CHAPTER II

VIADUCT BENTS

16. FIG. 9 shows a common form of bents of an elevated railway, the posts being riveted to the cross-girder on top and firmly anchored at the base. In this kind of construction the bending of the cross-girder due to loading is transmitted to posts, so that the latter are subjected to combined stresses.

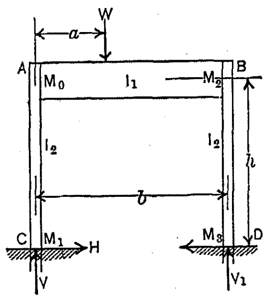

Fig. 9

The following designations will be used:

M_0, M_1, M_2, and M_3 . .	moments at points A, C, B, and D respectively.
I_1 and I_2	the moments of inertia of sections of cross-girder and posts respectively.
h	the height of neutral axis of the cross-girder above the plane of anchorage.
b	the distance apart of the axes of the posts.
V and V_1	the vertical reactions, positive upward, at C and D respectively.
H	the horizontal reaction at C or D, positive when directed toward right.
E	the modulus of elasticity assumed to be constant.

Calling those moments producing compression on the outside fibre of the structure *positive*, we have the following moments:

$M_1 - Hx$ . . . . in post CA at x from C.
$M_1 + Vx - Hh$ . . . . . . in cross-girder at x from A between A and W.
$M_1 + Vx - W(x-a) - Hh$ . . in cross-girder at x from A between W and B.
$M_3 - H(h-x)$ . . . . in post BD at x from B.

The total internal work of resistance in the members composing the bent, if we neglect the effect of all direct stresses as being inconsiderable when compared with that of the moment, would be,

$$\omega = \frac{1}{2EI_2}\left\{\int_0^h (M_1 - Hx)^2 dx + \int_0^h (M_3 - Hh + Hx)^2 dx\right\} + \frac{1}{2EI_1}\left[\int_0^a (M_1 + Vx - Hh)^2 dx + \int_a^b \{M_1 + Vx - W(x-a) - Hh\}^2 dx\right].$$

Noting that

$$V = \frac{W(b-a) + M_3 - M_1}{b}$$

the first derivatives of ω taken with respect to M_1, M_3, and H successively set equal to zero, will give the following equations of conditions:

$$\frac{d\omega}{dM_1} = M_1\left(\frac{h}{I_2} + \frac{b}{3I_1}\right) + M_3\frac{b}{6I_1} - Hh\left(\frac{h}{2I_2} + \frac{b}{2I_1}\right) + \frac{Wa}{6bI_1}(2b-a)(b-a) = 0.$$

$$\frac{d\omega}{dM_3} = M_3\left(\frac{h}{I_2} + \frac{b}{3I_1}\right) + M_1\frac{b}{6I_1} - Hh\left(\frac{h}{2I_2} + \frac{b}{2I_1}\right) + \frac{Wa}{6bI_1}(b^2 - a^2) = 0.$$

$$\frac{d\omega}{dH} = (M_1 + M_3)\left(\frac{h}{2I_2} + \frac{b}{2I_1}\right) - Hh\left(\frac{2h}{3I_2} + \frac{b}{I_1}\right) + \frac{Wa}{2I_1}(b-a) = 0.$$

Combining these equations, we get the following values of H, M_1, and M_3:

$$H = \frac{3I_2 a(b-a)}{2h(hI_1 + 2bI_2)}W \quad . . . . \quad (15)$$

$$M_1 = \frac{I_2}{2}\left\{\frac{1}{hI_1 + 2bI_2} - \frac{b-2a}{b(6hI_1 + bI_2)}\right\}a(b-a)W \quad . \quad (16)$$

$$M_3 = \frac{I_2}{2}\left\{\frac{1}{hI_1 + 2bI_2} + \frac{b-2a}{b(6hI_1 + bI_2)}\right\}a(b-a)W \quad . \quad (17)$$

Since

$$M_0 = M_1 - Hh \text{ and } M_2 = M_3 - Hh,$$

we get,

$$M_0 = -\frac{I_2}{2}\left\{\frac{2}{hI_1 + 2bI_2} + \frac{b-2a}{b(6hI_1 + bI_2)}\right\}a(b-a)W. \quad (18)$$

$$M_2 = -\frac{I_2}{2}\left\{\frac{2}{hI_1 + 2bI_2} - \frac{b-2a}{b(6hI_1 + bI_2)}\right\}a(b-a)W. \quad (19)$$

Again, since

$$V = \frac{1}{b}\{W(b-a) + M_3 - M_1\},$$

we now get

$$V = \left\{1 + \frac{I_2 a(b-2a)}{b(6hI_1 + bI_2)}\right\}\frac{b-a}{b}W \quad . . . . \quad (20)$$

It is evident that the maximum stress in post AC is caused by the combined action of V and the moment M_1 or M_0, whichever is greater.

Similarly, since

$$V_1 = \frac{1}{b}\{Wa + M_1 - M_3\},$$

we get

$$V_1 = \left\{1 - \frac{I_2(b-a)(b-2a)}{b(6hI_1 + bI_2)}\right\}\frac{a}{b}W.$$

17. If we represent by h_0 and h_1 the distances of the points of contraflexure in the posts (Fig. 10), since

$$M_1 - Hh_0 = 0,$$
$$M_3 - Hh_1 = 0,$$

we get from the preceding equations,

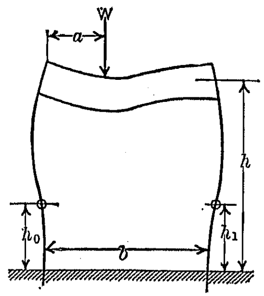

Fig. 10

$$h_0 = \frac{hI_1 + 2\,bI_2}{3}\left\{\frac{1}{hI_1 + 2\,bI_2} - \frac{b - 2\,a}{b\,(6\,hI_1 + bI_2)}\right\}h \quad . \quad . \quad (21)$$

$$h_1 = \frac{hI_1 + 2\,bI_2}{3}\left\{\frac{1}{hI_1 + 2\,bI_2} + \frac{b - 2\,a}{b\,(6\,hI_1 + bI_2)}\right\}h \quad . \quad . \quad (22)$$

For $a = \frac{b}{2}$, then,

$$h_0 = h_1 = \frac{1}{3}h,$$

showing that M_0 and M_2 will then be opposite and twice in amount of M_1 and M_3 respectively.

18. The same condition will obtain in case of loads symmetrically disposed with respect to the centre of the cross-girder. Thus in the case of a single-track bent (Fig. 11), Eqs. (16) to (19) will give for a loading of 2 W,

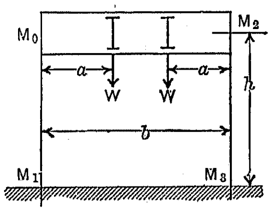

Fig. 11

$$M_0 = M_2 = -\frac{2\,I_2\,a\,(b - a)}{hI_1 + 2\,bI_2}W \quad . \quad . \quad . \quad . \quad (23)$$

$$M_1 = M_3 = \frac{I_2 a\,(b - a)}{hI_1 + 2\,bI_2}W \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

19. In case the posts are *hinged* at C and D (Fig. 12), M_1 and M_3 will disappear from the preceding equations, and

$$H = \frac{3\,I_2 a\,(b - a)}{2\,h\,(2\,hI_1 + 3\,bI_2)}W \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Consequently,

$$M_0 = -Hh = -\frac{3\,I_2 a\,(b - a)}{2\,(2\,hI_1 + 3\,bI_2)}W \quad . \quad . \quad . \quad (26)$$

If the posts were hinged at A and B, there would of course be no moment in the posts.

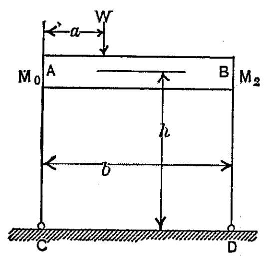

Fig. 12

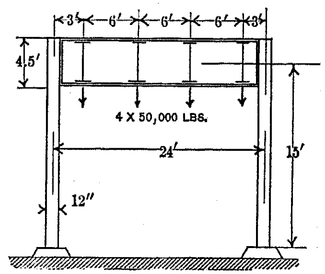

Fig. 13

Example.—In the elevated-railway bent of Fig. 13 to calculate the maximum stress in posts under full loading. Given,

$$I_1 = 24{,}000 \text{ in.}^4$$
$$I_2 = 1\,000 \text{ in.}^4$$
$$A_2 = 24 \text{ ins.}^2$$
$$W = 50{,}000 \text{ lbs.}$$

From Eqs. (23) and (24),

$$M_0 = -\frac{2\,I_2 W}{hI_1 + 2\,bI_2}\sum_0^{\frac{b}{2}} a\,(b - a) = -\,582{,}350 \text{ in.-lbs.}$$

$$M_1 = \frac{I_2 W}{hI_1 + 2\,bI_2} \sum_0^b a\,(b - a) = 291{,}170 \text{ in.-lbs.}$$

$$V = \sum_0^b \frac{b - a}{b}\,W = 100{,}000 \text{ lbs.}$$

The maximum stress in the post will then be,

$$\frac{582{,}350}{1000} \times 6 + \frac{100{,}000}{24} = 7660 \text{ lbs. per sq. in.}$$

Stresses due to moment in the longitudinal plane and those due to changes of temperature remain still to be provided for in the posts.

In case the lower end of each post is hinged, we get from Eq. (26),

$$M_0 = -\frac{3\,I_2 W}{2\,(2\,hI_1 + 3\,b\,I_2)} \sum_0^b a\,(b - a) = -\ 450{,}000 \text{ in.-lbs.},$$

so that the maximum stress in the post will be,

$$\frac{450{,}000}{1000} \times 6 + \frac{100{,}000}{24} = 6870 \text{ lbs. per sq. in.}$$

It will thus be seen that so far as the vertical loading is concerned, the stress in the post is increased by fixing the lower ends of the posts.

20. Wind pressure also produces moments in the posts and cross-girder of a bent. In the bent of Fig. 14, in which the posts are constrained at both ends, the wind pressures P and P_1, assumed to be acting as shown by arrows in the axis of the cross-girder, tend to deform the bent, as shown exaggerated in the figure. With the symmetrical disposition of materials and end conditions, as

is the case under consideration, it will be easy to see, without going into analytical works, that so long as we do not take into consideration the lengthwise deformation of the girder AB, which is generally inconsiderable when compared with the deflection of the posts, the relative position of points A and B would always remain unchanged, and as a consequence both posts would be equally bent by P and P_1, or, in other words, the reactions H and H_1 would be equal, and the points of contraflexure in the posts would be at the same height above C and D. Then, since equilibrium requires that

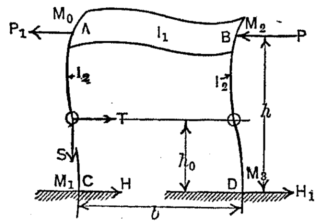

Fig. 14

$$P_1 + P - (H + H_1) = 0,$$

we may put

$$H = H_1 = \frac{P + P_1}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Passing a section through one of the points of contraflexure, and representing by S and T the direct and tangential stresses at the section of the upper portion (the opposite stresses of equal amount being assumed to be acting at the section of the lower portion), we have

$$H = T,$$

and taking moment at C (moments + when producing compression on the outside fibre as before),

$$M_1 = Hh_0\,.$$

In the upper side of the section, taking moments successively at A, o, B and D, we get,

$$M_0 = -H(h - h_0).$$
$$Sb = -(P_1 + P)(h - h_0).$$
$$M_2 = -T(h - h_0) - Sb = H(h - h_0) = -M_0.$$
$$M_3 = Th_0 - Sb - (P + P_1)h = -Hh_0 = -M_1.$$

The moment at any point of the frame may now be expressed as follows, the origin of x being taken at A, C, and D respectively:

Cross-girder, $M_0 + \dfrac{M_2 - M_0}{b}x = -H(h - h_0) + \dfrac{2H(h - h_0)}{b}x.$

Left post, $M_1 + \dfrac{M_0 - M_1}{h}x = H(h_0 - x).$

Right post, $M_3 + \dfrac{M_2 - M_3}{h}x = H(-h_0 + x).$

Neglecting the influence of direct stresses and shears as before, we obtain for the internal work:

$$\omega = \frac{H^2}{2EI_1}\int_0^b (h - h_0)^2\left(\frac{2x}{b} - 1\right)^2 dx + \frac{H^2}{EI_2}\int_0^h (h_0 - x)^2\,dx.$$

Putting the first derivative of ω with respect to h_0, equal to zero, we obtain,

$$h_0 = \frac{bI_2 + 3hI_1}{bI_2 + 6hI_1}h \quad \ldots\ldots\ldots \quad (28)$$

and consequently,

$$M_0 = -M_2 = -H(h - h_0) = -\frac{3hI_1}{bI_2 + 6hI_1}hH \quad . \quad (29)$$

$$M_1 = -M_3 = Hh_0 = \frac{bI_2 + 3hI_1}{bI_2 + 6hI_1}hH \quad \ldots\ldots \quad (30)$$

The *direct stress* in the post is S, being compression in AC and tension in BD.

$$S = \frac{(P_1 + P)(h - h_0)}{b} \quad \ldots\ldots \quad (31)$$

The posts are, therefore, subjected to bending and direct stress combined.

If the posts were *hinged* at the base, M_1 and M_3 would disappear, and as h_0 would then be equal to o, we get

$$M_0 = -M_2 = -Hh.$$

The reverse actions would take place if the girder were hinged to the posts while the latter are fixed at the base.

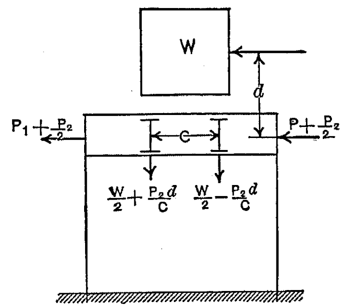

Fig. 15

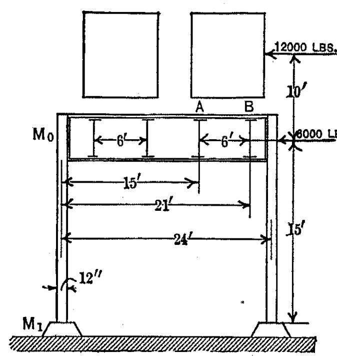

Fig. 16

21. In case the wind pressure P_2 acts on the train at a distance of d (Fig. 15) above the axis of the cross-girder in addition to P and P_1 acting on the structure as before, the bent will beside its own weight be subjected to forces due to the overturning moment of P_2, as shown in the figure. The moments as found by Eqs. (16) to (19) for vertical forces are in this case to be combined with those by Eqs. (29) and (30) for horizontal forces to obtain the resultant moments and reactions.

EXAMPLE.—In the bent of the preceding example, suppose wind pressures of 12,000 lbs. and 6000 lbs. be acting on the exposed surfaces of the train and viaduct respectively (Fig. 16).

To find the stresses produced in the posts due to these wind pressures only.

Considering the horizontal forces only, a pressure of 18,000 lbs. acting at the axis of the cross-girder will cause in lee-side post, according to Eqs. (29) and (30), the following moments:

$$M_0 = -\frac{45 \times 24{,}000}{24 \times 1000 + 90 \times 24{,}000} \times 15 \times \frac{18{,}000}{2} = -66{,}780 \text{ ft.-lbs.}$$

$$M_1 = \frac{24 \times 1000 + 45 \times 24{,}000}{24 \times 1000 + 90 \times 24{,}000} \times 15 \times \frac{18{,}000}{2} = 68{,}240 \text{ ft.-lbs.}$$

The overturning moment of wind pressure on the train produces one upward and another downward pressure of $\frac{12{,}000 \times 10}{6} = 20{,}000$ lbs. on the cross-girder at A and B, for which we get, from Eqs. (18) and (16),

$$M_0 = -\frac{1000}{2} \times 20{,}000 \left\{ \left(\frac{2}{408{,}000} + \frac{24 - 30}{52{,}416{,}000} \right) 15 \times 9 - \left(\frac{2}{408{,}000} + \frac{24 - 42}{52{,}416{,}000} \right) 21 \times 3 \right\} = -3580 \text{ ft.-lbs.}$$

$$M_1 = \frac{1000}{2} \times 20{,}000 \left\{ \left(\frac{1}{408{,}000} - \frac{24 - 30}{52{,}416{,}000} \right) 15 \times 9 - \left(\frac{1}{408{,}000} - \frac{24 - 42}{52{,}416{,}000} \right) 21 \times 3 \right\} = 1700 \text{ ft.-lbs.}$$

As to the direct stress in the posts, we get, from Eqs. (31) and (20),

$$V = \frac{18{,}000\,(h - h_0)}{24} + 5000 = 10{,}565 \text{ lbs.}$$

The maximum fibre stress in the left post due to wind pressure will therefore be a compression of

$$\frac{(66{,}780 + 3580)\,12}{1000} \times 6 + \frac{10{,}565}{24} = 5505 \text{ lbs. per sq. in.}$$

This amount of fibre stress is but little less than that due to full loading found in the preceding example.

In case the lower ends of the posts are hinged, M_0 would be

$$-9000 \times 15 = -135{,}000 \text{ ft.-lbs. nearly,}$$

and

$$V = \frac{12{,}000 \times 25 + 6000 \times 15}{24} = 16{,}250 \text{ lbs.,}$$

so that the maximum fibre stress will not be less than

$$\frac{135{,}000 \times 12 \times 6}{1000} + \frac{16{,}250}{24} = 10{,}397 \text{ lbs. per sq. in.}$$

These figures show that the decrease of moment due to vertical loading by hinging the lower ends of the posts is far more than neutralized by the increased moment caused in the same by wind pressure.

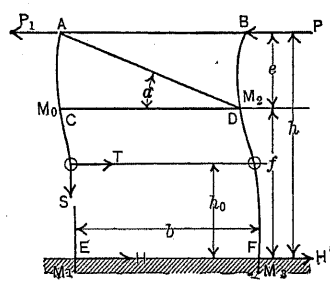

Fig. 17

22. In a bent with simple cross-bracing, such as shown in Fig. 17, acted on by wind pressures P and P_1, if we suppose all the joints of the bracing to be hinged, there would be two points of no moment in each post, viz.: A,B, and the points of contraflexure o, o. Also there would be no moment in any member of the bracing. Passing a section through o of the post AE, and denoting by T and S the tangential and direct stresses acting at the section of the upper portion, as in the preceding case, we have

$$T = H = \frac{P + P_1}{2}.$$

Taking moments at E and C,

$$\begin{aligned} M_1 &= Hh_0, \\ M_0 &= - T(f - h_0) = - H(f - h_0), \\ M_2 &= - M_0, \\ M_3 &= - M_1. \end{aligned}$$

Taking moment at o of the post BF,

$$- Sb - (P_1 + P)(h - h_0) = 0,$$

or

$$S = - \frac{(P_1 + P)(h - h_0)}{b} \quad . \quad . \quad . \quad . \quad . \quad (32)$$

Then at any point distant x from E we have the following moments in the post AE:

E to C, $\quad M_1 + \dfrac{M_0 - M_1}{f} x = H(h_0 - x).$

C to A, $\quad M_0 - \dfrac{M_0}{e}(x - f) = H(f - h_0)\left(\dfrac{x - f}{e} - 1\right).$

The corresponding moments in the post BF have simply the opposite signs.

Neglecting, then, the influence of all direct and tangential stresses both in the posts and bracing, we get for the internal work of moments in the posts:

$$\omega = \frac{H^2}{EI}\left\{ \int_0^f (h_0 - x)^2 \, dx + \int_f^h (f - h_0)^2 \left(\frac{x - f}{e} - 1\right)^2 dx \right\}.$$

Integrating, we obtain,

$$\omega = \frac{H^2}{EI}\left\{ f\left(h_0^2 - h_0 f + \frac{f^2}{3}\right) + \frac{e}{3}(f - h_0)^2 \right\}.$$

Differentiating ω with respect to h_0, and setting the differential coefficient equal to zero, we get

$$h_0 = \frac{f(2h + f)}{2(h + 2f)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

so that

$$M_1 = - M_3 = \frac{f(2h + f)}{2(h + 2f)} H \quad . \quad . \quad . \quad . \quad . \quad (34)$$

$$M_0 = - M_2 = - \frac{3f^2}{2(h + 2f)} H \quad . \quad . \quad . \quad . \quad (35)$$

To obtain direct stresses in the posts and braces, pass a section through AB, AD, and CD; then, considering the left portion of the section as far as to the point of contraflexure, the moment taken with respect to D will give, by calling, as before, compression — and tension +:

$$- Sb - T(f - h_0) - P_1 e + \overline{AB} \cdot e = 0,$$

from which

$$\overline{AB} = \frac{Sb + T(f - h_0) + P_1 e}{e} = -(P + P_1)\frac{(h - h_0)}{2e} + \frac{P_1 - P}{2}.$$

Taking moment at A, we get

$$-T(h-h_0)-\overline{CD}\cdot e=0,$$

from which

$$\overline{CD}=-\frac{T(h-h_0)}{e}=-\frac{(P+P_1)(h-h_0)}{2e}.$$

Further, since $\Sigma V=0$ at the section,

$$-S-\overline{AD}\sin\alpha=0.$$

Whence

$$\overline{AD}=-\frac{S}{\sin\alpha}=\frac{(P_1+P)(h-h_0)}{b\sin\alpha}.$$

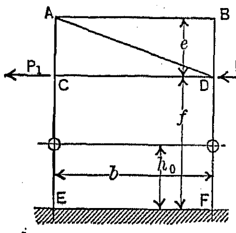

Fig 18

The direct stress in either post is nothing else than S, being compression in AE and tension in BF.

In case the wind pressures are supposed to be acting at points C and D (Fig. 18), the condition of affairs remains unchanged so far as moments in the posts are concerned. The only differences with the preceding case are in direct stresses. Since, here

$$S=-\frac{(P_1+P)(f-h_0)}{b},$$

we get for the direct stresses in posts and braces the following expressions:

$$\overline{AB}=-\frac{(P_1+P)(f-h_0)}{2e},$$

$$\overline{CD}=-\frac{(P_1+P)(h-h_0)}{2e}+P_1,$$

$$\overline{AD}=+\frac{(P_1+P)(f-h_0)}{b\sin\alpha}.$$

It is to be noted that the neglect of direct stresses in the calculation of internal work implies the indeformability of the frame $ABCD$, which is evidently not true, but the effect of its deformation is generally so small that the formulas deduced above will be practically correct.

This form of construction is more common in portal bracing of a metallic bridge than in viaduct bents, of which former, the following example furnishes a case.

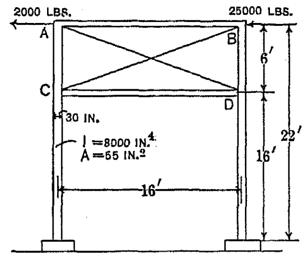

Fig. 19

EXAMPLE. — In the portal bracing of Fig. 19 find the stresses in braces and the greatest fibre stress produced in the posts in carrying the wind pressures as shown, down to the masonry.

From Eq. (33),

$$h_0=\frac{16\,(44+16)}{2\,(22+32)}=8.9\text{ ft.}$$

From (34) and (35),

$$M_1=13{,}500\times 8.9=120{,}150\text{ ft.-lbs.}$$
$$M_0=-13{,}500\,(16-8.9)=-95{,}850\text{ ft.-lbs.}$$

From Eq. (32),

$$S=-\frac{27{,}000\,(22-8.9)}{16}=-22{,}100\text{ lbs.}$$

Consequently the maximum fibre stress in the post will be

$$\frac{22{,}100}{55}+\frac{120{,}150\times 12}{8000}\times 15=3105\text{ lbs. per square in.,}$$

being tension in the right post and compression in the left one.

Supposing the diagonal braces to be capable of resisting tension only, only AD would be in action with the given direction of wind pressures, and we get,

$$\overline{AB} = -(2000+25{,}000)\,\frac{(22-8.9)}{2\times 6} + \frac{2000-25{,}000}{2} = -40{,}980 \text{ lbs.}$$

$$\overline{CD} = -\frac{(2000+25{,}000)\,(22-8.9)}{2\times 6} = -29{,}480 \text{ lbs.}$$

$$\overline{AD} = \frac{(2000+25{,}000)\,(22-8.9)}{16\times .35} = +63{,}160 \text{ lbs.}$$

23. In a bent with knee-bracings, such as shown in Fig. 20, assuming all the members to be hinged at their connections and the base of each post firmly anchored to the masonry, we have, as before,

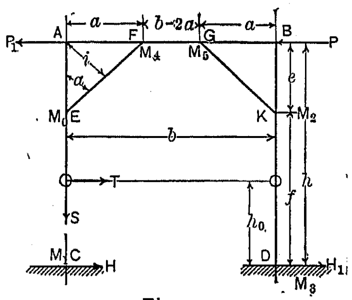

Fig. 20

$$H = H_1 = \frac{P+P_1}{2},$$

$$T = H,$$

$$S = -\frac{(P+P_1)\,(h-h_0)}{b},$$

$$M_1 = -M_3 = Hh_0,$$

$$M_0 = -M_2 = -H\,(f-h_0),$$

$$M_4 = -M_5 = -H\,(h-h_0)\left(\frac{b-2a}{b}\right),$$

h_0 representing the height of the points of contraflexure as in all the previous cases.

Considering bending moments only, we get for ω the following expression; the origins of x being taken at C, E, A, and F:

$$\omega = \frac{1}{EI_2}\int_0^f (M_1 - Hx)^2\,dx + \frac{1}{EI_2}\int_0^e \left(M_0 - \frac{M_0}{e}x\right)^2 dx$$
$$+ \frac{1}{EI_1}\int_0^a \left(\frac{M_4}{a}x\right)^2 dx + \frac{1}{2\,EI_1}\int_0^{b-2a}\left(M_4 - \frac{2\,M_4}{b-2a}x\right)^2 dx,$$

in which I_1 and I_2 represent the moments of inertia of the strut AB and posts AC and BD respectively. Since

$$\frac{dM_1}{dh_0} = H,\quad \frac{dM_0}{dh_0} = H,\quad \frac{dM_4}{dh_0} = H\left(\frac{b-2a}{b}\right),$$

we get for

$$\frac{d\omega}{dh_0} = 0,$$

$$\frac{1}{EI_2}\int_0^f 2\,H^2\,(h_0 - x)\,dx + \frac{1}{EI_2}\int_0^e -2\,H^2\,(f-h_0)\left(1-\frac{x}{e}\right)^2 dx$$
$$+ \frac{1}{EI_1}\int_0^a -2\,H^2\left(\frac{b-2a}{ab}\right)^2 (h-h_0)\,x^2\,dx$$
$$+ \frac{1}{2\,EI_1}\int_0^{b-2a} -2\,H^2\,(h-h_0)\left(\frac{b-2a}{b}\right)^2\left(1-\frac{2x}{b-2a}\right)^2 dx = 0,$$

from which

$$h_0 = \frac{bI_1\,(2h+f)\,f + I_2\,(b-2a)^2\,h}{2\,bI_1\,(h+2f) + I_2\,(b-2a)^2} \quad . \quad . \quad . \quad (36)$$

Knowing h_0, the moment at any point of the bent may at once be written. Thus,

C to E, $H\,(h_0 - x)$, origin of x taken at C,

E to A, $-H\,(f-h_0)\left(1-\dfrac{x}{e}\right)$, origin of x taken at E,

A to $F, -H\ (h - h_0)\left(\frac{b - 2a}{b}\right)\frac{x}{a}$, origin of x taken at A,

F to $G, -H\ (h - h_0)\left(\frac{b - 2a}{b}\right)\left(1 - \frac{2x}{b - 2a}\right)$, origin of x taken at F.

The moments in the right half of the frame are simply opposite in signs to those of the left. The direct stresses are obtained in the following manner:

Passing a section through A and EF, and considering the left portion above o, the moment taken with respect to A will give:

$$- T\ (h - h_0) - \overline{EF} \cdot i = 0,$$

or

$$\overline{EF} = - \frac{H\ (h - h_0)}{i} = - \overline{GK}.$$

In the same section, since $\Sigma V = 0$,

$$- S + \overline{AE} + \overline{EF} \cos \alpha = 0,$$

from which

$$\overline{AE} = S - \overline{EF} \cos \alpha = \frac{H\ (h - h_0)\ (b - 2a)}{ab} = - \overline{BK},$$

$$\overline{CE} = S = - \frac{2H\ (h - h_0)}{b} = - \overline{KD}.$$

Passing a section through AF and EF, and considering the left position of it, since Σ Horiz. forces $= 0$,

$$H - P_1 + \overline{AF} + \overline{EF} \sin \alpha = 0,$$

from which

$$\overline{AF} = \frac{P_1 - P}{2} + \frac{H\ (h - h_0)}{e}.$$

A vertical section through FG will give for the left half of the section,

$$H - P_1 + \overline{FG} = 0,$$

or

$$\overline{FG} = H + P_1 = \frac{P_1 - P}{2}.$$

At a section through GB and GK we have for the left portion of the same,

$$H - P_1 + \overline{GB} + \overline{GK} \sin \alpha = 0,$$

from which

$$\overline{GB} = \frac{P_1 - P}{2} - \frac{H\ (h - h_0)}{e}.$$

Like the last case, this form of construction is often used for portals, especially of a wooden bridge. The following example is a case of the latter.

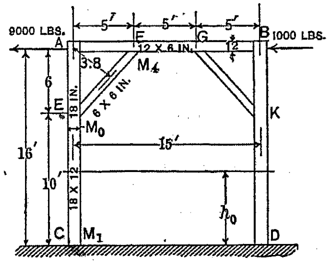

Fig. 21

EXAMPLE. — To calculate the stresses in the portal of a wooden bridge, of Fig. 21, due to wind pressures as shown.

Here
$$I_1 = \frac{6 \times 12^3}{12} = 864 \text{ in.}^4,$$

$$I_2 = \frac{12 \times 18^3}{12} = 5832 \text{ in.}^4$$

Substituting in Eq. (36) the values of several terms, we get,

$$h_0 = \frac{15 \times 864 \times 42 \times 10 + 5832 \times 5^2 \times 16}{2 \times 15 \times 864 \times 36 + 5832 \times 5^2} = 7.21 \text{ ft.}$$

Then, since $H = \dfrac{9000 + 1000}{2} = 5000$ lbs.,

$$M_1 = 5000 \times 7.21 = 36{,}050 \text{ ft.-lbs.}$$
$$M_0 = -5000\,(10 - 7.21) = -13{,}950 \text{ ft.-lbs.}$$
$$M_4 = -5000\,(16 - 7.21)\,\frac{5}{15} = -14{,}650 \text{ ft.-lbs.}$$

The following are direct stresses:

$$\overline{CE} = -\frac{10{,}000\,(16 - 7.21)}{15} = -5860 \text{ lbs.}$$
$$\overline{EF} = -\frac{5000\,(16 - 7.21)}{3.8} = -11{,}566 \text{ lbs.}$$
$$\overline{GK} = +11{,}566 \text{ lbs.}$$
$$\overline{AF} = \frac{9000 - 1000}{2} + \frac{5000\,(16 - 7.21)}{6} = +11{,}325 \text{ lbs.}$$
$$\overline{FG} = \frac{9000 - 1000}{2} = +4000 \text{ lbs.}$$
$$\overline{GB} = \frac{9000 - 1000}{2} - \frac{5000\,(16 - 7.21)}{6} = -3325 \text{ lbs.}$$

The following comparisons of maximum stresses in members, as calculated on assumptions of fixed and pivoted ends of end posts, show that there is a considerable margin of safety in the statical calculations as generally followed in designing a portal brace of this kind, when the post ends are really fixed.

MAXIMUM STRESSES IN LBS. PER SQ. IN.

End Conditions.	*AC.*			*EF.*	*AB.*		
	Direct.	Bending	Total.	Direct.	Direct.	Bending.	Total.
Fixed . . .	27	668	695	321	157	1221	1378
Hinged . .	49	926	1075	658	240	2223	2463