NUMERICAL ESTIMATION OF DYNAMIC DISPLACEMENTS AND THE FORCES ACTING ON THE MOORING SYSTEM OF SUBMERGED FLOATING BREAKWATER UNDER WAVE ACTION

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Abstract: 本研究は，数値波動水槽を利用して浮体の波浪動揺を解析することを目的とする．直接数値計算法による波動場の解析法に浮体を考慮し，浮体に作用する波力に対して浮体の運動方程式を解き，浮体の有限変位を考慮した動的応答を解く手法を開発した．また，本数値解析手法の妥当性を検証するために矩形断面を有する浮体を使った二次元水理模型実験を行って浮体の波浪応答や浮体による波変形，および緊張係留した係留索に作用する張力の計測を行った．数値計算結果と実験結果の比較より，浮体の波浪動揺，波変形，および係留張力の時間変化を本数値解析手法は良好に再現することが確認でき，本解析手法の有効性が示された．

Keywords: *Dynamic displacements; mooring forces; VOF simulation; submerged floating breakwater.*

1. INTRODUCTION

When designing a floating breakwater, knowledge of the internal forces is required for its structural design. Due to time varying nature of environmental loading, such as wind, waves, currents etc, a dynamic analysis is often required to compute these forces. Dynamic analysis also includes the evaluation of the motion responses of the floating body, such as sway, heave, roll etc, and the mooring line forces.

Some researchers have investigated the dynamics of floating body (Shirakura et el., 2000, Lau et el., 1990, Takaki et el., 2000). Seah and Yeung (2003) have studied the sway and roll hydrodynamics of cylindrical sections. Hur and Mizutani (2003) have developed a numerical model to estimate the wave forces acting on a three-dimensional body on a submerged breakwater. Sen (1993) developed a numerical method to simulate the motions of two-dimensional floating bodies. Most of the dynamics studies are done based on potential-flow theory in the frequency domain. Moreover, the boundary conditions are applied on the body surface at its initial position and this may cause the limitation of the model to simulate when the dynamic displacements are large. Also, the model can not simulate for time-marching irregular waves.

A two-dimensional numerical model is proposed here that combines the VOF method and the porous body model to simulate the nonlinear wave interaction with the floating body. A rectangular shaped pontoon type submerged floating breakwater supported by mooring chain is considered in the model. The alignment of mooring chains is considered for both vertical and inclined directions. The dynamics of the body is calculated considering its time-marching finite displacements. The time-marching boundary conditions enable it to consider the non-linear dynamic interaction between the waves and the floating body. Furthermore, the model can simulate for both regular and irregular waves. The numerical results reproduce well the ones measured by a two-dimensional experimental study. The details of the numerical and experimental studies are presented in this paper.

2. NUMERICAL INVESTIGATIONS

The numerical model consists of the continuity equation, the Navier-Stokes equation for incompressible fluid and the advection equation that represents the behavior of the free surface. The two-dimensional numerical domain is divided into staggered meshes in both horizontal (x-axis) and vertical (z-axis) directions. As there occurs the dynamic displacements (sway, heave and roll) of the breakwater due to wave action, the numerical cells can be classified into five types; a full cell filled with fluid, an empty cell occupied by air, a surface cell containing both fluid and air, an obstacle cell that represents the structure and the porous cell containing the fluid, the structure and/or air. So the continuity equation, Navier Stokes equations and the VOF function equation should be modified as below considering the effects of γ_x, γ_z and γ_v, where γ_x and γ_z represent the ratio of the permeable length to the cell length in vertical and horizontal directions respectively and γ_v represents the ratio of the permeability volume in a cell.

The continuity equation is,

$$\frac{\partial(\gamma_x u)}{\partial x}+\frac{\partial(\gamma_z w)}{\partial z}=q(x,z,t) \quad (1)$$

$$q(x,z,t)=\begin{cases} q^*(z,t)\ldots\ldots\ldots\ldots x=x_S \\ 0\ldots\ldots\ldots\ldots\ldots x\neq x_S \end{cases} \quad (2)$$

where u and w are the flow velocity of x and z direction respectively, q is the wave generation source with q^* as the source strength which is only located at $x = x_S$ and t is the time.

The Navier-Stokes equations,

$$\gamma_v\frac{\partial u}{\partial t}+\gamma_x u\frac{\partial u}{\partial x}+\gamma_z w\frac{\partial u}{\partial z}=-\frac{\gamma_v}{\rho}\frac{\partial p}{\partial x}+ \upsilon\left[\frac{\partial}{\partial x}\left\{\gamma_x\left(2\frac{\partial u}{\partial x}\right)\right\}+\frac{\partial}{\partial z}\left\{\gamma_z\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right\}\right] \quad (3)$$

$$\gamma_v\frac{\partial w}{\partial t}+\gamma_x u\frac{\partial w}{\partial x}+\gamma_z w\frac{\partial w}{\partial z}=-\frac{\gamma_v}{\rho}\frac{\partial p}{\partial z}+ \upsilon\left[\frac{\partial}{\partial x}\left\{\gamma_x\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\right\}+\frac{\partial}{\partial z}\left\{\gamma_z\left(2\frac{\partial w}{\partial z}\right)\right\}\right]+\frac{1}{3}\upsilon\frac{\partial q}{\partial z}-\gamma_v g \quad (4)$$

where p is the pressure, υ is the kinematic viscosity, ρ is the fluid density and g is the gravitational acceleration.

The advection equation of VOF function F,

$$\gamma_v\frac{\partial F}{\partial t}+\frac{\partial(\gamma_x uF)}{\partial x}+\frac{\partial(\gamma_z wF)}{\partial z}=Fq \quad (5)$$

As suggested by Brorsen and Larsen (1987), the flux density of wave generation source q is gradually intensified for initial three wave periods from the start of computation for stable wave generation. The mesh sizes are used as $\Delta x = 2$ cm and $\Delta z = 1$ cm. An added dissipation zone method is used to treat the open boundaries. The pressure in the full cell can be calculated by means of the SOLA scheme. However, in surface cells, different procedures are required because the locations of the pressure points in the staggered mesh generally differ from the actual locations on the free surface. Therefore, the linear interpolation or extrapolation between the pressure on the free surface (atmospheric pressure) and the pressure of the adjacent full cell is used to calculate the pressure of the surface cell. Another boundary condition is applied to the cells adjacent to the obstacle faces. That is, the water particle velocities are set to equal as the velocities of the moving breakwater in respective directions.

It is considered that the floating body is of very light weight compared to the buoyancy forces acting on it vertically. This assumption results no slack state in the mooring lines that causes no impulsive force on it. The dynamics of the floating body due to wave action are calculated as shown in Fig. 1. The horizontal displacement (sway), vertical displacement (heave) and the rotational movement (roll) of the body are calculated with respect to its center of gravity. Referring to Fig. 1, the dynamics of the body at any instantaneous position can be calculated using following equations.

The wave forces acting on the body H3, H4, H5, H6, V3, V4, V5, and V6 are calculated by integrating forces acting on the respective surface of the body which are estimated from the pressure of the respective cells. The pressure calculation considers both the static and dynamic pressures of the water particle. As the pressure acts at the direction of normal to the body surface, so the wave forces mentioned above are calculated by taking the components of the normal forces in the respective directions of x and z.

Applying the Newton's second law, the horizontal motion equation of the body can be written as below:

$$\sum F_X = m.a_x$$

$$or,\ H3+H5-H4-H6-2.T3\cos\theta 3+2.T4\cos\theta 4 = ma_x \quad (6)$$

where, a_x denotes the acceleration of the body in horizontal direction and T3 and T4 represent the offshoreside and onshoreside mooring force respectively

The similar equation for vertical motion of the body can be written as below:

$$\sum F_Z = m.a_z$$

$$or,\ V4+V6+W-V3-V5+2T3\sin\theta 3+2T4\sin\theta 4 = m.a_z \quad (7)$$

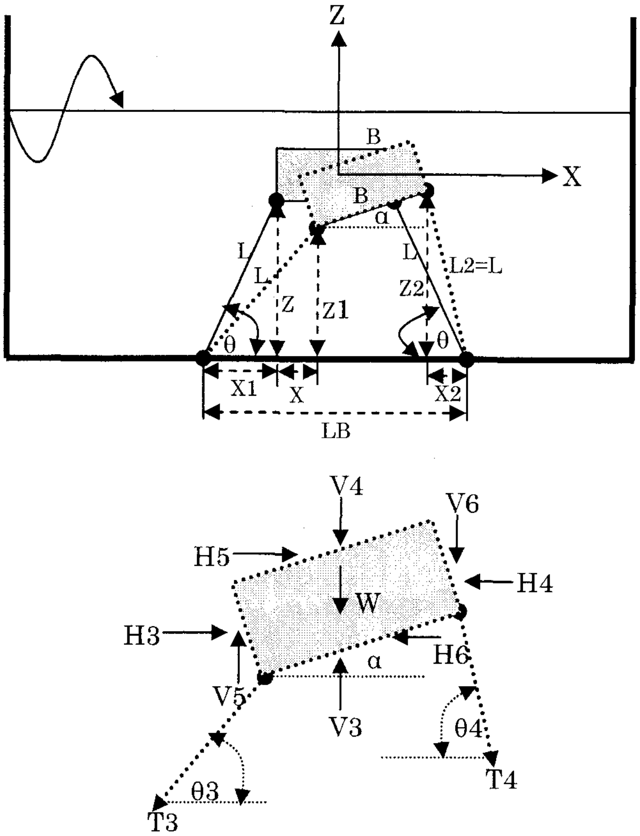

Fig. 1 Calculation of floating body dynamics

where, a_z denotes the acceleration of the body in vertical direction and W is the weight of the body. Again, the rotation of the body is governed by the moments acting on the body. Taking the moment acting of the center of gravity of the body in anti-clockwise direction, following equation is found.

$$\sum M_{Cg.X} + \sum M_{cg.Z} + \sum M_{cg.T} = I\alpha \qquad (8)$$

where, $\sum M_{Cg.X}$, $\sum M_{Cg.Z}$, $\sum M_{Cg.T}$ represent the summation of moment governed by the horizontal wave forces, vertical wave forces and the mooring lines forces respectively, I is the mass moment of inertia of the body and α is the roll displacement of the body.

Also, considering no slack condition, from the geometry of the Fig. 1, we write Eq. (9) and (10).

$$Z1 = \sqrt{L^2 - (X1 + X)^2} \qquad (9)$$

$$(LB - X1 - X - B\cos\alpha)^2 + (Z1 + B\sin\alpha)^2 = L^2 \qquad (10)$$

By solving above five simultaneous equations (Eq. (6) to Eq. (10)), the unknown parameters a_x, a_z, α, T3 and T4 are calculated. Finally the sway and heave displacements are estimated using a_x and a_z values.

Mizutani et el. (2004) developed a VOF simulation model to study the dynamics of the floating breakwater considering finite displacement. They solved the dynamics equations explicitly and the model considers only vertical moorings. In the present model, we consider both vertical and inclined mooring alignments and the simultaneous equations are solved implicitly.

3. LABORATORY EXPERIMENTS

A set of experiments is done in a two-dimensional wave tank at Nagoya University. The detail experimental setup is shown in Fig. 2. The wave tank is 30m long, 0.7m wide and 0.9m deep. The floating breakwater considered in this study is pontoon type and is made of acrylic plate. The floating body is 30.4 cm long, 68 cm wide and 13.7 cm deep. The body is anchored to the bottom of the tank with mooring chain with 90 degree and 60 degree inclinations. The floating body is always anchored so that its top surface was at the height of 62 cm from the bottom of the wave tank. Water depth in the wave tank was varied as 62 cm, 65 cm

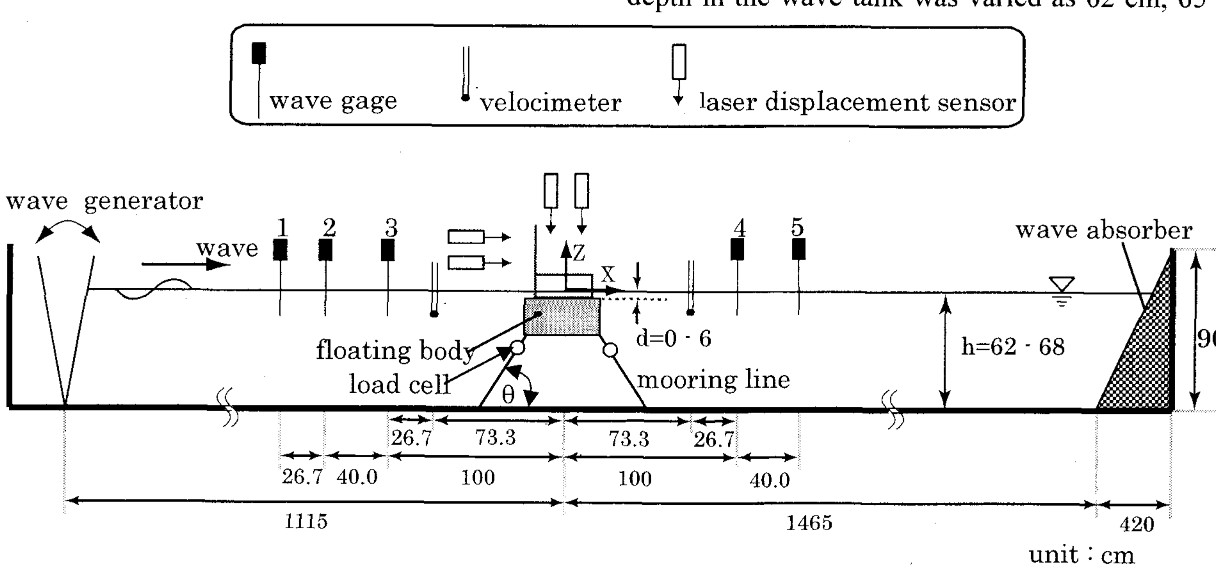

Fig. 2 Detail experimental setup

and 68 cm. Regular waves are generated from the wave generator and the wave steepness (H/L) is varied as 0.01, 0.02 and 0.03. Five wave gauges and two velocity meters are set up in front and behind the floating body to measure the water surface elevation and the water particle velocity during the experiment. Because of the two-dimensional experiment, the sway (Δx), heave (Δz) and the roll (α) of the body occurs due to the wave action and these displacements of the floating body are measured with the laser system composed of four displacement meters. Two vertical laser rays and two horizontal laser rays are focused on a white paper box that is set up at the top surface of the floating body. The mass of the box is much less than that of the floating body and it is assumed not to affect the motion of floating body. To measure the tensile forces acting on the mooring chains supporting the floating body that receives the wave action, two ring type load cells are connected with the mooring chains, one in offshore side and another in onshore side. Other two dummy cells are connected with other two mooring chains to make symmetry.

4. RESULTS AND DISCUSSION

This section discusses about the numerical model results and its performance compared to that of the experimental ones. The results are discussed in the following sub-sections:

(a) Estimation of water surface profiles

The time series water surfaces at five points (Fig. 2) of offshore and onshoresides measured during the experiment are compared with numerical simulation values and are shown in Fig. 3. It represents the conditions of 3.4 cm initial wave height, 1.6 seconds wave period, 68 cm water depth which means that the top surface of the body is submerged at 6 cm below the still water level and vertical mooring lines. The comparison shows good agreement of numerical simulations of water surface profiles at offshore side (wave gage 1, 2 and 3) with the measured data. The water surfaces at onshoreside of the breakwater (wave gage 4 and 5) show inconsistent variation in both estimated and measured values. Also the height of the transmitted waves are seen smaller that that of the initial wave height. These happen due to the breaking of the incoming waves by the breakwater. The good estimation of the water surfaces at both upstream and downstream sides of the floating breakwater by the developed numerical model demands the good performances of the VOF method in evaluating the free surfaces.

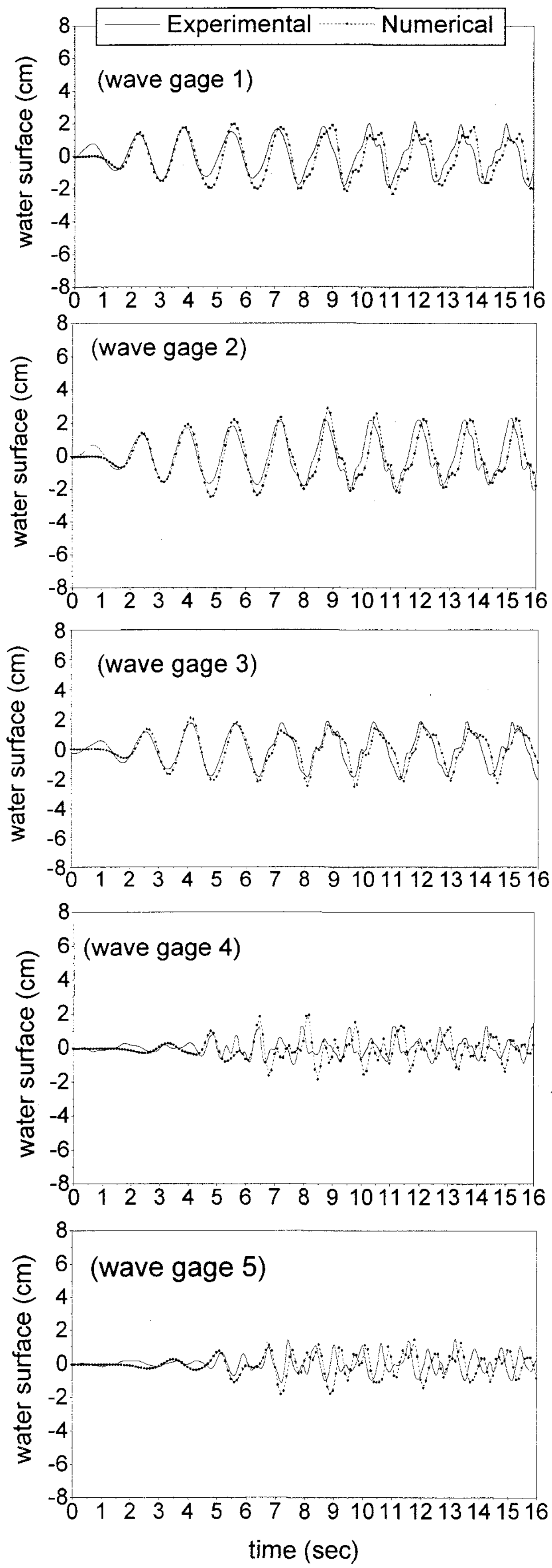

Fig.3 Comparison between numerical and experimental results of the water surface profiles at both upstream and downstream sides of the breakwater (H=3.4 cm, T=1.6 sec, h=68cm, d=6 cm, θ=90 degree)

(b) Estimation of displacements of floating body

Fig. 4 shows the comparison between the estimated time series values of the sway and heave displacements of the breakwater with the measured values. In this case the breakwater top surface coincides with the still water level (h=62 cm, d=0 cm) and breakwater is moored with vertical mooring (θ=90 degree). The figure shows very good agreement between the estimated and measured values. The positive values of the sway represent the horizontal displacements of the body along the onshoreside, whereas, the positive values of the heave represent the downward displacements of the body with respect to its initial position at still water level. Due to light weight of the breakwater compared to the buoyancy force acting on it that causes the mooring lines always in tension, as mentioned earlier, and due to the vertical mooring lines, there occur no roll displacement of the breakwater. But the roll is seen in Fig. 5 where the mooring lines are anchored to the tank bottom at 60 degree inclination. In this figure the very good estimation of the roll values are observed, whereas, there are little bit differences between the estimated of measured values of the sway and heave displacements.

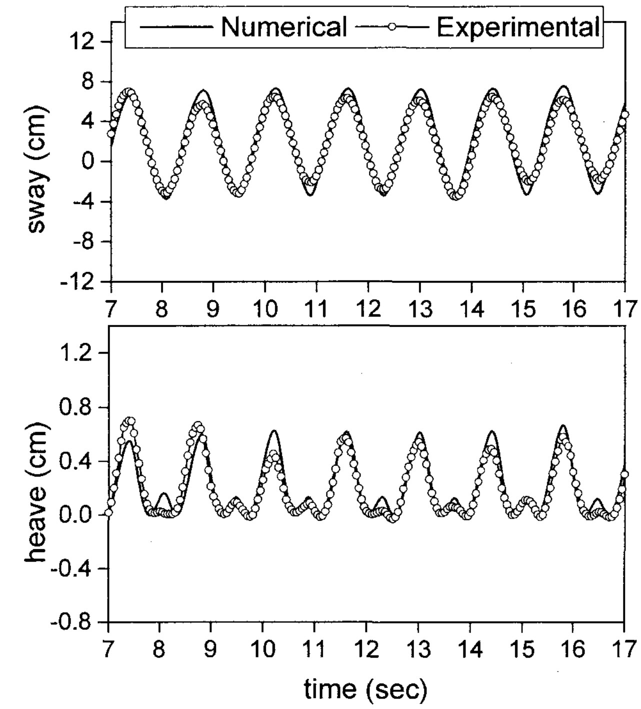

Fig. 4 Comparison between numerical and experimental results of the dynamics of the floating body supported by vertical moorings (H=8.2 cm, T=1.4 sec, h=62 cm, d=0 cm, θ=90 degree)

(c) Estimation of mooring lines forces

The time series estimation of the forces acting on the onshoreside and offshoreside mooring lines are compared with the measured values using load cells during the experiment. The comparisons for two cases are shown in Fig. 6 and in Fig. 7. Fig. 6 represents the condition of vertical mooring lines (θ=90 degree), zero submergence depth (h=62 cm, d=0 cm), H=8.2 cm and T=1.4 seconds, whereas, Fig. 7 represents the condition of inclined mooring lines (θ=60 degree), 3 cm submergence depth (h=65 cm, d=3 cm), H=3.8 cm and T=0.9 second. It is seen good agreement between the numerical model results and experimentally measured values of both the onshoreside and offshoreside mooring line forces in case of Fig. 6. In this figure it is also seen that the minimum value of the measured offshoreside mooring line force is 145 gm-force, which is above the self weight of the mooring chain (about 60 gm-force). It demands that the overall conditions keep the mooring lines always in tension and there occur no slack state in them.

Fig. 7 shows little bit difference in the magnitudes of offshoreside and onshoreside mooring line forces in their comparison of estimated and measured values. It suggests that more improvement of the numerical model, especially in case of the breakwater is moored with inclined mooring chain, should be done.

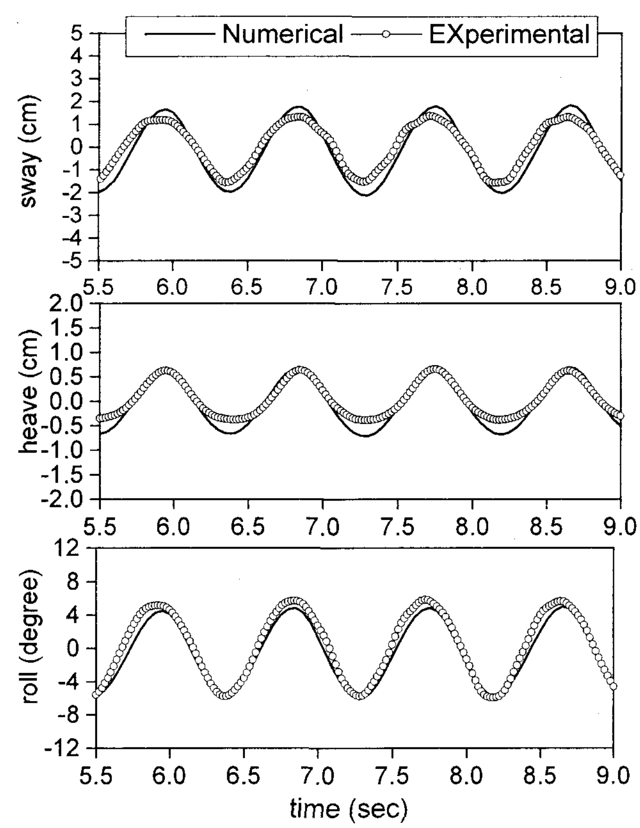

Fig. 5 Comparison between numerical and experimental results of the dynamics of the floating body supported by inclined moorings (H=3.8 cm, T=0.9 sec, h=65 cm, d=3 cm, θ=60 degree)

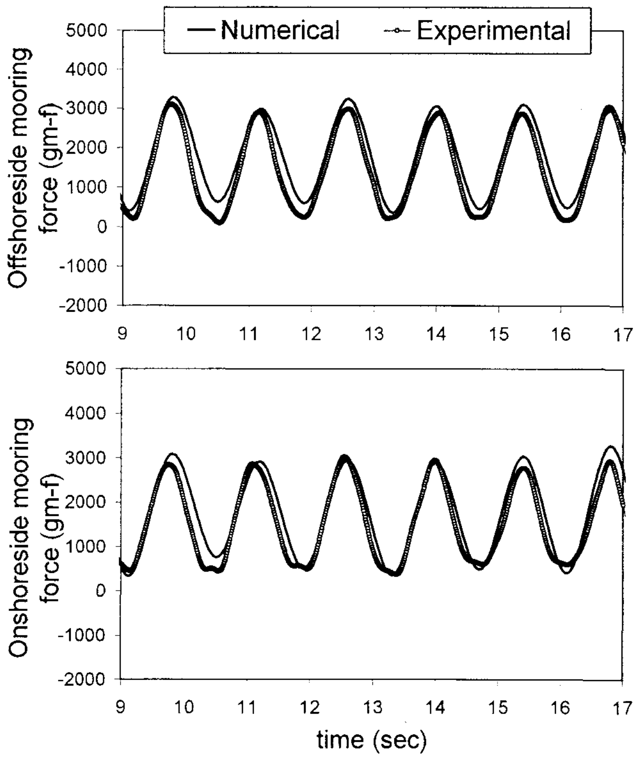

Fig. 6 Comparison between numerical and experimental results of the forces acting on the vertical mooring lines anchoring the breakwater (H=8.2 cm, T=1.4 sec, h=62 cm, d=0 cm, θ=90 degree)

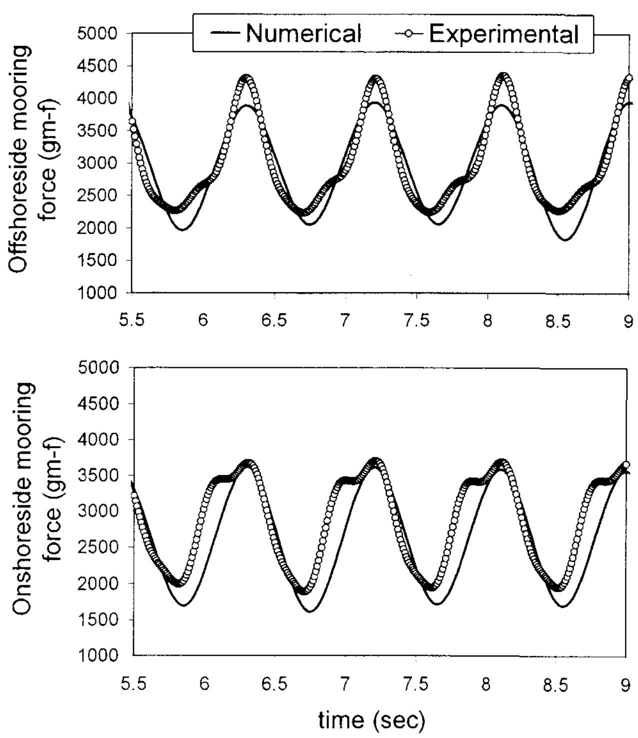

Fig. 7 Comparison between numerical and experimental results of the forces acting on the inclined mooring lines anchoring the breakwater (H=3.8 cm, T=0.9 sec, h=65 cm, d=3 cm, θ=60 degree)

5. CONCLUSION

A two-dimensional numerical model is developed to estimate the dynamics of the pontoon type rectangular shaped submerged floating breakwater which is anchored with mooring chain. Also, the offshoreside and onshoreside mooring line forces can be estimated by this model. The model can estimate the displacements (sway, heave and roll) of the breakwater under regular wave action and the forces acting on its mooring lines for both vertical and inclined mooring. The model combines the VOF method and the porous body model to simulate the nonlinear wave deformation including wave breaking and its interaction with the floating body. Using the model, the free surfaces are treated with VOF method and its comparison with experimental values shows good performance of this method to treat the free surfaces. Also, the estimated values of the sway, heave, roll, offshore and onshoreside mooring line forces are compared with the experimentally measured data. The comparison shows good agreement between them. So the developed numerical model demands good performance in estimating water surface profiles, dynamic displacements and mooring forces of submerged floating breakwater.

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