UNSTEADY RADIAL FLOW OF OIL BEING DISCHARGED FROM A SOURCE ON THE OCEAN

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SUMMARY

The problem considered is that of calculating the uncoupled stratified flow associated with a suddenly initiated and uniformly sustained source of mass flux and/or energy flux. A solution is constructed from critical flow conditions, providing maximum discharge for given head, using the method of characteristics, and it is remarked that this solution tends towards a steady state defined simply by constant energy head. A trial calculation suggests that this steady state solution can be employed for finite time, to a good approximation. The approximate solution then consists of a centred simple wave extending from the line $r=r_0$ where r_0 is the radius at which the flow becomes critical for given mass and energy flux, to a line $r/t=\sqrt{3}\ \sqrt{g(1-\lambda)h_0}$, where λ is a relative density of the oil and h_0 is the critical depth of the oil layer.

INTRODUCTION

The problem posed is the following : to compute the plane radial flow caused by a suddenly initiated and uniformly sustained source of mass flux and/or energy flux. The application envisaged is that ofan uncoupled surface layer of oil spreading over water. This case may be realised immediately after the holing of a tanker.

The equations employed are those of mass and momentum [1,3] :

$$r\frac{\partial h}{\partial t}+\frac{\partial}{\partial r}(uhr)=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+g(1-\lambda)h\frac{\partial h}{\partial r}=0 \qquad (2)$$

where u and h are the velocity and depth in the surface layer, r is the radial distance from the source, t is time and λ is the density of the surface fluid as a fraction of the density of the lower fluid. In the event that applications with mixing are envisaged, eq. (1) must be modified to the extent of a mixing term on the right, while shear stresses (due to wind or friction) and Coriolis forces may be added to the right of eq. (2). These terms then simply modify the quasi-invariants of eqs. (1) and (2) as outlined later.

By the use of eq. (1), eq. (2) can be written, like eq. (1), in pseudo-conservation form :

$$r\frac{\partial}{\partial t}(uh)+\frac{\partial}{\partial r}\left[r\left\{u^2h+\frac{g(1-\lambda)h^2}{2}\right\}\right]=0 \qquad (3)$$

so that $\left\{u^2h+\frac{g(1-\lambda)h^2}{2}\right\}$ is the momentum flux density (per unit span normal to the radii) as in rectilinear flows. Equations (1) and (2), (3) are equivalent to the energy equation.

By the use of eq. (1), eq. (2) can also be written in the form :

$$r\frac{\partial}{\partial t}\left\{\frac{hu^2}{2}+g\frac{(1-\lambda)h^2}{2}\right\}+\frac{\partial}{\partial r}\left[uhr\left\{\frac{u^2}{2}+g(1-\lambda)h\right\}\right]=0 \qquad (4)$$

so that $\left\{\frac{hu^2}{2}+g\frac{(1-\lambda)h^2}{2}\right\}$ is the energy level and $\left\{\frac{u^2}{2}+g(1-\lambda)h\right\}$ is the energy flux density per unit mass or "energy head", again as in rectilinear flows.

All of these equations lead to characteristics defined by

$$\dot{r}_\pm=u\pm\sqrt{g(1-\lambda)h} \qquad (5)$$

and quasi-invariants

$$\left[u\pm2\sqrt{g(1-\lambda)h}\right]_{t_1}^{t_2}=\pm\int_{t_1}^{t_2}\frac{u\sqrt{g(1-\lambda)h}}{r}dt=\mp\phi_\pm \qquad (6)$$

along $\pm$ characteristics.

SOURCE CONDITIONS

We require a source condition whereby no info-

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rmation is fed back from the varying main flow into the source, while one point information is fed, from the source into the main flow. This condition is attained if and only if the flow is critical at the source, i.e. $u_0=\sqrt{g(1-\lambda)h_0}$, where the suffice 0 stands for a source condition. For any given mass flux and energy flux this condition determines a radius from which a computation of the flow can be initiated, namely that radius r_0 for which

$$u_0h_0r_0=\text{constant}=\text{say } \alpha$$

$$\frac{u_0^2}{2}+g(1-\lambda)h_0=\text{constant}=\text{say } \beta$$

so that $r_0=g(1-\lambda)\alpha(3/2\,\beta)^{3/2}$

Inspection of the quasi-invariants in the hodograph $(u\sim2\sqrt{g(1-\lambda)h})$ plane shows that the flow thus initiated will be supercritical for all $r>r_0$.

FRONT CONDITIONS

As a front condition we take the coincidence of C_+ and C_- characteristics, as corresponds to a zero fluid depth. Such a front is sometimes called a "St. Venant Front", and is still used in certain computations. However, as it leads to an infinite Froude number at the front, we doubt whether it can ever develop fuyll without the prior intervention of instability. This notwithstanding, the solution may still be useful as a first approximation to a physically realistic flow (e.g. [1,2])

CONSTRUCTION OF THE SOLUTION

We first remark that, for points sufficiently close to r_0, the flow behaves initially as a rectilinear flow. The solution is therefore the usual centred wave solution of rectilinear flow (e.g. [3] p. 102). The points of the initial solution are determined in the hodograph plane by the line

$$u+2\sqrt{g(1-\lambda)h}=u_0+2\sqrt{g(1-\lambda)h_0}$$

and the slopes of the corresponding C_- characteristics. This determination is illustrated by points A_0, B_0, C_0, etc. in the example illustrated, for

$$u_0=\sqrt{g(1-\lambda)h_0}=1\text{ m/s}$$

$$\text{at } r_0=2\text{ m}$$

in Figs. 1 and 2.

From the points of the initial solution, the subsequent solution is constructed by the graphical method of quasi-invariants as illustrated in Figs. 1 and 2. It is seen that the unsteady state solution in the limit, $t\to\infty$, tends towards a steady state solution defined simply by the condition that the mass flux and energy flux density should be constant through successive anulli. The front is then seen to start with a celerity of $u_0+2\sqrt{g(1-\lambda)h_0}$ and to approach, asymptotically, a celerity of $\sqrt{\frac{u_0^2}{2}+g(1-\lambda)h_0}$.

Now it is observed from the hodograph characteristics that the system tends ratherrapidly to the lim-

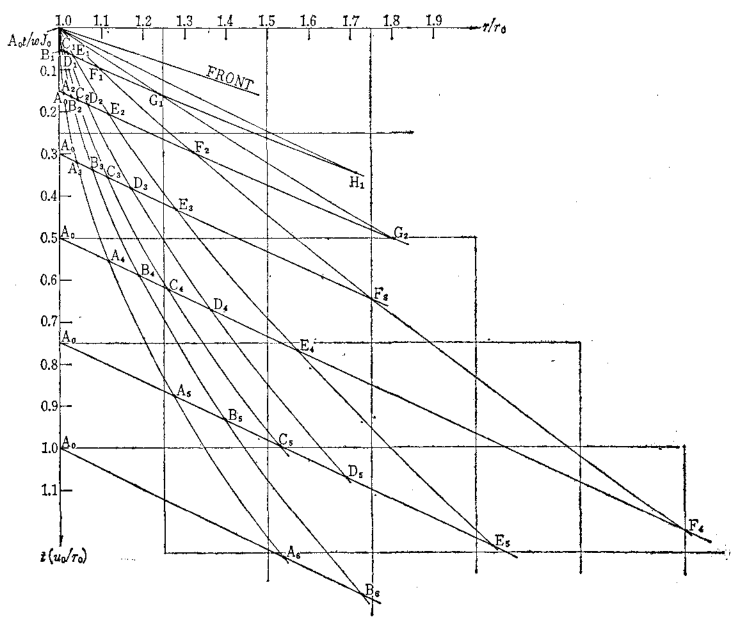

Fig. 1

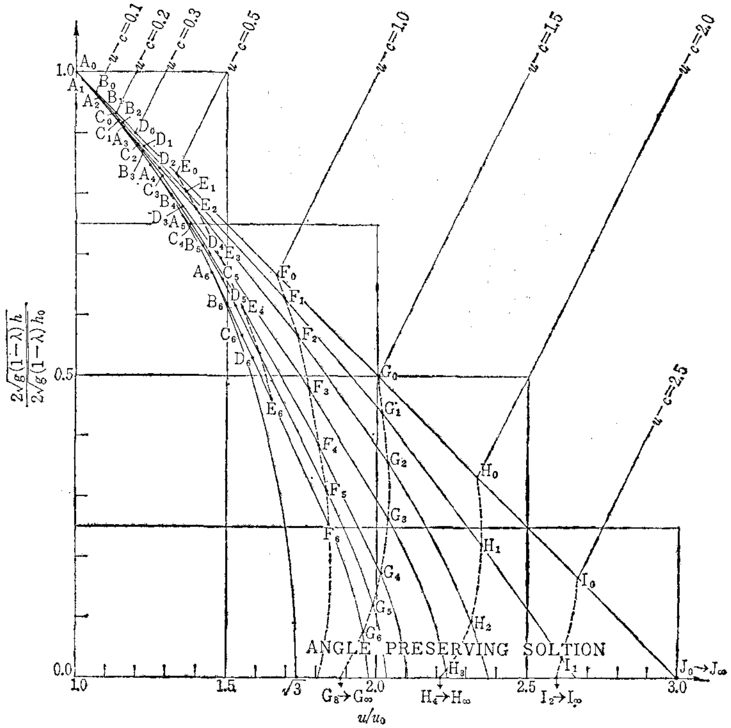

Fig. 2

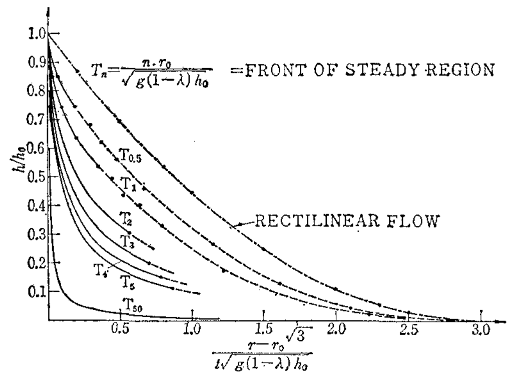

Fig. 3

iting steady case. This observation suggests that, although this limiting steady case corresponds properly only to $t=\infty$, it could be used as a "quasi-steady" approximation for fiinte t. In this case the solution would again consist of a centred simple wave with C_- celerities defining the flow values on the curve $\frac{u^2}{2}+g(1-\lambda)h_0=\frac{u_0^2}{2}+g(1-\lambda)h_0$ in the $u \sim 2\sqrt{g(1-\lambda)h}$ plane.

In order to facilitate comparison of the quasi-steady radial solution and the rectilinear solution, they have been plotted non-dimensionally in Fig. 3. In this figure we have introduced a static head h_p defined by $2\sqrt{g(1-\lambda)h_p}=u_0+2\sqrt{g(1-\lambda)h_0}$ as corresponds to the still water level upstream from the critical flow in the rectilinear case. In the case of radial flow, of course, this head has no corresponding physical significance, although it can be calculated from the same formula.

We remark that in the event of mixing or energy diffusion by friction, Coriolis acceleration, etc., the computation remains identical except for the inclusion of additional terms under the time integrals in eq. (6). The present solution then forms the limiting case of all such flows.

CONCLUSIONS

A radial flow analagous to the rectilinear St. Venant flow can be easily constructed using the method of characteristics. The flow front has celerity $2\sqrt{g(1-\lambda)h_p}$ at $t=0$, tending asymptotically to $\sqrt{3} \cdot \sqrt{g(1-\lambda)h_p}$ at $t=\infty$. A trial computation suggests that the solution rapidly approaches the steady state, properly corresponding to $t=\infty$, so that the steady state can be employed approximately for finite t. In the resulting approximation the solution forms a centerd simple wave defined by a constant total head, but at the same time quite close to the centred simple wave of rectilinear flow with constant Riemann invariant. However instead of a front defined for all t by $r/t=2\sqrt{g(1-\lambda)h_p}$ we now have one defined by $r/t=\sqrt{3}\sqrt{g(1-\lambda)h_p}$ for all t.

REFERENCES

[1] M.B.Abbott : On the spreading of one fluid over another, La Houille Blanche, 5, 6, 1961.

[2] M.B. Abbott : Les critières de l'instabilité des fluides incompressibles aux surfaces libres, Societé Hydrotechnique de France, VIII èmes Journées d'Hydraulique, Lille, 1964.

[3] M.B. Abbott : An Introduction to the method of characteristics, Thames and Hudson (London), 1966; American Elseviers (New York), 1966.