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CREEP OF CONCRETE IN THE LIGHT OF HYDRATION OF CEMENT AND VISCOSITY OF INTERNAL WATER

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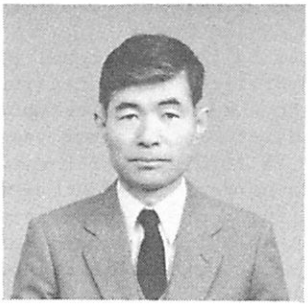

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SYNOPSIS

A microscopic creep model of concrete was presented by taking account of the internal structure of hardened cement paste and hydration of cement, thereby deriving creep equations. Then they were fitted to the measured creep curves of concrete for famous high dams in Japan, and concrete for reactor vessels. The creep curves express the influence of the age at loading, water-cement ratio, etc. The presented creep law includes time-hardening and strain-hardening laws as its extreme cases. These equations contain temperature-dependent parameters expressing the process of cement hydration, and these parameters have been obtained by measuring bound water content in cement paste at various temperatures and at various ages.

1. INTRODUCTION

Creep of concrete must be grasped quantatively and its general representation must be given in order to analyse and evaluate the behaviour and safety of important concrete structures, such as large concrete dams, long span prestressed concrete bridges, prestressed concrete reactor vessels, etc.

To this end, we must pay attention to microscopic mechanisms of creep, while accumulating experimental data steadily. From this point of view, the authors presented, about ten years ago, a creep model of concrete taking account of the viscosity of internal water and hydration of cement in connection with an experiment on creep of concrete at high temperatures [1].

Since then, measurements of bound water of sealed cement paste have been undertaken continually, thus making it possible to analyse creep data of sealed concrete in a concrete form. The result will be reported in the following.

Formally, a review of the state of the art of the studies on creep of concrete must be given here, but it is entrusted to the voluminous book by A.M. Neville [5]. The only thing that the present authors would like to mention is that they were deeply impressed by the description of internal structures of cement paste shown by T.C. Powers, and that they are aware of very few creep studies based on the similar perspective presented here.

2. HYDRATION OF CEMENT

From the measured values of bound water of cement paste (moderate heat Portland cement) cured in water, the hydration of cement in water at an ordinary temperature could be expressed as follows [1],

$$\frac{dC_H(t)}{dt} = k_0(1-n)t^{-n}(W-\gamma C_H(t))\ (C-C_H(t)) \quad \cdots\cdots\cdots \quad (1)$$

with $C_H(0)=0$, $0<n<1$,

where

t	:	age of cement paste (days)
C	:	cement content per 1 m^3 of cement paste (kg/m^3)
W	:	water content per 1 m^3 of cement paste (kg/m^3)
$C_H(t)$	:	content of hydrated cement per 1 m^3 of cement paste (kg/m^3)
k_0, n	:	constants,
γ	:	required water-cement ratio (not expressed by percent) for complete hydration of cement.

The solutions of the above equation (1) have been shown as follows [1],

$$\frac{C_H(t)}{C} = R_H(t) = \begin{cases} \dfrac{1-\exp\{(\gamma C-W)k_0t^{1-n}\}}{1-\gamma\frac{C}{W}\ \exp\{(\gamma C-W)k_0t^{1-n}\}} \quad \cdots\cdots\cdots \quad (2) \\ \qquad (W/C\neq\gamma) \\ \dfrac{\gamma k_0 t^{1-n}}{\frac{1}{C}+\gamma k_0 t^{1-n}} \quad \cdots\cdots\cdots \quad (3) \\ \qquad (W/C=\gamma) \end{cases}$$

where $R_H(t)$ is degree of hydration.

The weight measurement of bound water for normal and moderate heat Portland cements paste cured in sealed test tubes, at the temperatures with the range of 20°C to 80°C were undertaken. Measurement conditions are shown in table-1. The outline of the measurements is as follows; the pastes cured under sealed condition were crushed in a mortar at specified ages, sieved with #20 mesh sieve, dried under reduced pressure below 6 mmHg in a desiccator for about 4 days, and then heated at 950°C. The lost mass during this heating was regarded as bound water.

The necessary values of perfect bound water content γ to fit the curves (2) or (3) to experimental data were assumed to be $(\gamma=)0.37$ and 0.25. The former value ($\gamma=0.37$) was suitable for the cement paste cured in water [1] [2], and the latter ($\gamma=0.25$) was the upper limit of the recently derived data on the cement paste cured in test tubes.

As a result of this analysis, the values of k_0 and n in expression (1) showed the following temperature dependence:

$$k_0 = \text{COFFK}\ \exp\ (\text{CLINK}/\text{Tk}), \quad \cdots\cdots\cdots \quad (4)$$

$$n = \text{COFFN}\ \exp\ (\text{CLINN}/\text{Tk}), \quad \cdots\cdots\cdots \quad (5)$$

in which COFFK(>0), CLINK(>0), COFFN(>0), CLINN(<0) are constants;
Tk : absolute temperature (°K), (see figure-1, and table-2). The data at 30°C

Table 1 : Experimental Conditions of Measurements of Bound Water of Cement Pastes

Kind of Cement	Normal Portland Cement	Moderate Heat Portland Cement
Plant	Saitama Plant of Nihon Cement Co.	Fujiwara Plant of Onoda Cement Co.
Water-cement Ratio (W/C)	40%	
Curing method	Cured in sealed test tube	
Curing temperature (°C)	20, 30, 40, 50, 60, 70, 80	
Standard age at measurement (days)*	3, 5, 7, 14, 28, 56, 91, 98, 182, 365, 546, 553, 735	

* At later ages, the higher the temperature, the more test tubes bursted by the pressure caused by the expansion action of chemical ractions. These cases were omitted.

Table 2 : Temperature Dependence of Hydration Parameters of Cement*

[m, kg, days]

		Normal Portland Cement		Moderate Heat Portland Cement	
Assumed perfect bound water content (%)		25	37	25	37
k_0	(CONSTK)	(-1.6512)	(-3.5284)	(0.070604)	(-2.1608)
	COFFK	0.19182	0.02935	1.0732	0.11523
	CLINK	-1399.3	-978.97	-1985.0	-1450.5
n	(CONSTN)	(0.97127)	(0.61146)	(1.4727)	(0.97453)
	COFFN	2.6413	1.8431	4.36099	2.6499
	CLIN	-374.70	-234.24	-537.53	-354.27

* CONSTK = ln (COFFK), ONSTN = ln (COFFN)

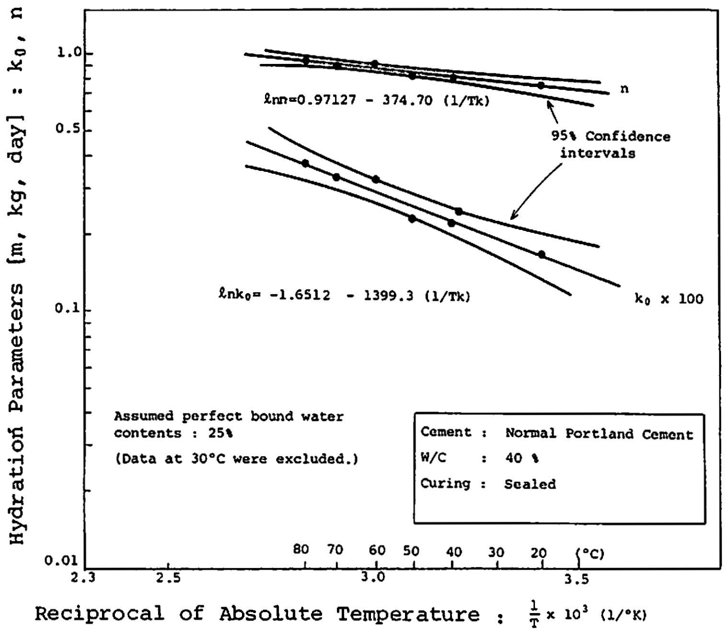


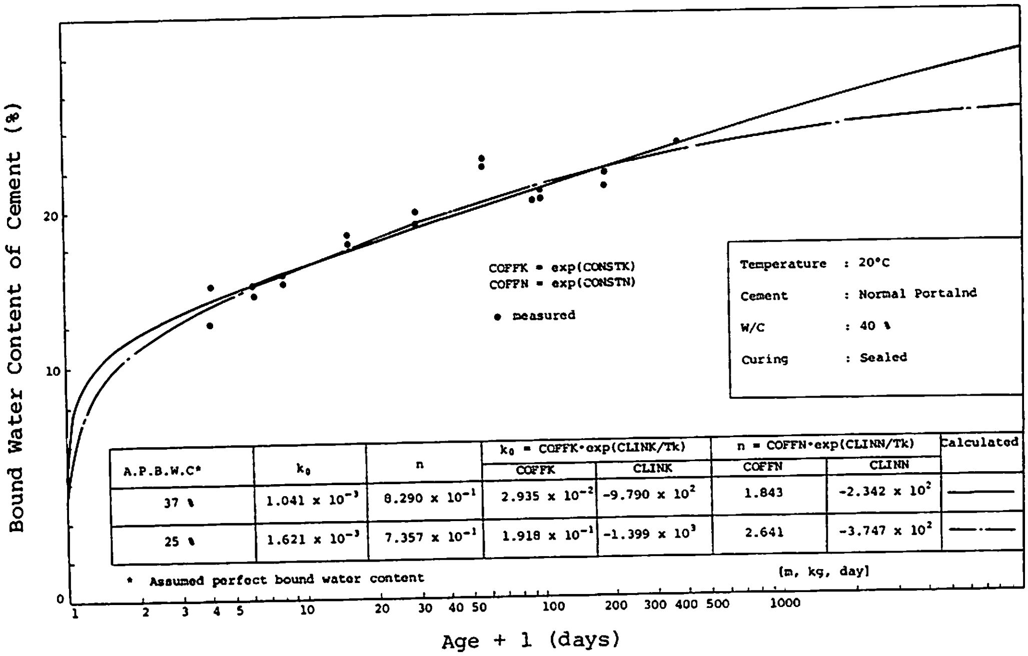
Fig. 1 : Temperature Dependence of Hydration Parameters of Cement (Normal Portland Cement)

was excluded, because the air conditioned box for that temperature was not installed with a refrigerator, and so the inside temperature sometimes exceeded 30°C during the summer. Figure-2 shows the fitted results at 20°C. Figures-3 and 4 show the calculated values at all the measured temperature conditions. These results show that the effect of curing at high temperatures is remarkable at the initial stage, but the hydration at lower temperatures gets ahead at later ages. They also indicate that the hydration of normal Portland cement is faster than that of moderate heat Portland cement.

If we define "the equivalent hydration age (t_{eq}) at T_2°C" as the necessary age to reach the same degree of hydration which the cement paste cured at a standard temperature T_1°C (20°C for example) reached in t_1 days, then

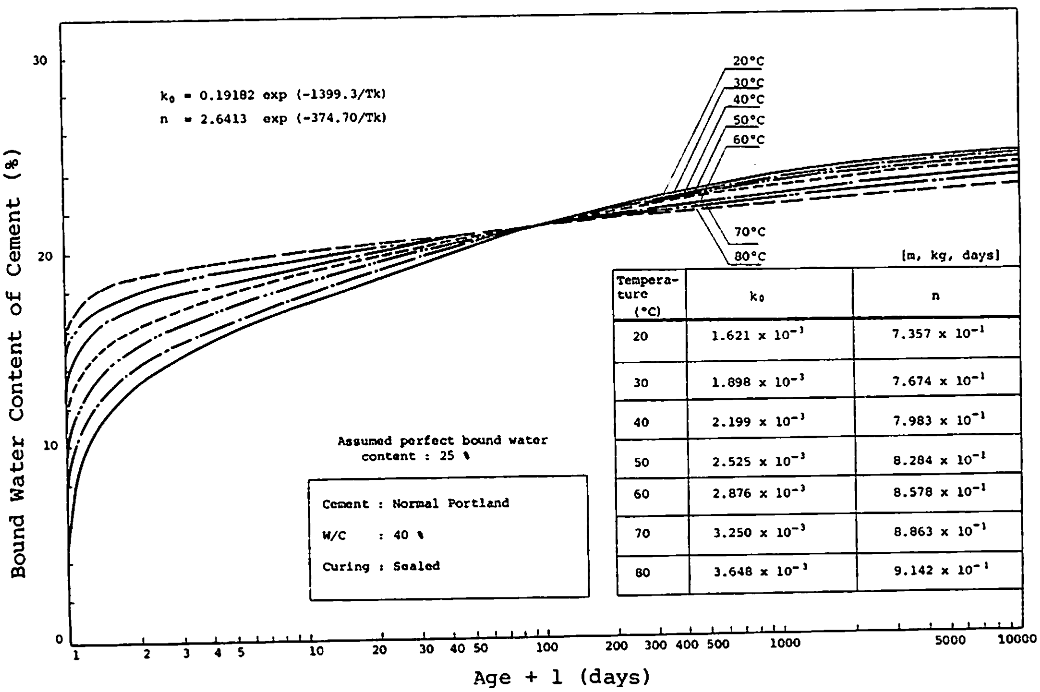
$$t_{eq} = \left(\frac{k_1}{k_2} \right)^{\frac{1}{1-n_2}} t_1^{\frac{1-n_1}{1-n_2}} , \qquad (6)$$

where k_1 and n_1, and k_2 and n_2 are hydration parameters at T_1 and T_2°C, respectively. These relations are shown in figure-5.



A.P.B.W.C*	k_0	n	k_0 = COFFK·exp(CLINK/Tk) COFFK	CLINK	n = COFFN·exp(CLINN/Tk) COFFN	CLINN	Calculated
37 %	1.041×10^{-3}	8.290×10^{-1}	2.935×10^{-2}	-9.790×10^{2}	1.843	-2.342×10^{2}	———
25 %	1.621×10^{-3}	7.357×10^{-1}	1.918×10^{-1}	-1.399×10^{3}	2.641	-3.747×10^{2}	—·—

Fig. 2 : Hydration of Cement at 20°C



Temperature (°C)	k_0	n
20	1.621×10^{-3}	7.357×10^{-1}
30	1.898×10^{-3}	7.674×10^{-1}
40	2.199×10^{-3}	7.983×10^{-1}
50	2.525×10^{-3}	8.284×10^{-1}
60	2.876×10^{-3}	8.578×10^{-1}
70	3.250×10^{-3}	8.863×10^{-1}
80	3.648×10^{-3}	9.142×10^{-1}

Fig. 3 : Hydration of Cement at Various Temperaturs (a)
(Normal Portland Cement)

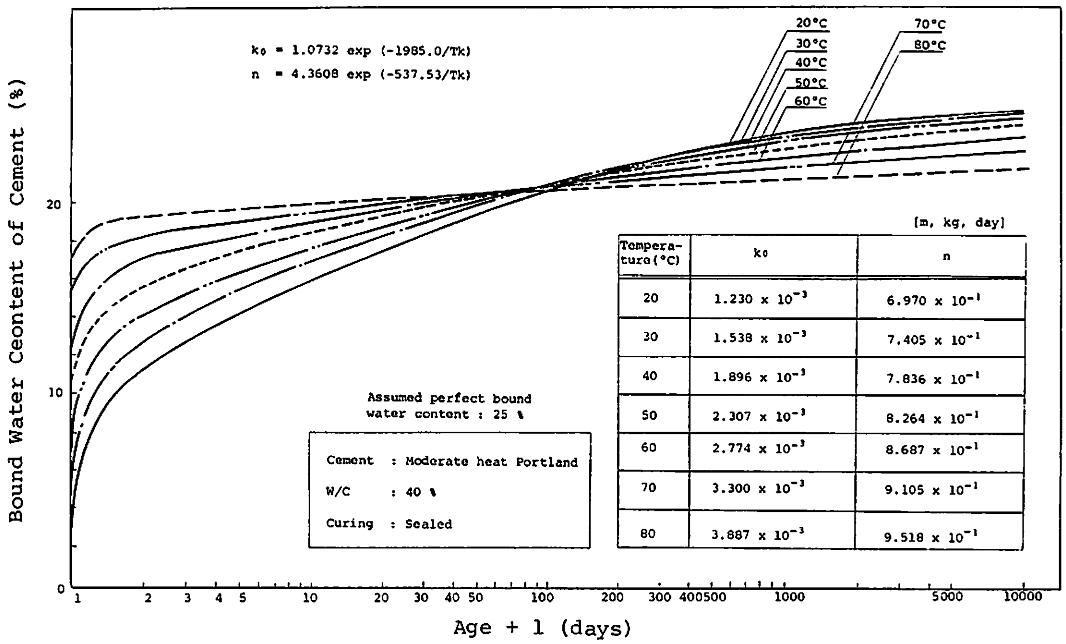


Fig. 4 : Hydration of Cement at Various Temperatures (b)
(Moderate Heat Portland Cement)

The experimental hydration data under a stepwise temperature rise could not be followed accurately by the calculated values of equation (1) when the integration was carried out with the explicit inclusion of time t. Therefore, using the solutions (2) and (3), and excluding t from the right side of equation (1), we got the following equations (7) and (8), which are expected to be used for the hydration under varying temperatures:

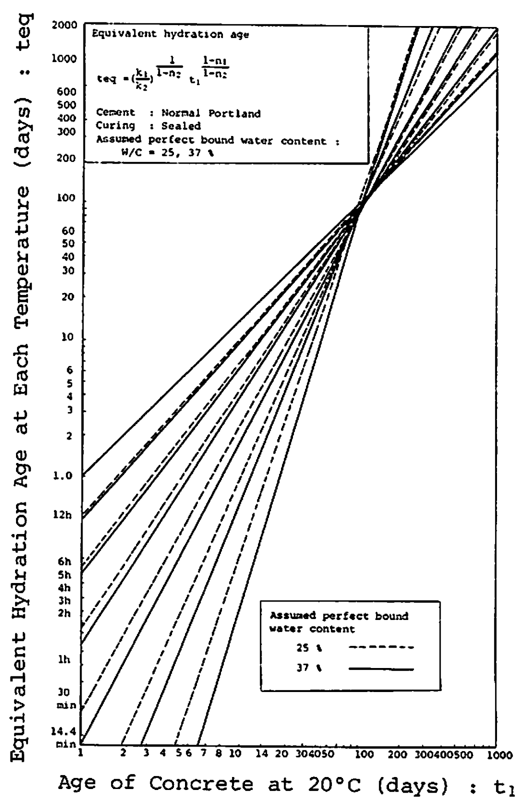


Fig. 5 : Equivalent Hydration Age at Each Temperature to a Standard Temperature

$$
\frac{dC_H(t)}{dt} = \begin{cases} k_0^{\frac{1}{1-n}}(1-n)\left[\frac{1}{(\gamma C-W)}\ln\left(\frac{1-C_H(t)/C}{1-\gamma C_H(t)/W}\right)\right]^{-\frac{n}{1-n}} \\ \quad \times\ [W-\gamma C_H(t)]\ [C-C_H(t)], \\ \qquad (W/C \neq \gamma) \quad \cdots\cdots\cdots\cdots \quad (7) \\ k_0^{\frac{1}{1-n}}(1-n)\left[\frac{C_H(t)}{W}\right]^{-\frac{n}{1-n}} [W-\gamma C_H(t)]\ [C-C_H(t)]^{\frac{1}{1-n}}, \\ \qquad (W/C = \gamma) \quad \cdots\cdots\cdots\cdots \quad (8) \end{cases}
$$

Solving these equations under the condition that

$$T = T_1^\circ \quad (k_0=k_1,\ n=n_1) \text{ when } 0 \leqq t < t_1,$$

and

$$T = T_2^\circ \quad (k_0=k_2,\ n=n_2) \text{ when } t \geqq t_1,$$

we get the following solutions for $t \geqq t_1$;

$$
\frac{C_H(t)}{C} = R_H(t)
$$

$$
= \frac{1-\exp[(\gamma C-W)k_2(t-t_1+t_{eq})^{1-n_2}]}{1-\gamma\frac{C}{W}\exp[(\gamma C-W)k_2(t-t_1+t_{eq})^{1-n_2}]}
$$

$$(W/C \neq \gamma), \quad \cdots\cdots\cdots\cdots \quad (9)$$

where t_{eq} is the equivalent hydration age defined by expression (6). Figure-6 shows the calculated hydration processes when the temperature was raised stepwise at various ages. The validity of these equations must be examined after further accurate data of hydration under these conditions are obtained.

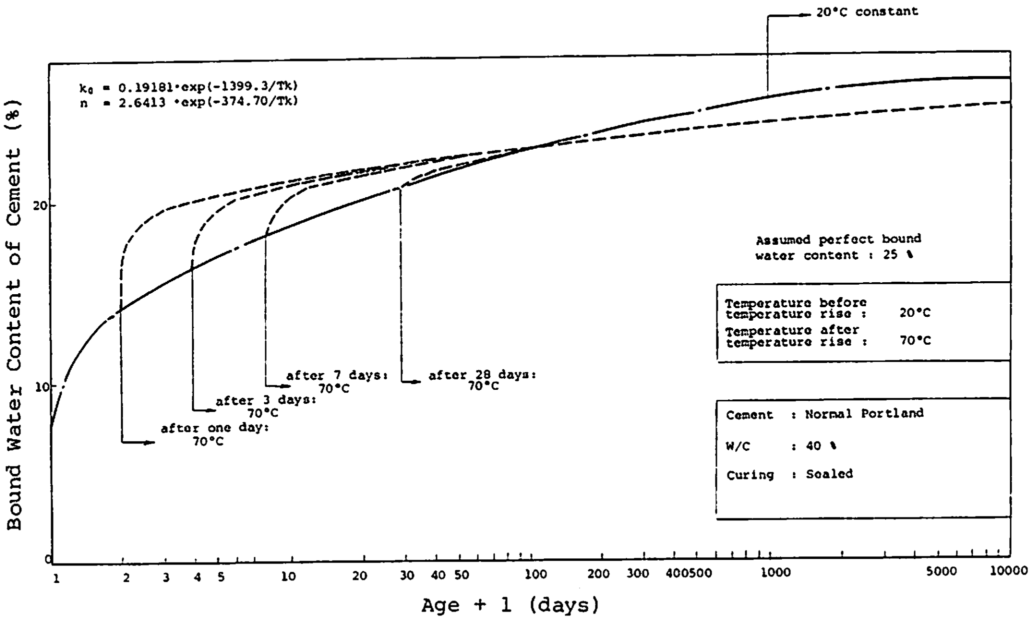


Fig. 6 : Hydration of Cement Under a Stepwise Temperature Rise

3. A CREEP MODEL TAKING ACCOUNT OF VISCOSITY OF WATER AND HYDRATION OF CEMENT

With the postulation of the model in figure-7, a rheology equation of sealed concrete was derived on the following assumptions [1] :

1) The needle shaped crystal of calcium silicate hydrate is approximated by a cylinder. During the process of hydration, number of crystals increases but their size remains the same. (This has been proven by Powers and Brownyard through their experiments [3].)

2) Increment of creep strain ($d\varepsilon_c$) during a small interval of time is proportional to stress (σ), and becomes larger as the thickness (2d) of the water membrane between crystals increases. On the other hand, the creep strain increment becomes smaller when the contact length (ℓ) of crystals, the number of crystals (m), and the viscosity of water (η) increases.

3) Creep of concrete increases approximately proportional to the volume of cement paste in concrete.

The problem of creep recovery was left for the future. This is because we did not have sufficient recovery data, and the recovery strain seems relatively small according to Kimishima & Kitahara's experimental data [4] [5]. Also, the pure physicists like Wittmann [5] and Parrot [6] who are very familiar with cement chemisty regard that creep strain of concrete is essentially irrecoverable. When ample recovery data are obtained, the above model will be improved by use of the thermodynamically recoverable component proposed by Powers [9] and Z.P. Bažant [10].

On the above assumptions, we have

$$d\varepsilon_c \propto d^p \sigma^q dt / \eta^s \ell^a \ .$$

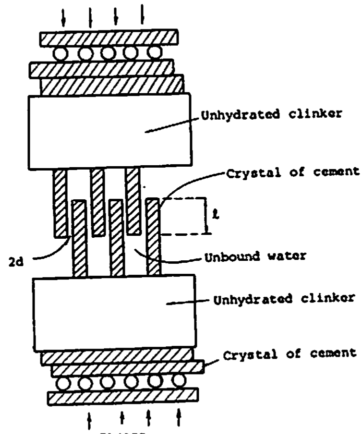


Fig. 7 : One Dimensional Rheological Model of Cement Paste Taking Account of Viscosity of Water and Hydration of Cement

(In the previous paper [1], only the case where q = 2 was considered.) On the other hand, with a rough approximation, (h : length of a crystal, r : its radius)

$$d \propto \frac{V_{UM}}{2\pi rhm},$$

$$m \propto V_{HC} + V_{HW} \propto V_{HC},$$

and

$$\ell \propto \varepsilon_c,$$

where V_{UW} : volume concentration of unbound water,

V_{HC} : volume concentration of hydrated cement,

V_{HW} : volume concentration of hydrated (or bound) water.

We assume that creep strain is proportional to stress when the stress is constant. (This is observed experimentally when stress is not excessive.) Then we have

$$a + 1 = q,$$

moreover, we chose the parameter "a" so that this model may coincide with a Maxwell model when q = 1. Then we have

$$q = s.$$

Next, we define $\eta_*(t) \equiv \eta[V_{HC}/V_{UW}]^{p/q}/(qA_1)^{1/q}$ .... (10)

where A_1 is a constant which depends on the amount of paste and the type of aggregates in concrete. This parameter $\eta_*(t)$ becomes the coefficient of viscosity when q = 1, so we term this as "pseudo-viscosity" when $q \neq 1$. When the elasticity of sealed concrete changes with age, the response of instantaneous strain to a stepwise stress change is also stepwise, and concrete does not repel the applied load even when the modulus of elasticity increases over time. From this fact the instantaneous strain component should be regarded as hypoelestic. Then assuming that compressive stress is positive in sealed concrete, we get

$$\varepsilon(t-t_1) = \int_{t_1}^{t} \frac{1}{E_i(s)} \frac{d\sigma(s)}{ds} ds + \left[\int_{t_1}^{t} \left(\frac{\sigma(s)}{\eta_*(s)}\right)^q ds\right]^{1/q}, \quad \ldots(11)$$

where
t : age of concrete (days),
t_1 : age at loading (days),
$\varepsilon(t-t_1)$: sum of elastic and creep strains after $(t-t_1)$ days of initial loading,
$\sigma(s)$: stress at the age of s days (kgf/cm^2),
$E_i(s)$: instantaneous modulus of elasticity at the age of s days (kgf/cm^2) (this modulus is calculated using instantaneous strain after loading).

The explicit expressions of $\eta_*(t)$ at constant temperatures are

$$\eta_*(t) = \begin{cases} \dfrac{\eta}{(qA_1)^{1/q}}\left(\dfrac{P_W}{P_C}\right)^{P/q}\left[\dfrac{1-\exp\{(\gamma C-W)k_0 t^{1-n}\}}{W/C-\gamma}\right]^{P/q} \\ \qquad (W/C \neq \gamma), \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (12) \\ \dfrac{\eta}{(qA_1)^{1/q}}\left(\dfrac{P_W}{P_C}\right)^{P/q} (k_0 C)^{P/q}\, t^{(1-n)P/q} \\ \qquad (W/C = \gamma), \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13) \end{cases}$$

where P_W and P_C are densities of water and cement, respectively. In the above expressions, the change of water viscosity with temperature is expressed as

$$\eta = A_v \exp(E/RT), \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

where
T : absolute temperature (°k),
E : activation energy of viscosity of interstitial water,
R : gas constant,
A_v : a constant.

Examining this model leads to a conclusion that the reason superposition method overestimates creep recovery is because this method includes nonlinear viscous strain component to stress in Boltzmann's hereditary integral, and thus the essentially irrecoverable component is regarded as recoverable.

4. CREEP CURVES DERIVED FROM THE MODEL

Putting $\sigma=1$ $(t \geqq t_1)$ in expression (11), we get the following expressions (15) to (17) which show the creep strain per unit stress $\varepsilon_c(t-t_1)$ after $(t-t_1)$ days of initial loading:

$$\varepsilon_c(t-t_1) = \begin{cases} A_4\exp(-\frac{E}{RT})\left\{\int_{t_1}^{t}\left[\frac{W/C-\gamma}{1-\exp\{(\gamma C-W)k_0 s^{1-n}\}}\right]^P ds\right\}^{1/q}, \\ \qquad (W/C \neq \gamma) \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (15) \\ A_4\exp(-\frac{E}{RT})\ (\frac{1}{k_0 C})^{P/q}\left[\frac{1}{(1-n)P-1}\{t_1^{1-(1-n)P}-t^{1-(1-n)P}\}\right]^{1/q}, \\ \qquad (W/C=\gamma,\ \text{and}\ (1-n)P-1 \neq 0), \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (16) \\ A_4\exp(-\frac{E}{RT})\ (\frac{1}{k_0 C})^{P/q}\left[\ell n(1+\frac{t-t_1}{t_1})\right]^{1/q}, \\ \qquad (W/C=\gamma,\ \text{and}\ (1-n)P-1=0), \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (17) \end{cases}$$

where t : age of concrete (days),
T : absolute temperature of concrete (°K),
C : content of cement per 1 m^3 of cement paste (kg/m^3),
W : content of water per 1 m^3 of cement paste (kg/m^3),
A_4, P, q: creep constants.

A_4 is regarded as a function of the type of aggregates used and the paste content, and the effect of temperature on this quantity is considered insignificant.

We use expression (17) as an approximate expression even when $W/C \neq \gamma$, and call is "the simplest approximate creep curve". We call the most general expression (15) as "integral type". Figure-8 shows the result of the creep curves approximated by the above mentioned simplest approximate creep curves. In the figure,

$$As = A_4\exp(-E/RT)\ (1/k_0 C)^{P/q} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (18)$$

The above expressions (15) to (17) are closely related to the power type creep expression as will be explained later, and the results of the following expression (19) are shown in figure-9:

$$\varepsilon(t-t_1)=1/Ei+a(t-t_1)^b \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (19)$$

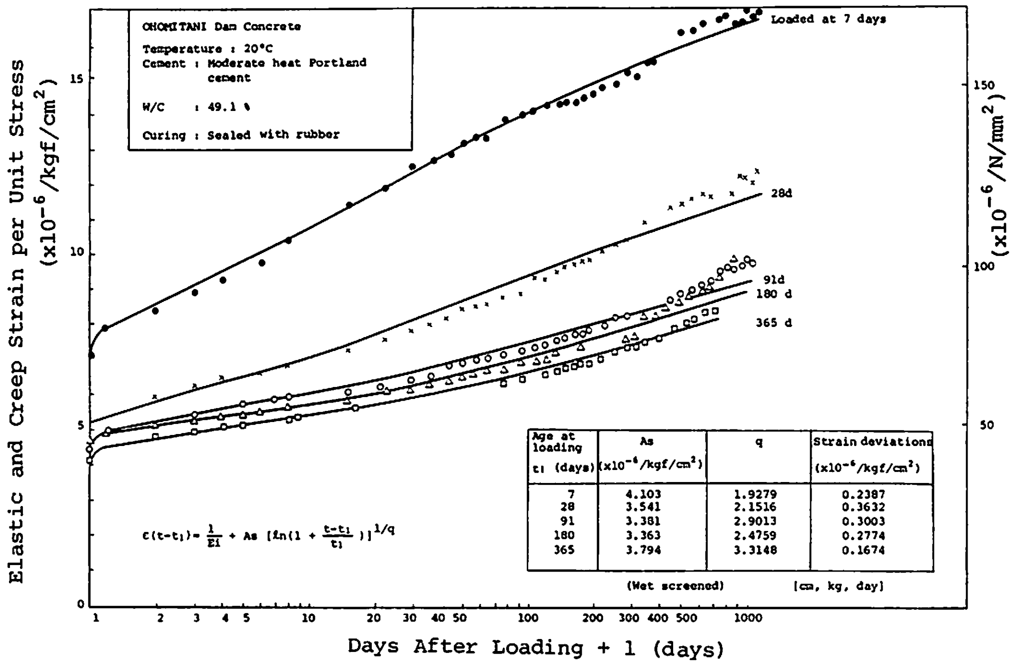


Fig. 8 : Fittings of the Simplest Approximate Creep Curves

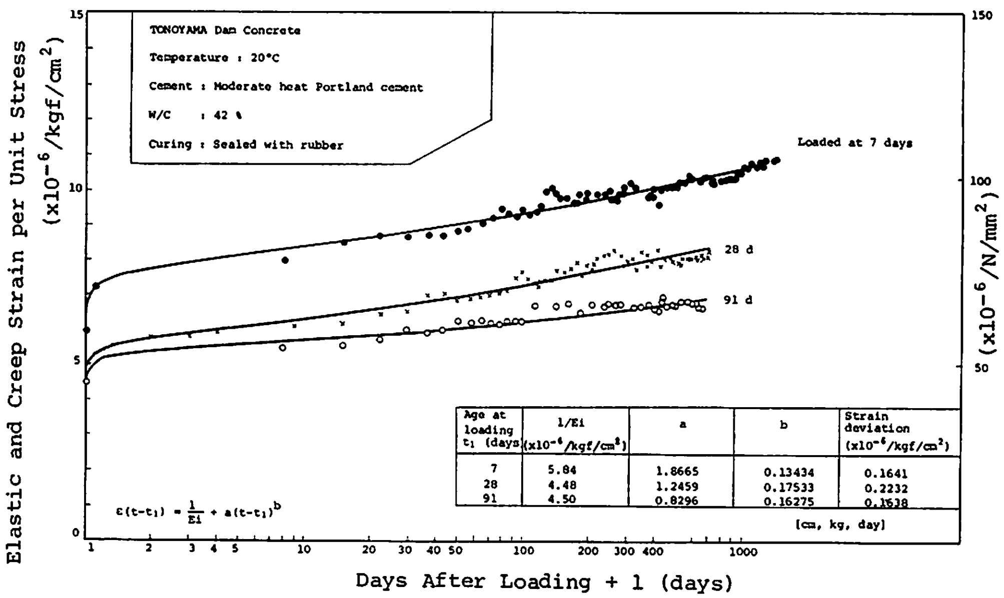


Fig. 9 : Fitting of Power Function Type Creep Curves

where a : a parameter depending on the age at loading,
b : a constant.

These two kinds of curves (17) and (19) express the creep curves with a good degree of accuracy, and these curves were used to derive differential values of creep curves, which are necessary in the process of deriving the integral type creep curves..

Figure-10 shows the integral type creep curves fitted to Kurobe No. 4 Dam concrete creep data. The necessary constants k_0 and n were obtained from hydration experiment. The initial value of q was from the simplest approximate curve, the differential values were selected from either the power type or the simplest type, on the basis of better approximation to creep data. The remaining unknown constants A and P ($A = A_4 \exp(-E/RT)$) were determined by the method of least squares. Then the creep strains (15) were calculated using Gauss's numerical integral method, and the deviations were calculated. Afterwards, a minor change was given to q to calculate next deviations, and the new deviations were compared with the first ones. This process was repeated until the optimum parameters A, P and q were obtained to minimize the sum of the squares of deviations. Using double precision calculation, q was determined to the six significant figures. The degree of fit was compared using the following d_{crp} which we termed "the index of strain deviations",

$$d_{crp} = \sqrt{\sum_{i=2}^{N} [C_i(t_i) - C_{cali}(t_i)]^2/(N-1)} \qquad (20)$$

where $C_i(t_i)$: measured creep strain at the age of t_i days,
$C_{cali}(t_i)$: calculated creep strain at the same age,
N : number of data.

$C_i(t_i)$'s were calculated by subtracting instantaneous strain from measured strain at each age which had been corrected with autogeneous strain. In this method, the instantaneous strain was assumed to have no error.

The specimens were cylinders, 15 cm in diameter and 60 cm long. These specimens were cast in neoprene covers inserted in steel mould. 25 cm long Carlson type electric strain meters were embedded at the centers of specimens in axial direction. The top and bottom were attached to 1 cm thick steel discs and sealed with hose bands and a cohesive agent. The load was applied to each specimen with a hydraulic jack connected to hydro-pneumatic accumulator and booster. The load was generated by a motor powered pump and kept constant automatically. The temperature was maintained at 20°C. The mean mechanical strain was calculated using the strains of two loaded specimens and two control specimens.

As expression (15) shows the effect of water-cement ratio upon creep, creep test with various water-cement rations was conducted keeping the cement paste content constant. Figure-11 shows the results of these tests. The calculated creep strains with W/C=35 and 50 % in this figure were obtained by using the creep parameters A, P, and q of concrete with W/C=40 %. In this case, the specimens were sealed with 0.2 mm thick copper plates.

Now, the expression (17) becomes logarithmic when q=1, which resembles the famous creep curves adopted by the U.S. Bureau of Reclamation [7]. But actually, q is about 2 to 3 as we have seen previously.

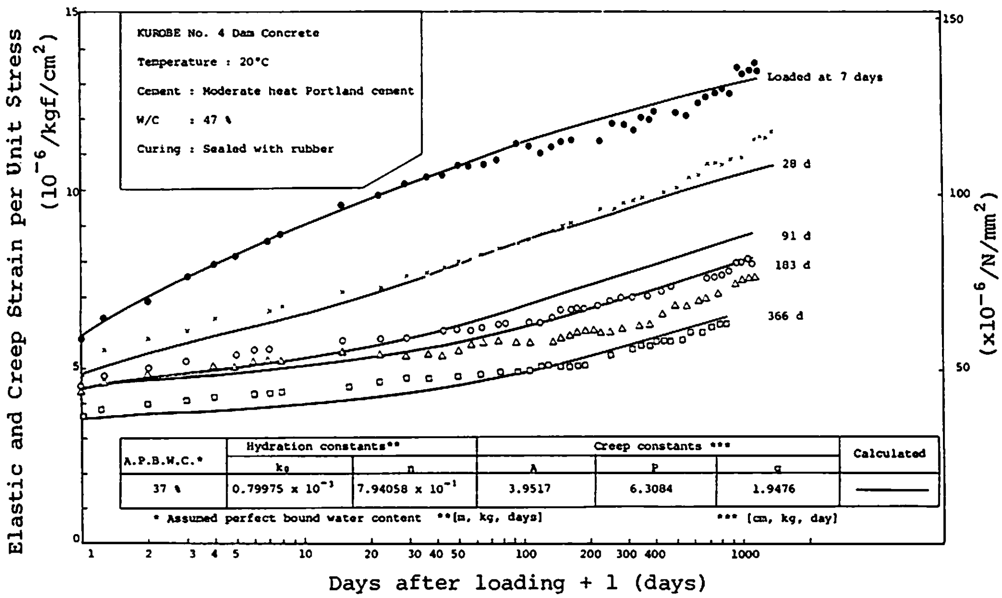


Fig. 10 : Fitting of Hydration Viscosity Creep Curves of Integral Type

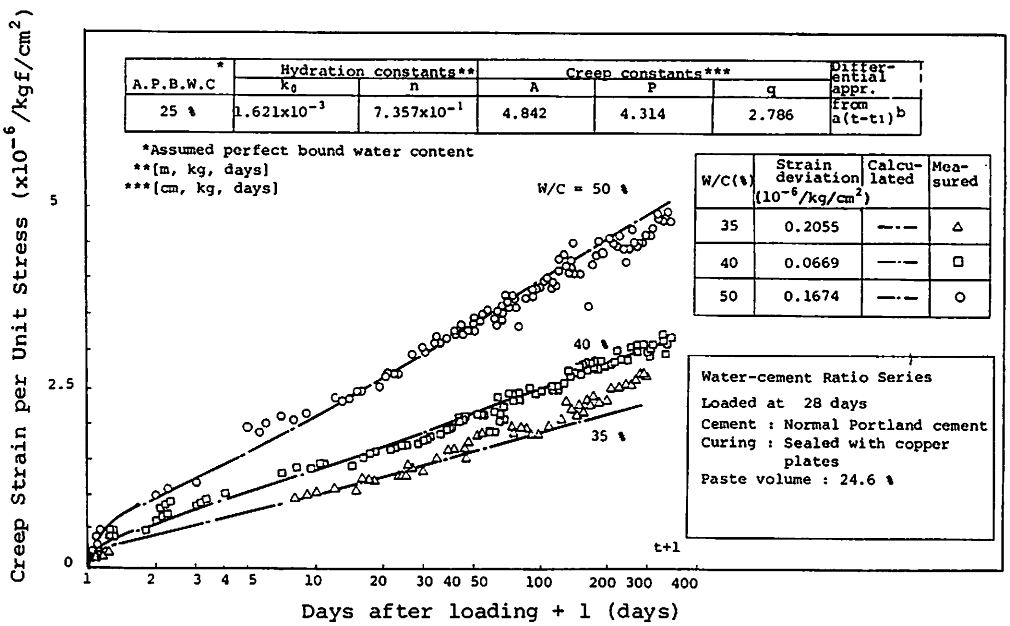


Fig. 11 : Creep of Concrete with W/C=35 and 50% Calculated with A, P, q of W/C=40% Concrete (Integral type)

Creep curves in special cases are as follows:

1) Right after loading; creep strains are proportional to $(t-t_1)^{1/q}$ in all cases.

2) When long period has elapsed after loading,
   a) if cement is in excess (i.e. $\gamma C-W>0$), then creep strains stop to increase,
   b) if water is in excess (i.e. $\gamma C-W<0$), then

$$\varepsilon_c(t-t_1) \rightarrow \frac{\sigma_0}{\eta_*(\infty)} (t-t_1)^{1/q} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (21)$$

   c) if $\gamma C-W=0$, then the creep strains tend to a constant value (see eq. (16)) or the creep rates tend to zero (see eq. (17), experimentally, $(1-n)P-1>0$ seems to be the general case).

According to this model, the final creep behaviour is determined by the condition whether the concrete contains unhydrated water in the structure or not. This conclusion coincides with the Hannant's experimental result [5] that perfectly dried concrete shows little creep.

From the above examinations, we see that the creep curves from this model are very closely related to power type expressions. In fact, these curves have been used by Straub (1930) [5], Shank (1935) [5], Browne and Blundell (1969) [5], etc. Recently, Bažant and Osman [8] proposed the following expression and have been using it frequently :

$$\varepsilon(t-t_1) = \frac{1}{E_0} + \frac{\phi_1}{E_0} t_1^{-m_B} (t-t_1)^{n_B} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (22)$$

where ϕ_1, m_B, n_B : constants,
t : age of concrete,
t_1 : age at loading.

This expression can be derived analytically from the present model, and also the restriction upon this expression can be deduced (that is, $n_B=1/q$, $m_B=(1-n)P/q$, m_B shows the dependence of creep strain on the age at loading).

5. RELATION OF THE DERIVED CREEP CALCULATION METHOD TO THE OTHER CONVENTIONAL METHODS

The calculation methods based on the present model include both the time-hardening [11] and the strain-hardening methods[11] for special cases.

The time-hardening method assumes that hardening of a material is caused by the lapse of time. And this is related to the hardening of concrete resulting from hydration.

On the other hand, the strain-hardening concept regards that the increase of strain hardens the material and arrests the further growth of creep strain. This indicates that further creep is inhibited when hardened cement paste is compressed, and the increase in intertwining of microscopic needle shaped crystals caused by the hydration of cement occurs.

Therefore it is reasonable to deduce that these two phenomena are taking place simultaneously.

In the following passages, we examine the relationship between the method from the present model and the other methods of creep analysis.

5.1 Time-hardening Method, Rate of Creep Method and the Model

According to Odqvist and Hult [11], time-hardening is expressed by the following equation:

$$d\varepsilon c/dt = A_H \sigma^{\ell_H} t^{-\lambda} , \qquad (23)$$

where A_H, ℓ_H, λ are constants. It is quite reasonable, in the metal theory, to assume that $\lambda < 1$ in order to integrate the expression (23), but as for concrete, we must consider the case where $\lambda \geq 1$. In fact we sometimes encounter the case $\lambda = (1-n)P/q \geq 1$ when we put $q = 1$ into equation (13). In this case, the equation (23) is integrable because the lower limit of the integral : t_1 can not become zero (concrete has not hardened at $t_1 = 0$!). In expression (11), if we put $q = 1$, and $\eta_*(t) = t^\lambda/A_H$, this model coincides with the result of time-hardening model with $\ell_H = 1$. In this case, creep strains become proportional to the stress when the stress in kept constant. This is the case $W/C = \gamma$, and moreover $\eta_*(t)$ can be expressed by equation (13). Extending the definition of time-hardening, and putting $d\varepsilon c/dt = g(\sigma,t)$, our model coincides with the time-hardening model as long as $q = 1$, even when $W/C \neq \gamma$.

Adopting Neville's notation [5], and assuming that the instantaneous strain component is hypoelastic, the rate of creep method is expressed by the following equation (24) :

$$\varepsilon(t-t_1) = \int_{t_1}^{t} \frac{1}{E_i(s)} \frac{d\sigma}{ds} ds + \int_{t_1}^{t} \sigma \frac{dC_{sp}}{dt} ds , \qquad (24)$$

where C_{sp} is the creep strain per unit stress.

Differentiating the above expression with respect to time, and putting $dC_{sp}/dt = 1/\eta(t)$, we find that equation (24) is same as the Maxwell model with time-varying rheological parameters. Also this is just the expression (11) whose q is taken as unity. This has been also called as Dischinger's equation.

5.2 Strain-hardening Method and the Present Model

According to Odqvist and Hult [11], the strain-hardening law is expressed by

$$d\varepsilon c/dt = B_H \sigma^{m_H} \varepsilon^{-\mu} , \qquad (25)$$

where B_H, m_H, μ are constants. Rewriting equation (25), we have

$$\frac{d}{dt}(\varepsilon_c^{\mu+1}) = (\mu+1)B_H\sigma^{\mu+1} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (26)$$

In equation (26), it is assumed that $m_H = \mu + 1$ in order to make creep strain proportional to the applied constant stress. Putting $\eta_*(t)$ = constant in expression (11) and comparing this with equation (26), we see that when $\eta_*(t)$ = constant, that is, when hydration is over, the present model reduces to the strain-hardening model.

The applicability of strain-hardening method to concrete structures was examined by G. L. England [12] and actually applied to the creep analysis of the prestressed concrete pressure vessels at Oldbury Nuclear Power Station by the engineers of the U. K. Central Electricity Generating Board [13] and compared with the measured strain in the real structure.

From the above mentioned §5.1 and §5.2, it can be seen that when q = 1, the present model expresses the rate of creep method (special case of time hardening method) while hydration is in progress, and it changes to express strain hardening method after hydration is completed. Of course, the hydration-viscosity model, or expression (11), can be used without any restriction regarding the hydration of cement, or its age.

Figure-12 shows the measured and calculated strains of concrete subjected to step-wise stress variations. A, P, q and instantaneous moduli of elasticity used for the calculation had been obtained from the creep tests with a constant stress : σ = 75 kgf/cm^2 (7.355 N/mm^2) loaded at 29,50, and 71 days of age respectively. The strain in this case, for example, after t_3 = 71 days, with t_1 = 29, and t_2 = 50 days, is given by

$$\varepsilon(t-t_1) = \frac{\sigma_1}{E_i(t_1)} + \frac{\sigma_2 - \sigma_1}{E_i(t_2)} + \frac{\sigma_3 - \sigma_2}{E_i(t_3)} + \left[\int_{t_1}^{t_2}\left(\frac{\sigma_1}{\eta_*(s)}\right)^q ds + \int_{t_2}^{t_3}\left(\frac{\sigma_2}{\eta_*(s)}\right)^q ds + \int_{t_3}^{t}\left(\frac{\sigma_3}{\eta_*(s)}\right)^q ds\right]^{1/q},$$

$$(t>t_3). \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (27)$$

Figure-13 shows the interrelation between various creep calculation methods.

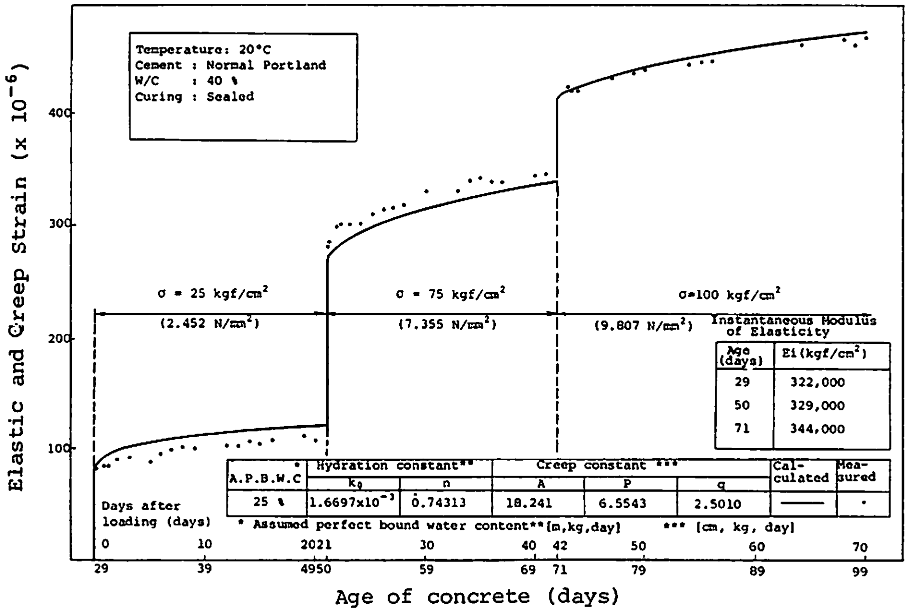


Fig. 12 : Creep Under Stepwise Stress Increases

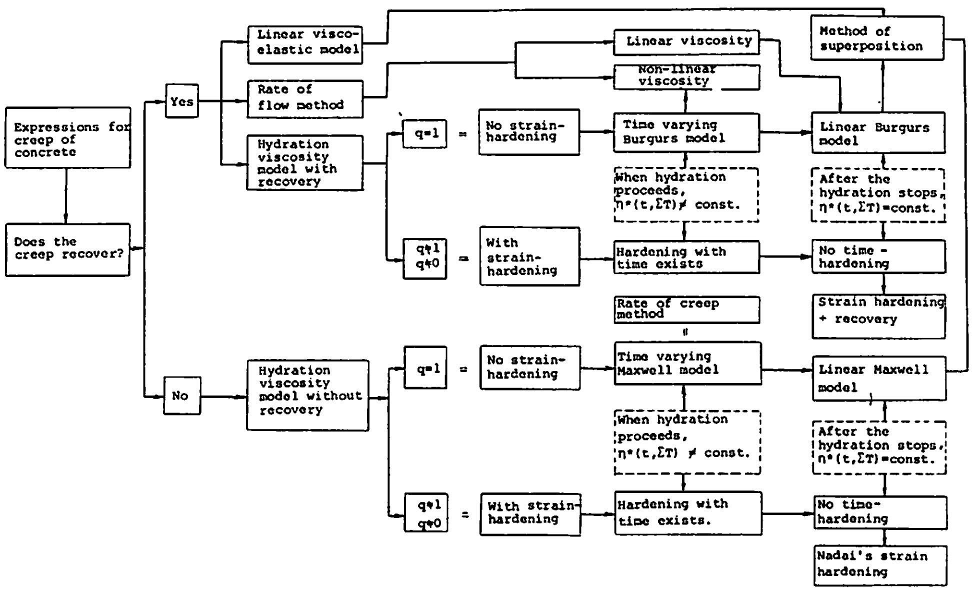


Fig. 13 : Inter-relations Between Various Creep Expressions and Methods of Calculation

6. RELATION BETWEEN THE HYDRATION-VISCOSITY MODEL AND TEMPERATURE

Experimentally, the variation of creep strain due to temperature is in same order as the variation of the reciprocal of coefficient of water viscosity (see table-3). In the present model, creep strain is inversely proportional to the coefficient of water viscosity after hydration is over. This is easily seen in expressions (11) to (14). This model consistently accounts for the observed phenomena in which creep is accelerated, although the increase of temperature is supposed to accelerate the hydration of cement and make concrete harden more rapidly. If the expression representing the hydration process under arbitrary varying temperatures is established, we can calculate the strain variation under such conditions automatically, using expression (11).

Table 3 : Comparison of the Reciprocals of the Coefficients of Viscosity of Water and Creep Strains of Sealed Concrete Loaded at the Intended Loading Age of 98 Days at Each Temperature [1].

(Relative values)

Temperature (°C)	20	40	70
Reciprocal of the coefficient of viscosity (η^{-1})	1.00	1.68	2.70
Creep strain after 71 days of loading	1.00	1.79	2.80

7. EXTENSION TO CREEP IN TENSION

Thus far in the study, the initial contact length between crystals has been assumed to be zero (i.e. $\ell=0$). So, if the model is subjected to tensile load at the initial stage, it will collapse instantly, as can be seen from figure-7. To extend the theory to tensile creep, we assume that $\ell=\ell_1$ at $t=t_1$, and corresponding "latent strain" to be ε_0. In that case, we also assume that $d\varepsilon c \geqq 0$ when $\sigma(t) \geqq 0$ (stress in compression), and $d\varepsilon c \leqq 0$ when $\sigma(t) \leqq 0$ (stress in tension). Considering that $\varepsilon c+\varepsilon_0 \geqq 0$, and moreover $\varepsilon c(0)=0$, we get

$$\varepsilon(t-t_1) = \int_{t_1}^{t} \frac{1}{E_i(s)} \frac{d\sigma(s)}{ds} ds - \varepsilon_0 + \left\{ \varepsilon_0{}^q + \int_{t_1}^{t} \mathrm{sgn}(\sigma) \frac{|\sigma(s)|^q}{\eta_*{}^q(s)} ds \right\}^{1/q} + \sum_{i=1}^{m} \frac{1}{\eta_{vi}} \int_{t_1}^{t} \sigma(s) \exp\left[-(t-s)/\lambda_i\right] ds, \quad \ldots\ldots (28)$$

where

$$\mathrm{sgn}(\sigma) = \begin{cases} -1, & (\sigma<0) \\ 0, & (\sigma=0) \\ 1, & (\sigma>0). \end{cases} \quad \dots\dots\dots \quad (29)$$

We added in expression (28) m pieces of Voigt elements in series formally to express recoverable components of creep strain. In the above expression (28), λ_i is the relaxation time of the i-th Voigt element ($=\eta_{vi}/E_{vi}$), and E_{vi} and η_{vi} are modulus of elasticity and coefficient of viscosity in the i-th Voigt element, respectively.

If t_1 is very large, $\eta_*(t) \fallingdotseq \eta_*(\infty)$=constant, as $t>t_1$. And if the stress is in compression, we put $\sigma(t)=\sigma_+$=constant>0, ($t \geqq t_1$) in expression (28), and neglecting recoverable components, we have

$$\varepsilon(t-t_1)= \frac{\sigma+}{E_i(\infty)} -\varepsilon_0 + \left[\varepsilon_0{}^q + \left(\frac{\sigma+}{\eta_*(\infty)} \right)^q (t-t_1) \right]^{1/q}. \quad \dots\dots\dots \quad (30)$$

If the stress is in tension:

$$\sigma(t)=\sigma-=\mathrm{constant}<0\,(t \geqq t_1),$$

we have

$$\varepsilon(t-t_1)= \frac{\sigma-}{E_i(\infty)} - \varepsilon_0 + \left[\varepsilon_0{}^q - \left(\frac{|\sigma-|}{\eta_*(\infty)} \right)^q (t-t_1) \right]^{1/q} \quad \dots\dots\dots \quad (31)$$

Figure-14 shows the difference in shape of tensile and compressive creep strains. According to the present model, the initial rate of creep under compression is finite, and this rate decreases with time. On the other hand, the creep rate in tension increases with time and finally tends to infinity when the strain reaches $-\varepsilon_0$, and this is the collapse. In fact, these two curves are two branches of a creep curve in which the initial resistance is not present. Figure-15 shows the experimental results by Brooks & Neville [14]. If we add recoverable creep strain components to the curves in figure-14, we get very similar curves to these curves.

In this generalized model, creep is not proportional to applied stress σ even when it is kept constant. Under compression, when $(t-t_1) \to 0$, the strain is proportional to σ^q, but the influence of initial resistance disappears and the strain becomes linear to stress with the lapse of time. Under tension, when $(t-t_1) \to 0$, the strain is also proportional to σ^q, and non-linear behaviour is magnified with the lapse of time, and the specimens collapse in order of the magnitude of applied stress, when it reaches a certain strain level ($-\varepsilon_0$).

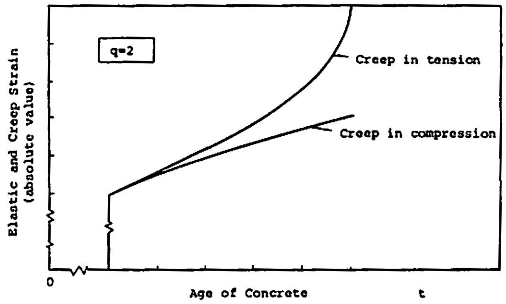


Fig. 14 : Difference of Creep Curves in Tension and Conpression. (Without recoverable creep strain)

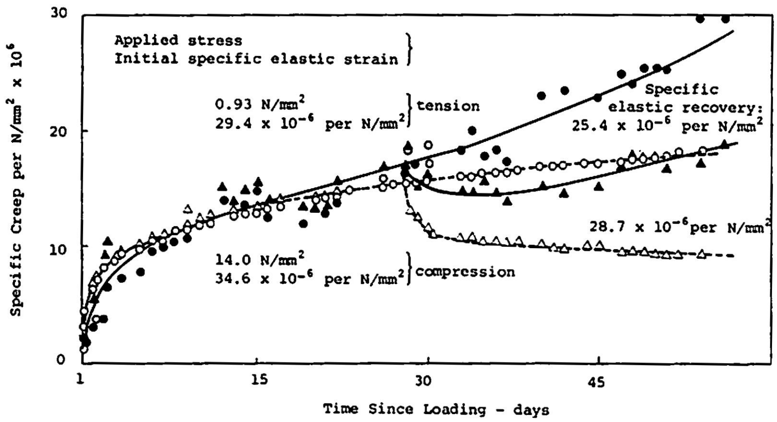


Fig. 15 : Experimental Results by Brooks & Neville [14] Showing the Difference of the Shapes of Creep Curves in Compression and in Tension (Cured in water. Loaded at 28 days and unloaded at 56 days.)

The creep rupture time (or critical time in Odqvist and Hult's terminology [11]) after the completion of hydration is

$$t-t_1=\eta_*{}^q(\infty)\varepsilon_0{}^q/\sigma^q \qquad (32)$$

The exactly same expression has been derived in the field of metal creep based on the different idea and has been widely accepted [11].

The consistency of expression (28) under the combination of the change of stress in compression and tension is also easily verified.

8. CONCLUSIONS

A creep model of concrete taking account of cement hydration and viscosity of internal water seems capable of explaining a somewhat wide range of creep phenomena of this material, and in some respects (e.g. the influence of the age at loading and water-cement ratio) quantitative agreements were obtained between experiments and calculations. Furthermore, the proposed model is closely related to the various conventional methods of creep analysis, and strain-hardening and rate of creep methods are the special cases of the generalized method deduced from this model.

Thus the introduction of the microscopic perspective has been proven to be essential in the study of concrete creep.

Recently, the amount of pomp-crete used in civil engineering works has increased, and the creep problem will hold an increasingly important position in ensuring the safety of these structures.

The authors wish to extend the theory to cover drying creep, creep recovery, etc., after accumulating more basic experimental data.

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