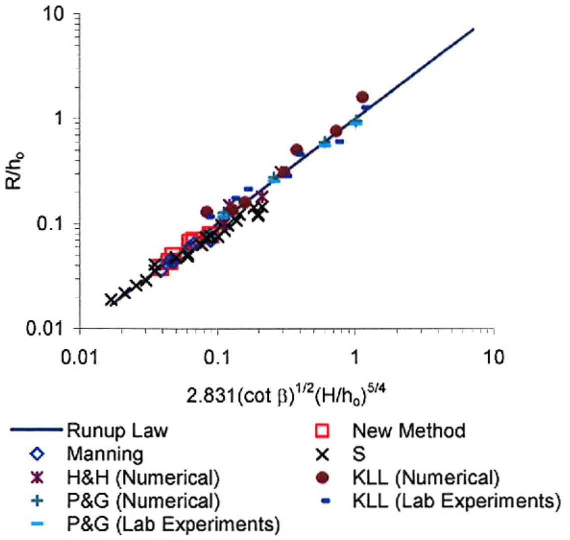


Fig.6 Run up height comparison.

Table 1 Run up height comparison

β	R/h$_o$			Square Error	
	RL	MM	NM	MM	NM
19.85	0.089	0.069	0.078	0.000404	0.000129
11.43	0.068	0.065	0.069	0.000009	0.000004
10	0.063	0.060	0.066	0.000008	0.000007
5.67	0.048	0.043	0.049	0.000017	0.000003
5	0.045	0.044	0.044	0.000001	0.000001
3.73	0.039	0.035	0.038	0.000010	0.000000
			Average	0.00007	0.00003

**RL = Run up law; MM = Manning Method; NM = New Method*

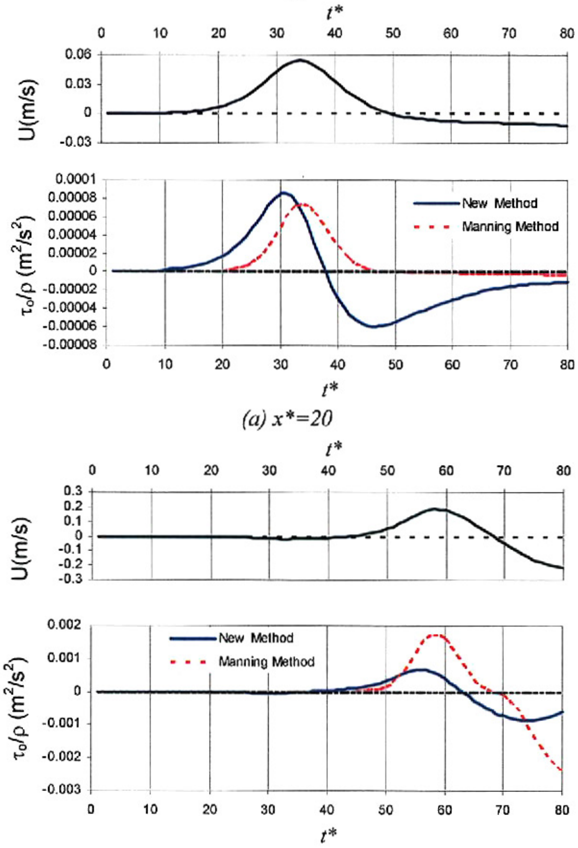
Bed stress accumulation analysis is conducted in order to understand the bed stress behavior during long wave run up. The following parameters are introduced for the analysis .

$$\overline{\tau_{0(-)}} = \sum_{t=1}^{T} \tau_0(t)_{(-)} / N_{(-)} \tag{12a}$$

$$\overline{\tau_{0(+)}} = \sum_{t=1}^{T} \tau_0(t)_{(+)} / N_{(+)} \tag{12b}$$

$$\overline{\tau_0} = \sum_{t=1}^{T} \tau_0(t) / N \tag{12c}$$

where $\overline{\tau_{0(-)}}$ is the average value of negative bed stress, $\overline{\tau_{0(+)}}$ is the average value of positive bed stress, $\overline{\tau_0}$ is the total average bed stress, $\tau_0(t)$ is the recorded bed stress at time t with $\tau_0(t)_{(-)}$ and $\tau_0(t)_{(+)}$ corresponds to negative and positive bed stress value respectively. N is the total number of time with $N_{(+)}$ and $N_{(-)}$ is the total number of time where the bed stress is positive and negative respectively. Negative and positive sign denotes the direction towards or leaving shoreline respectively. The maximum ($\tau_{0(\max)}$) and minimum ($\tau_{0(\min)}$) value of bed stress is also investigated.



(a) x=20*

(b) x=2*

Fig.7 Bed stress comparison.

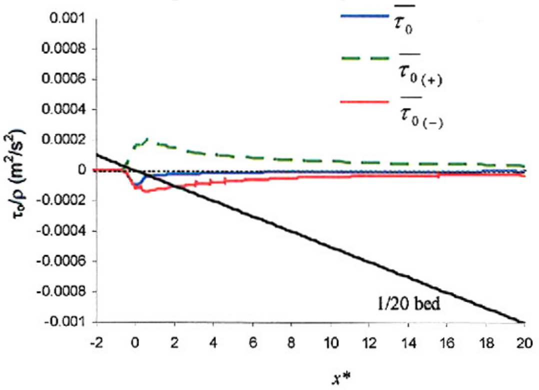


Fig.8 Bed stress accumulation (New Method).

It can be expected in the case of periodical waves that the sediment transport budget moving towards and leaving the shoreline to be equal. However, it was found that it does not imply to the case of long wave. Analyses of bed stress accumulation (**Fig. 8** and **Fig. 9**) shows that total average bed stress $\overline{\tau_0}$ with negative value is more dominant. Furthermore, the average positive value is higher than the negative value with the minimum bed stress magnitude is higher than the maximum (**Fig. 10**). Thus, sediment transport moving towards the shoreline will occurs in less frequency with relatively high magnitude and the sediment transport leaving the shoreline occurs longer with a short period of high magnitude.

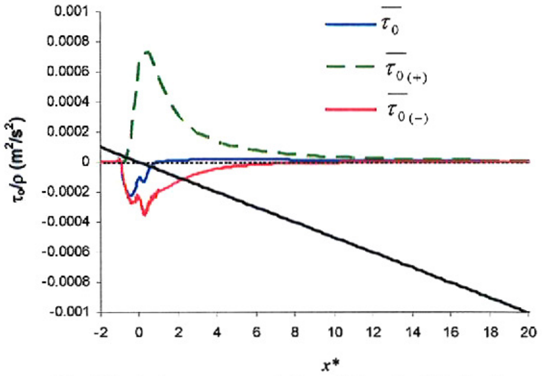


Fig.9 Bed stress accumulation (Manning Method).

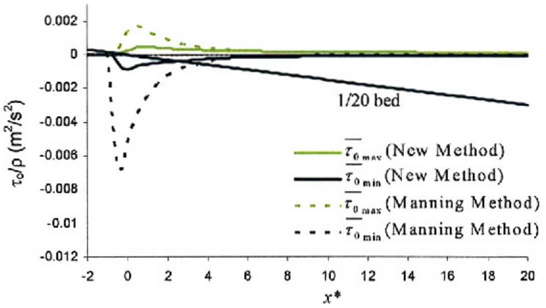


Fig.10 Maximum and minimum bed stress.

4. CONCLUSION

A new method for wave run up simulation has been developed in this study. The shallow water equation model is upgraded by introducing boundary layer approach for bed stress assessment. The shallow water equation and k-ω model are coupled and solved simultaneously at each time step. The free stream velocity and surface are obtained from the shallow water equation. The free stream velocity is used in the k-ω model for bed stress assessment. The assessed bed stress is used in the momentum equation.

The new method is applied to canonical problem of wave run up. The results show that the new method has improved the accuracy of the shallow water equation model. Bed stress comparison shows that the new method is able to reproduce the effect of overshooting. Furthermore, phase shift between the velocity and bed stress is also observed. The manning method does not able to assess these phenomenons. Bed stress accumulation analysis during the wave motion reveals that negative bed stress will have more influence in the process.

Overall, the proposed calculation method has shown promising results. It has shown to be efficient with higher accuracy than the conventional Manning method. Further improvement and application to various cases with different bed slope and different wave height (breaking wave case) should be conducted.

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