…ALLY-INDETERMINATE STRESSES-HIROI

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THE

STATICALLY-INDETERMINATE

STRESSES

IN FRAMES COMMONLY USED FOR

BRIDGES

BY

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94 ILLUSTRATIONS

SECOND EDITION
REVISED AND ENLARGED

NEW YORK
D. VAN NOSTRAND COMPANY
25 PARK PLACE
1915

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Stanhope Press
F. H. GILSON COMPANY
BOSTON, U.S.A.

PREFACE TO THE SECOND EDITION

In the present edition, corrections are made of errors which were found in the first issue of the work. The temperature stress in viaduct bents, which the author neglected to work out in the previous edition, is made the subject of an appendix. A proof of the theorems of Castigliano is also appended, forming a supplementary article to the introductory chapter.

I. H.

February, 1915.

登　録	昭和 61年 6月 5日
番　号	第 32327 号
社団法人	土木学会
附属	土木図書館

PREFACE TO THE FIRST EDITION

THE present work is the outgrowth of a series of lectures given to the students of Civil Engineering in the Tokyo Imperial University. It contains the solution of those problems most commonly met in the practice of a bridge engineer, the aim of the author being to save time and labor of those intent on a more rational design of the class of the structures treated, than is generally followed, by furnishing them with necessary formulas for which rough approximation or even guess-work frequently forms a substitute.

For different treatment of some of the cases discussed in this work, readers may do well to compare the works of Professors Burr, Greene, Du Bois and Johnson, and also those of Professors Engesser, Résal, Winkler, Melan, Müller-Breslau, Steiner, etc.

The author acknowledges his indebtedness for valuable assistance in preparing the volume, to his colaborer Assistant-Professor H. Kimishima.

TOKYO IMPERIAL UNIVERSITY,
August, 1904.

I. H.

CONTENTS

INTRODUCTORY CHAPTER

GENERAL PRINCIPLES

CHAPTER I

TRUSSED BEAMS

CHAPTER II

VIADUCT BENTS

CHAPTER III

Continuous Girders

CHAPTER IV

Arches with two Hinges

CHAPTER V

Arches without Hinges

CHAPTER VI

Suspension Bridges

Trusses with Redundant Members

CHAPTER VII

Secondary Stresses due to Rigidity of Joints

APPENDIX I

APPENDIX II

INTRODUCTORY CHAPTER

GENERAL PRINCIPLES

1. Most of the cases of statically-indeterminate stresses occurring in the practice of a bridge engineer can be solved in several different ways; but in this, the author has made the exclusive use of the method of work as the simplest and the most direct way for arriving at the results.

It is a well-known principle in mechanics that when external forces act on an elastic body, the latter undergoes deformations, which, according to the Hooke's law, are proportional to the stresses causing them, — the deformations assumed to be disappearing the moment the forces are taken off. The work thus performed in the body while being acted on by external forces, we call the *work of resistance*. This internal work, which we shall henceforth designate with ω, may be expressed in the following manner, for different kinds of stresses.

2. Direct Stress. — Suppose a straight bar having a cross-section A, length L, and modulus of elasticity E, be subjected to tension or compression increasing from o to S. Assuming the strain to be proportional to stress, the bar would undergo, at any moment when the stress is s, a deformation of

$$\frac{sL}{AE}.$$

Since work equals the force into its displacement the increment of work performed in the bar at the moment will be

$$\frac{sL}{AE}ds,$$

so that for the total work of resistance in the bar we get

$$\omega = \int_0^S \frac{sL}{AE}ds = \frac{S^2L}{2AE} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

3. Normal Stress. — If the bar be a curved one with a developed length of L', then representing by N the normal stress acting at any section distant c — measured along the axis of the bar — from one end, we have, by the same reasoning as before, for the work of resistance in the elementary length dc,

$$\frac{N^2dc}{2AE},$$

and for the total internal work in the bar due to N,

$$\omega = \int_0^{L'} \frac{N^2dc}{2AE} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

4. Bending Moment. — Let Fig. 1 represent the portion of a beam, subjected to bending moment M; then in any elementary length dx, at a distance of y from the neutral axis NA, will be found taking place a deformation of

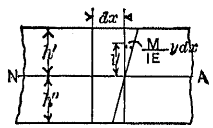

Fig. 1

$$\frac{M}{IE}ydx$$

in the elementary length of the fibre; and at the farthest fibre,

$$\frac{M}{IE}h'dx.$$

Representing by I the moment of inertia of the section, and by b the width of the beam at y, we get for the stress acting in the elementary section bdy, the expression

$$\frac{M}{I}bydy,$$

and consequently, for the work of resistance in the same,

$$\frac{1}{2}\frac{M}{IE}ydx \cdot \frac{M}{I}bydy = \frac{1}{2}\frac{M^2}{I^2E}bdxy^2dy;$$

so that for the total work in the elementary length dx we get

$$\frac{1}{2}\frac{M^2}{I^2E}dx\int_{-h''}^{h'} by^2dy;$$

and as

$$\int_{-h''}^{h'} by^2dy = I,$$

the total work of resistance due to the moment in length l of the beam will be

$$\omega = \int_0^l \frac{M^2dx}{2IE} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

5. Tangential Stress. — The deformation of a beam due to shear is generally so insignificant when compared with that due to the bending, that it may be totally neglected without sensible error in the calculation of internal work. In passing, however, the expression for the work will be given.

Let

T = tangential stress acting at any point distant x from one end of the piece.
G = modulus of elasticity for shear.
A = cross-section of the piece.

Since the action of the tangential stress in the elementary length dx (Fig. 2) is to produce the angular change γ, for which, were T uniformly distributed over the cross-section, we would have,

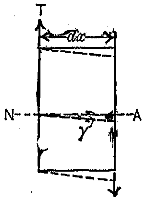

Fig. 2

$$\gamma = \frac{T}{GA},$$

and for the work performed in dx,

$$\tfrac{1}{2} T\gamma dx = \frac{T^2}{2\,GA} dx.$$

Since, however, the intensity of shear at different points of the cross-section differs with the form of the latter, we have for the internal work due to shear,

$$\omega = \int \frac{\alpha T^2 dx}{2\,GA} \quad \ldots\ldots\ldots \quad (4)$$

in which

$$\alpha = \frac{A}{T^2}\int_{-h''}^{h'} \tau^2 dA,$$

a quantity always greater than 1.

$$\tau = \frac{T}{bI}\int_{h}^{h'} y dA,$$

h representing the distance of fibres above the neutral axis where τ is to be found, and b, h', h'' and y the same as in Art. 4.

6. Theorems of Castigliano. — The fundamental principle of the method of work has been enunciated by Castigliano in following words: *

I. "*The displacement of the point of application of an external force acting on a body — caused by the elastic deformation of the latter — is equal to the first derivative of the work of resistance performed in the body, with respect to the force.*"

* "Theorie des Gleichgewichtes elastischer Systeme," von Castigliano.

II. "*The partial derivatives of the work of resistance with respect to statically-indeterminate forces which are so chosen that the forces themselves perform no work are equal to zero.*"

In order to make these enunciations clearly understood, an application of the theorems will be made to a simplest case of statically-indeterminate forces. In Fig. 3 let 1 and 2 represent two columns with a length of L, cross-sections of A_1, A_2 and moduli of elasticity of E_1, E_2 conjointly sustaining a load of W. The latter produces reactions

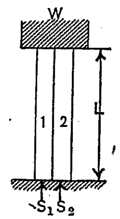

Fig. 3

$$S_1 \text{ and } S_2 = W - S_1,$$

which are at the same time stresses in the columns. Referring to Eq. (1) we get for the internal work in the columns the following expression:

$$\omega = \frac{S_1^2 L}{2\,A_1 E_1} + \frac{(W - S_1)^2 L}{2\,A_2 E_2}.$$

If we represent by δ the sinking of the load due to compression of the columns, then, according to the first theorem,

$$\frac{d\omega}{dW} = \delta = \frac{(W - S_1)\,L}{A_2 E_2},$$

and according to the second, since the bases of the columns are assumed to be immovable,

$$\frac{d\omega}{dS_1} = 0 = \frac{S_1 L}{A_1 E_1} + \frac{(S_1 - W) L}{A_2 E_2},$$

from which

$$S_1 = \frac{A_1 E_1}{A_1 E_1 + A_2 E_2} W.$$

7. The second theorem of Castigliano is a direct consequence of the first one, and concerns a special case in which the displacement of the external force is zero. In other words, according to this theorem, a statically-indeterminate force makes the work of resistance a minimum or a maximum. That it is a minimum can be seen by taking the second differential coefficient of ω with respect to the force having a certain amount of displacement. Since the latter will increase with every increment of the force, the second differential will be always positive. For this reason, this theorem is otherwise known as the *principle of least work*, which enunciates that the work of a system of forces acting on an elastic system of construction will be the least possible which is necessary to maintain equilibrium, or, in other words, the external forces so adjust themselves as to develop internal forces in the structure which will make the total work of resistance in the latter a minimum. The principle is a fundamental one in the economy of nature and is applicable to all cases of statically-indeterminate forces in which the forces under question undergo no displacements. For this purpose we have but to express ω in terms of external forces and to differentiate it successively with respect to the forces to be found. The differential coefficients thus obtained, set equal to zero, furnish as many equations

of conditions as there are unknown quantities. The rest of the operation for reduction is a simple algebraic work.

8. It is to be borne in mind that in all forms of structures to be hereafter treated, the joints of every piece, and the piece itself, are assumed to be free from all initial restraints.