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METHODS OF ESTIMATING CHEMICAL PRESTRESS AND EXPANSION DISTRIBUTION IN EXPANSIVE CONCRETE SUBJECTED TO UNIAXIAL RESTRAINT

(Reprint from Transactions of JCI, Vol.4, 1982)

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SYNOPSIS

In Japan expansive concretes are used for various types of structure both in shrinkage-compensating concretes and in chemically prestressed concretes. An important problem in these applications is an accurate estimation of chemical prestress and expansive strain. In this paper, the estimating method is proposed which is based on the concept of work quantity performed by expansive concrete on restraint. And the method is applicable to the cases not only of restraint by reinforcing bars, but also of steel, foundation ground or existing concrete structures.

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TRANSACTIONS OF THE JAPAN CONCRETE INSTITUTE VOL. 4, 1982

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METHODS OF ESTIMATING CHEMICAL PRESTRESS AND EXPANSION DISTRIBUTION IN EXPANSIVE CONCRETE SUBJECTED TO UNIAXIAL RESTRAINT

Yukikazu TSUJI*

ABSTRACT

Methods of estimating distributions of chemical prestress and expansive strain produced in concrete members using expansive concrete are proposed. These methods are based on the concept of work quantity that expansive concrete performs against restraint and do not include constants such as modulus of elasticity and creep coefficient of expansive concrete. In these methods two hypotheses such as the ones below are set. a) Expansive strains are linearly distributed in the direction of cross-sectional height. And, b) The work quantity U that expansive concrete performs against restraint per unit volume, is a constant value regardless of the degree of restraint by external restraining objects. It is possible to easily estimate the distributions of members under various conditions of restraint such as reinforcing bars, steel members, soil foundations and reinforced concrete structures by merely measuring expansive strains of standard uniaxially restrained specimens with a sufficient accuracy for practical purposes.

INTRODUCTION

In order to apply expansive concrete to various structures and improve their mechanical characteristics, it will be of importance to quantitatively grasp the chemical prestresses and expansive strains produced in the structures. For the cases of restraints only in uniaxial directions,there have been useful estimation methods proposed up to now. However, constants used for these methods, such as the modulus of elasticity and creep coefficient of expansive concrete which influence the accuracies of the estimates have not sufficiently been examined. At present, it appears to be a fairly difficult matter to accurately estimate these constants.

The author has proposed from some time ago a very convenient method based on the conception of work quantity and not involving such constants1), and a general method for making estimations for the cases of applying expansive concrete to reinforced concrete2). These methods are capable of making estimations by measuring work quantities of uniaxially restrained specimens with one kind of restraining steel ratio to be the reference for reinforced concrete members having reinforcing bars arranged asymmetrically in the

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† Japan Concrete Journal, Vol. 19, No. 6 (1981)

direction of cross-sectional height. The present study expands these methods, and proposes a general estimation method which can be applied not only to reinforcing bars, but also to the cases of restraint from steel, foundation ground or existing concrete structures.

1. BASIC EQUATION FOR ESTIMATION

The cross section to be considered for expansive concrete restrained in a uniaxial direction, as shown in Fig.1(a), is a case of a single symmetrical axis existing in the direction of cross-sectional height. Expansive concrete is restrained by reinforcing bars placed internally and by external restraining objects such as steel, foundation ground or existing concrete structures. And, hypotheses such as the ones below are set.

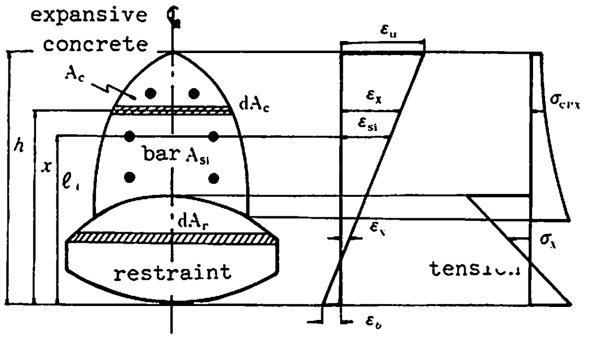
a) Expansive strains are linearly distributed in the direction of cross-sectional height(see Fig.1(b)). Accordingly, the cases considered are those when misalignments are not produced at the respective boundaries between expansive concrete and reinforcing bars or external restraining objects.

b) The work quantity U that expansive concrete performs against restraint per unit volume, is a constant value regardless of the degree of restraint by external restraining objects.

Firstly, from hypotheses a), the expansive strain ε_x at a distance x from the bottom fiber is given by the equation below.

$$\varepsilon_x=\varepsilon_b+(\varepsilon_u-\varepsilon_b)x/h=\varepsilon_b+\varepsilon_d x/h \qquad (1)$$

where, ε_u: expansive strain at the top fiber of cross section
ε_b: expansive strain at the bottom fiber of cross section(a negative value in case of shrinkage as shown in Fig. 1(b))
ε_d: difference in expansive strains between the top and bottom fibers of the cross section
h: the height of the cross section



(a) Cross section (b) Expansive strain (c) Chemical prestress

Fig. 1. Explanatory Diagram of General-type Cross Section

For the expansive strain ε_{si} of reinforcing bars of cross-sectional area A_{si} at a distance of l_i from the bottom figer, it will suffice to substitute l_i into x of Eq.(1).

Next, applying hypothesis b) to a minute area dA_c of Fig. 1(a), this hypothesis may be expressed as Eq. (2), and therefore, Eq. (2)'.

$$\frac{1}{2}\varepsilon_x\sigma_{cpx}dA_c=UdA_c \tag{2}$$

$$\varepsilon_x\sigma_{cpx}=2U \tag{2'}$$

where, σ_{cpx} : chemical prestress transferred to expansive concrete at a location a distance of x from the bottom fiber

From the conditions of the equilibrium of forces and moments in the direction of restraint, the following respective equations are obtained:

$$\int_{A_c}\sigma_{cpx}dA_c-\int_{A_r}\sigma_x dA_r-\sum A_{si}\varepsilon_{si}E_s=0 \tag{3}$$

$$\int_{A_c}\sigma_{cpx}x dA_c-\int_{A_r}\sigma_x x dA_r-\sum A_{si}\varepsilon_{si}E_s l_i=0 \tag{4}$$

where, σ_x: chemical prestress produced in external restraining object ($=\varepsilon_x E_r$)

E_r: Young's modulus of external restraining object

E_s: Young's modulus of reinforcing bar

Substituting Eqs.(1) and (2)' into Eqs. (3) and (4), for example, and rearranging for ε_b and ε_d , the simultaneous equations(5) and (6) constituting the basis of the estimation including these unknowns are obtained.

$$\int_{A_c}\frac{2U}{\varepsilon_b+\varepsilon_d x/h}dA_c-\int_{A_r}(\varepsilon_b+\varepsilon_d x/h)E_r dA_r -\sum A_{si}(\varepsilon_b+\varepsilon_d l_i/h)E_s=0 \tag{5}$$

$$\int_{A_c}\frac{2Ux}{\varepsilon_b+\varepsilon_d x/h}dA_c-\int_{A_r}(\varepsilon_b+\varepsilon_d x/h)xE_r dA_r -\sum A_{si}(\varepsilon_b+\varepsilon_d l_i/h)l_i E_s=0 \tag{6}$$

Since the above equations generally have a solution other than $\varepsilon_d=0$, by computing ε_b and ε_d satisfying both of the equations, the expansive strains produced at the various locations in the cross section are given by Eq. (1). The chemical prestresses transferred to the concrete are given by Eq. (2)', and the distribution will be a hyperbolic one as shown in Fig. 1(c).

The work quantity U performed by expansive concrete is determined by the expansive capability of the concrete produced in the object structure, and it will be difficult to actually estimate the value with accuracy.

Using expansive strain ε_{ss} obtained from the restrained specimen by Method A prescribed in Ref. 1, "Method of Testing Restrained Expansion and Shrinkage of Expansive Concrete" in JIS A 6202, "Expansive Additive for Concrete," (hereafter called uniaxially restrained standard specimen), a method employing the work quantity U_s computed by Eq. (7) below corrected by taking into account the shape and dimensions of the structure to be estimated, curing conditions, etc. may be considered. In effect, since the temperature hysteresis and the condition of moisture supply in an actual structure differ compared with a JIS Method A uniaxially restrained standard specimen, it becomes necessary to establish the value of work quantity of these factors on work quantity. It is desirable for full-size experiments to be repeated for this purpose. With work quantity established, it is possible to compute by the basic equations (5) and (6) the degree of influence on chemical prestress and expansion distribution of the cross-sectional dimensions and the shape of expansive concrete, and the restraining actions of reinforcing bars, steel, foundation ground and existing concrete.

$$U_s=\frac{1}{2}p_sE_p\varepsilon_{ss}^2 \qquad (7)$$

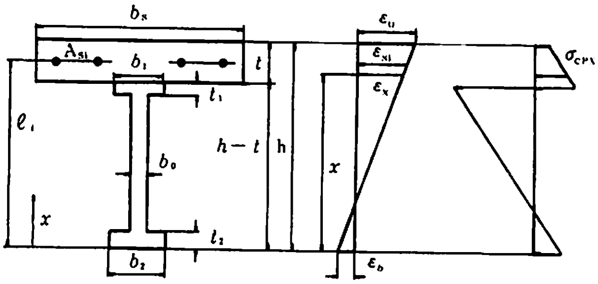
where, P_s: restraining steel ratio of uniaxially restrained standard specimen(0.96% to be used)

E_p: Young's modulus of restraining steel(200,000 N/mm^2 to be used)

2. METHOD OF ESTIMATION WHEN RESTRAINED BY STEEL GIRDER

In a case of using expansive concrete for the concrete deck slab of a steel composite girder, the chemical prestress and the distribution of expansion in the axial direction of a steel girder having the cross-sectional specifications given in Fig. 2(a) may be obtained by solving the simultaneous equations below considering the steel girder to be an external restraining object.

$$\begin{aligned}&(C_c+3\sum A_{si}l_i^2)\varepsilon_d^2+3h(C_b+2\sum A_{si}l_i)\varepsilon_b\varepsilon_d\\&+3h^2(C_a+\sum A_{si})\varepsilon_b^2-6h^2b_stU/E_s=0\end{aligned} \qquad (8)$$



(a) Cross section (b) Expansive strain (c) Chemical prestress

Fig. 2. Explanatory Diagram of Case of Restraint by Steel Girder or Existing Structure

$$(C_b+2\sum A_{si}l_i)\varepsilon_d{}^2+2h(C_a+\sum A_{si})\varepsilon_b\varepsilon_d$$
$$-4b_sh^2(U/E_s)\ln[(\varepsilon_b+\varepsilon_d)/\{\varepsilon_b+(1-t/h)\varepsilon_d\}]=0 \qquad (9)$$

provided that,

$$C_a=(b_2-b_0)t_2+(b_0-b_1)(h-t-t_1)+b_1(h-t) \qquad (10)$$

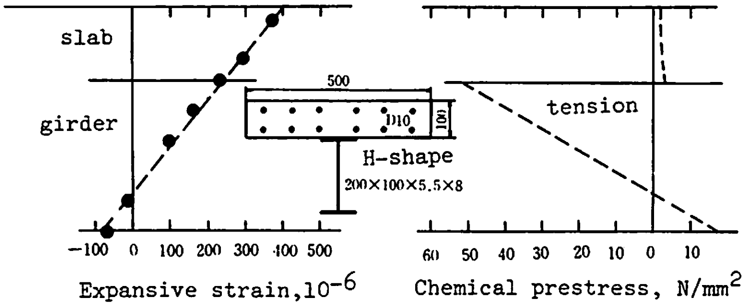
$$C_b=(b_2-b_0)t_2{}^2+(b_0-b_1)(h-t-t_1)^2+b_1(h-t)^2 \qquad (11)$$

$$C_c=(b_2-b_0)t_2{}^3+(b_0-b_1)(h-t-t_1)^3+b_1(h-t)^3 \qquad (12)$$

b_s, b_1, t_1, b_0, b_2, t_2: symbols given in Fig. 2(a)
The Young's moduli of the steel girder and reinforcing bars are taken as equal in the above equations.

A number of methods for obtaining the unknowns ε_b and ε_d from the above equations can be considered. Firstly, ε_b is assumed and the quadratic equation of Eq. (8) is solved to obtain ε_d , and these values are substituted into the left side of Eq. (9), and a comparison is made to see whether zero is sufficiently approached. When the result is not acceptable, a method of obtaining the solution repeating the above procedure is considered to be practical. In this case, ε_b , as shown in Fig. 2(b) becomes shrinkage strain and is of negative sign. If expansion distribution is obtained from Eqs. (8) and (9), the chemical prestress σ_{cps} transferred to concrete may be computed by Eq. (2)' and the chemical prestresses produced in reinforcing bars and the steel girder by multiplying their respective strains by the Young's modulus E_s (see Fig. 2(c)).

Examples of expansion and shrinkage strains at various locations in reinforcing bars and steel girder cross sections obtained by wire strain gauges pasted on are shown by black dots in Fig. 3. In the same figure, expansion



(a) Expansive strain (b) Chemical prestress

Fig. 3. Examples of Distribution of Expansive Strains and Chemical Prestresses in Case of Restraint by Steel Girder

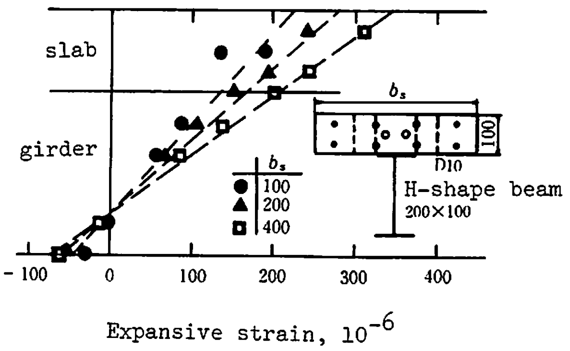


Fig. 4. Influence of Slab Width on Expansion Distribution (Reinforcement Ratio Constant)

distribution and chemical prestress distribution estimated by the above equations directly using the work quantity U_s obtained by Eq. (7) from the expansive strain $\varepsilon_{ss}=592\mathrm{x}10^{-6}$ of the uniaxially restrained standard specimen are indicated by broken lines. In case the boundary surfaces of steel girder and concrete should get out of line, as adopted in the hypothesis a) in section 1., expansive strain is linearly distributed in the direction of the height of the cross section, and at the bottom fiber of the steel girder it changes to the side of shrinkage. The estimated values also agree well with measured values, while chemical prestresses are estimated to be distributed as shown in Fig. 2(b).

The measured values of expansive strains in case of arranging reinforcing bars in slabs arranged symmetrically in the concrete cross section, maintaining the cross-sectional area ratio constant at 1.43%, and varying the width of the slab in three stages are shown in Fig. 4. The estimated value is calculated from the above equations employing the expansive strain of $500\mathrm{x}10^{-6}$ of the uniaxially restrained standard specimen without alteration. Expansive strain and shrinkage strain are respectively increased with increase in slab width, while the expansion distribution indicates a steep gradient. The estimated values also show this trend and agree well with the measured values.

Fig. 5 shows the expansion distribution when reinforcing bars and steel girder are maintained constant and only the width of the concrete of deck slab is doubled. As for Fig. 6, it shows the case of reinforcement at the upper level of the deck slab increased to approximately 4 times. Based on these figures, it is thought that for practical purposes the degrees of restraint by reinforcing bars and steel girder can be evaluated with adequate accuracy by Eqs. (8) and (9) proposed above.

3. METHOD OF ESTIMATION WHEN RESTRAINED BY EXISTING STRUCTURE

The use of expansive concrete on foundation ground or an existing concrete structure is often adopted in actual work. The chemical prestress and expansive strain produced in such a case may be estimated basically by the method described in 2. considering the foundation ground and existing concrete structures as external restraining objects and using a different value from the reinforcing bars and steel girder for the Young's modulus E_r of these external restraining objects.

In case the external restraining object is of a rectangular cross section, the estimate can be made by changing the coefficients C_a, C_b, and C_c in Eqs. (8) and (9) expressed by Eqs. (10) through (12) to C_a', C_b', and C_c'

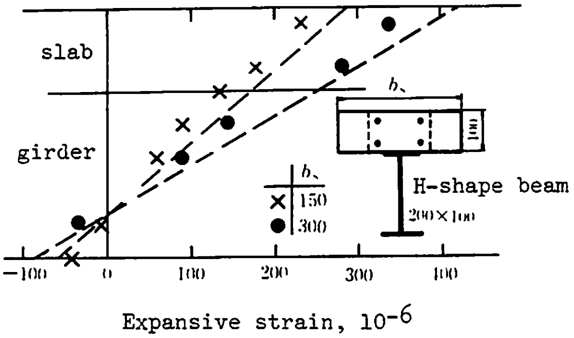


Fig. 5. Influence of Slab Width on Expansion Distribution (Reinforcement Quantity Constant)

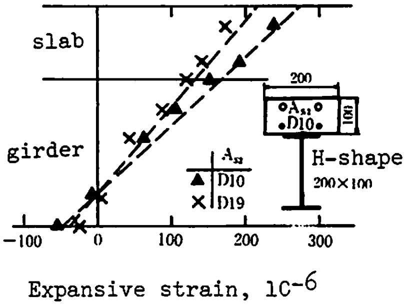


Fig. 6. Influence of Reinforcement Quantity on Expansion

expressed respectively by the equations below (see Fig. 2)

$$C_a' = b_0(h-t)/n' \quad (13)$$

$$C_b' = b_0(h-t)^2/n' \quad (14)$$

$$C_c' = b_0(h-t)^3/n' \quad (15)$$

where, $n' = E_s/E_r$.

The results of model tests when expansive concrete is placed on existing reinforced concrete members are shown in Fig. 7. This figure shows the expansion distributions with the widths of both expansive and existing concretes equal and constant at 15 cm, while the height of the expansive concrete is varied as 10 cm and 20 cm. The reinforcing bars in the expansive concrete were arranged symmetrically in the cross section equally with the reinforcement ratio of 0.95%. High-early-strength portland cement was used for the existing normal concrete, and expansive concrete was placed on top at the age of 5 weeks. Since compressive Young's moduli of normal concrete obtained at the ages of 5 and 8 weeks were 30,000N/mm^2 and 32,000N/mm^2, respectively, the average of 31,000N/mm^2 was used as the Young's modulus E_r of the external restraining object. The estimated values computed using unaltered the work quantity U_s obtained by Eq. (7)form 450x10^{-5}, the expansive strain ε_{ss} of the uniaxially restrained standard specimen, are indicated by broken lines.

When the proportion of expansive concrete becomes large the expansive strains produced in the expansive concrete are large, and in addition, show a steep expansion gradient. The estimated values also show this trend, and they are recognized as agreeing well with the measured values. Further, the estimated values of chemical prestress are also shown in broken lines in Fig. 7. At the boundary planes of the expansive concrete of 10 cm and 20 cm deep, it is estimated that tensile stresses of 3.8 N/mm^2 and 5.0 N/mm^2, respectively, will be produced in the normal concrete. If these were to be

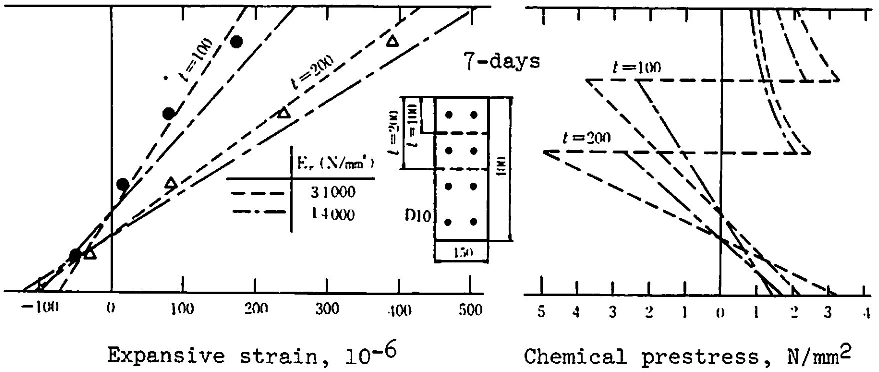


Fig. 7. Expansive Strain and Chemical Prestress Distributions in Case of Restraint by Existing Concrete

produced, the possibility of crack formation can be considered, but actually, cracking was not recognized. This was because of creep, and as a simple method of considering this influence the concept of effective Young's modulus is employed. As an example, the estimated value, when effective Young's modulus is reduced to 14,000 N/mm^2, is shown with a dot-dash line. When the Young's modulus of the external restraining object is decreased a reduction effect in bending stiffness appears,and in general, the expansive strain of expansive concrete is large, the expansion distribution becomes of steep gradient, while conversely, the chemical prestress is reduced. Although differing according to the shape and dimensions of the cross section, the differences with measured values do not become so large even when the Young's modulus of the external restraining object is decreased in this way. However, the tensile stresses produced at the boundary plane of normal concrete mentioned previously are estimated respectively as 2.3 N/mm^2 and 2.7 N/mm^2, for a reduction to approximately 55 to 60%. When subjected to restraint by an existing structure in this manner the evaluation of the Young's modulus considering the plastic deformation of the restraining object becomes important, and it is intended to carry out further studies regarding this point.

4. METHOD OF ESTIMATION FOR RECTANGULAR REINFORCED CONCRETE CROSS SECTION

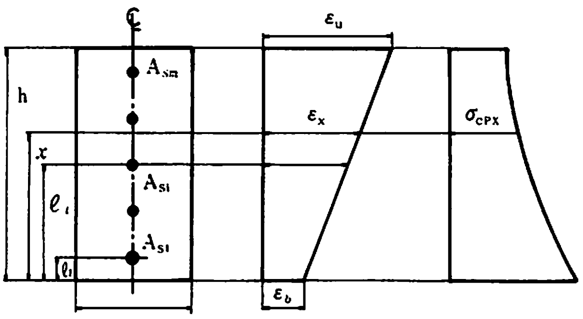
It has previously been reported regarding a case of reinforcing bars arranged in m layers symmetrically to right and left in a rectangular cross section with no external restraining object as shown in Fig. 8(a)2). The following equations reported can also be deduced from the basic equations (5) and (6).

$$\sum_{i=1}^{m}\frac{A_{si}E_s}{2bhU}\left(\frac{l_i^2}{h^2}\varepsilon_d^2+\frac{2l_i}{h}\varepsilon_d\varepsilon_b+\varepsilon_b^2\right)-1=0 \qquad (16)$$

$$\ln\left(1+\frac{\varepsilon_d}{\varepsilon_b}\right)-\sum_{i=1}^{m}\frac{A_{si}E_s}{2bhU}\left(\frac{l_i}{h}\varepsilon_d^2+\varepsilon_b\varepsilon_d\right)=0 \qquad (17)$$

The unknowns ε_d and ε_b can be determined similarly to the case of 2. In this case, ε_b generally becomes expansive strain as shown in Fig. 8(b), and is of positive sign.

Fig. 9 shows the relationship between the estimated and measured values of expansive strains produced at the locations of in the three kinds of cross sections, where, as shown in the diagrams, for tension reinforcement ratios of 0.62% and 1.1%, the ratios of cross-sectional areas of compression and tension reinforcement are varied between 0.28 and 1.0. With the amount of expansive admixture used at two levels, for expansive strains ε_{ss} of the uniaxially restrained standard specimen of 492×10^{-6} and 823×10^{-6}, estimates were made using the work quantities U_s respectively obtained by E_q. (7). In this cross section C with reinforcing bars arranged the most eccentrically, the expansive strain of the upper layer reinforcing bar is estimated to be slightly smaller than the measured value, and it is seen that the greater part of the errors for the various sections are within 10%. However, when examined in detail, with a cross section where the gradient of expansion distribution is steep, a trend of estimation for a slightly gentle gradient compared with measured values is seen. This is because this estimation method does not consider the influences of the differences between



(a) Cross section (b) Expansive strain (c) Chemical prestress

Fig. 8. Explanatory Diagram of Estimation of Rectangular Cross Section of Reinforced Concrete

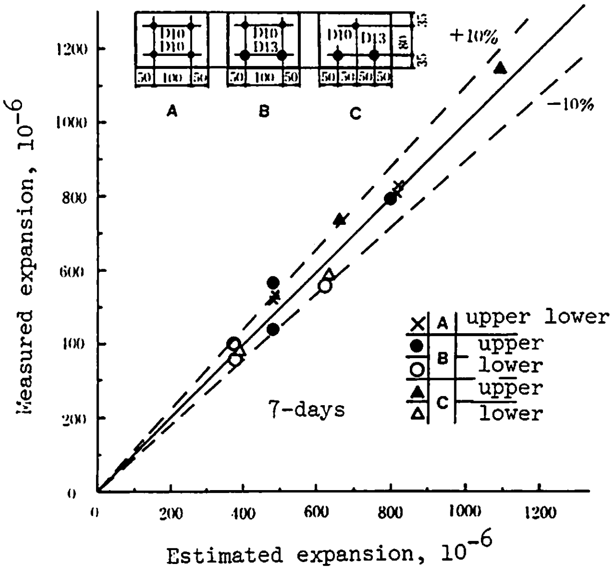


Fig. 9. Relationship Between Estimated and Measured Values of Rectangular Reinforced Concrete Cross Section

creep and elastic deformation of concrete due to chemical prestress.

The comparisons of the measured values of three different cross sections having the location of D16 arranged in the middle layer varied are shown in Fig. 10. The unit expansive admixture content is maintained constant and the water-binder ratio varied as 0.40, 0.50 and 0.60. The work quantities U_s obtained from expansive strain ε_{ss} of uniaxially restrained standard specimens at 535×10^{-6}, 772×10^{-6} and 661×10^{-6}, respectively, were estimated similarly to Fig. 9 and Fig. 10. Although the estimating accuracy appears to be slightly inferior compared with Fig. 10, in this case also the greater part had deviations from measured values within approximately 20%, and it is thought to be adequately applicable for actual use.

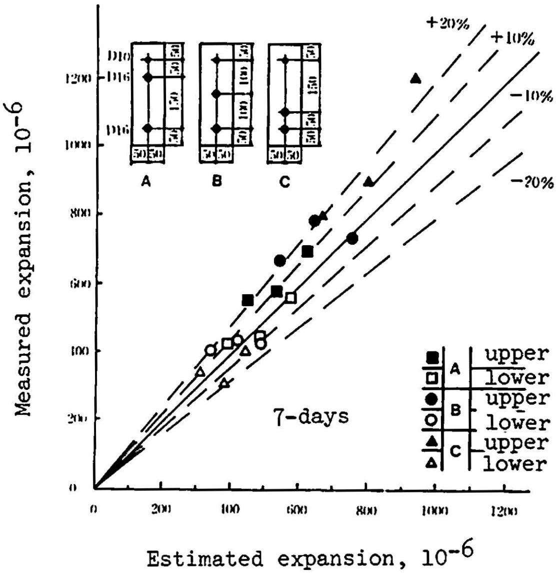


Fig. 10. Relationship Between Estimated and Measured Values of Rectangular Reinforced Concrete Cross Section (Influence of Location of Reinforcing Bar Arranged at Middle Layer)

CONCLUSIONS

A method of estimating the chemical prestress and expansion distribution which is one of the important problems in improving the mechanical properties of various structures applying expansive concrete in them has been proposed. This method is based on the concept of the quantity of work performed by expansive concrete on restraint, and is applicable to the cases not only of restraint by reinforcing bars, but also of steel, foundation ground or existing concrete structures. The estimated values were compared with the experimental results, and the following conclusions were obtained.

(1) Estimation of chemical prestress and expansion distribution, in general is to solve the simultaneous equations (5) and (6).

(2) Since there is only a small quantity of measured values for the cases of restraint by steel girders and existing concrete, the accuracy of estimation is not something that can be discussed, but by solving the simultaneous equations (8) and (9), it will be possible to estimate the chemical prestress and the expansion distribution.

(3) The chemical prestress and expansion distribution in a reinforced concrete member of rectangular cross section, may be estimated for practical purposes by solving the simultaneous equations (16) and (17) if a suitable value of work quantity is given, and with the exception of the cases of especially great eccentricity.

(4) In estimating the values for actual structures, the work quantity which is the basis will vary according to shape and dimensions of the structure, the curing method, etc. even though the concrete mix proportions are the same. Therefore, the value obtained by Eq. (7) on the uniaxially restrained specimen specified in JIS cannot be directly employed, and there is a necessity for corrections to be made. Further accumulation of data is intended to be carried out.

ACKNOWLEDGEMENTS

In conducting the study, the application of the concept of work quantity to minute parts was in accordance with the suggestions of Dr. Tadaaki Tanabe, Associate Professor, Nagoya University. Much valuable advice was received from Dr. Masatane Kokubu, Professor, Musashi Institute of Technology and Professor Emeritus, University of Tokyo, and Dr. Hajime Okamura, Professor, University of Tokyo in connection with implementation of the study. The author wishes to express his sincerest gratitude for this guidance.

This paper is a compilation of a part of the results of studies supported by the Grant-in-Aid for Scientific Research No. 575302 from the Japanese Ministry of Education while the author was at Ashikaga Institute of Technology, which is hereby gratefully acknowledged.

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APPENDIX: MIX PROPORTIONS AND CURING METHODS OF CONCRETE

The mix proportions and curing methods of concrete concerning Figs. 3 to 10 in the paper and the expansive strains ε_{ss} of uniaxially restrained standard specimens made the basis for estimations are given in the annexed table. Although differing depending on curing method, ε_{ss} was in a range of 450×10^{-6} to 820×10^{-6}, and there were specimens included having expansive capacities exceeding the applicable range given in 'Appendix 3, Procedure for designing Chemically Prestressed Concrete (Proposed)" of "Guide to Design and Construction of Expansive Concrete (Proposed)" of the Japan Society of Civil Engineers.

The expansive admixture used was Denka CSA#20 manufactured by Denki

Kagaku Kogyo Co., Ltd. Cements used were high-early-strength portland cements manufactured by Chichibu Cement Co., Ltd. for Mix E, while for all other mixes ordinary portland cement manufactured by Chichibu Cement Co., Ltd., was used. Aggregates used were river sand and river gravel from the Watarase River. Maximum size of aggregate was 25 mm for all the cases.

Annexed Table Mix Proportions and Curing Methods of Concrete, and Expansive Strain of Uniaxially Restrained Specimen

Mix name	$\frac{W}{C+E}$ (%)	$\frac{s}{a}$ (%)	Unit content (kg/m^3)*					Slump (cm)	ε_{ss}** (10^{-6})	Curing°	Ref.
			W	C	E	S	G				
A	50	40	165	280	50	755	1137	8.5	592	m	Fig. 3
B	50	39	165	280	50	728	1160	8.0	500	m	Figs.4,5,6
C	50	40	165	280	50	755	1137	8.5	614	m	Fig. 5
D	50	39	165	280	50	728	1160	9.4	450	m	Fig. 7
E	50	39	165	330	0	729	1162	10.7	-	m	
F	50	40	165	285	45	755	1137	8.3	492	m	Fig. 9
G	50	40	165	275	55	754	1136	8.5	823	m	
H	40	40	165	368	45	727	1099	7.2	535	m	
I	50	40	165	285	45	755	1137	8.8	772	w	Fig. 10
J	60	40	165	230	45	764	1169	9.1	661	m	

* 1 kilogram(kg) = 9.806 Newtons(N)

** expansive strain obtained from the uniaxially restrained standard specimen specified in JIS A 6202

° m: moist curing, w: water curing, at the temperature of 20°C