ERRATA.

PAGE

13, 14, etc., for *Sx*, *tx*, *ty*, read S_x, t_x, t_y.

23, 16th line, for *both* read *both ends*.

32, 2d line, for *on* read or.

34, in the equation, for w^x read w_x.

37, for 490 lbs. = W read 490 lbs. = w.

Insert pages 57 (*beginning at the* 3d *line*), 58, 59, and 60 between 8th and 9th *lines of page* 62.

80, 7th line, for *movement* read *increment*.

85, for $g \times g'$ read *g and g'*.

91, for *thin holes* read *the holes*.

91, for 4 *plates* $54' \times \frac{3}{8}'' \times 25'-7\frac{1}{2}''$ read 4 *plates* $54'' \times \frac{3}{8} \times 10'-8''$, 4 *plates* $54'' \times \frac{3}{8} \times 15'-0\frac{1}{2}''$.

PLATE

11, *cancel Web plate* $4'-6'' \times \frac{3}{8} \times 25'-6\frac{1}{2}''$.

" for $9-6'' \times \frac{3}{8} \times 15'-0''$ read $4'-6'' \times \frac{3}{8} \times 15'-0''$.

PREFACE.

For railway as well as highway bridges, there is probably no other form of girders that are more extensively used and daily being constructed than plate-girders. The reason for this lies mainly in the simplicity of their construction and their stiffness as compared with open-girders. That the construction of a plate-girder is simple, is, however, no reason to suppose that the stresses produced in it by external forces are, also, simple. On the contrary, to determine actual stresses in every part of a plate-girder is one of the most complicated problems that can come in the way of bridge engineers.

The abundance of plate-girder drawings in almost every engineering office, no matter whether their designs are correct or not, has usually been a source to many draughtsmen to "design" a new girder, and indeed even in regular bridge works in the rush of work, as is usual in those places, is there hardly any opportunity for draughtsmen to examine

whether each part is rightly proportioned or not, unless they are thoroughly posted in the ground principles upon which rest every part of the design of a structure, and, which latter being not always the case, as a consequence plate-girders are often designed and constructed in a most careless manner, no particular attention being paid to the proportion and arrangement of parts, the spacing of rivets, etc., every one of which forms the most important factor in the strength of a girder; merely showing that the fact that a structure is standing is not the indication of the correctness of its design.

It is the aim of the writer, in the little volume now given to the public, to present, in as simple a manner as possible, a somewhat rational mode of designing girders of this class with special reference to American practice. And in the absence of any particular treatise on the subject within the reach of everyone as yet, it is hoped that it may be of some help to beginners in bridge designing.

I. H.

WEB-STRESS.

While in trusses without superfluous members, or those supposed to be so, to determine the stresses acting in their members is quite a simple matter, we are in much ignorance as to the nature and amount of stresses in certain parts of plate-girders, which are usually considered to be the simplest forms of girders, although they have been the subject of elaborate mathematical investigations, especially by Prof. Airy and Mon. Bresse.

In trusses like ordinary lattice girders, any one can see at once that the bending moment produces two parallel stresses resisted by two chords, and that the shear produces in the web-members stresses whose directions correspond to those of the resisting members themselves, in order to form proper reactions at both ends of the girder. The amounts of all

these stresses can be easily calculated by well-known methods. But not so with continuous web-plate-girders. Even after we consider, as is most frequently done, as taken off from the web the function of taking part in resisting flange stresses, and consider flanges alone as taking care of the bending moment, the ambiguity of stresses in webs still remains. Prof. Airy communicated the result of his investigation to the Royal Society in 1862, under the title, "On the Strains in the Interior of Beams." Starting from the consideration that a beam is composed of laminæ in a vertical plane, and that any number of forces acting at a point may always be replaced by two forces at right angles to each other, he derived equations of equilibrium, with which he determined the magnitudes and directions of these two forces in several parts of a beam under various modes of loading and supporting. His investigation verified the already existing notion that stresses, compression and tension take place in the web at the angle of 45°, a fact which

Stephenson more than 40 years ago found from his experiment on the model of the Britannia Tubular Bridge. The days for Britannia and Conway tubes are now gone, but in those days there was much controversy as to the relative merits of plate and open-girders for large spans. Starting from the result of his experiment, Stephenson argued that since plate-girder is nothing more than a lattice-girder with web bars put close together, and that the bar in tension may at the same time be subjected to compression in the direction of its width, one-half the amount of metal can be saved if the web were a continuous plate. On the other hand, it has been rightly argued that, on account of the impossibility of accurate determination of stresses in the web, and of other practical causes, larger amount of material is required to cover that ambiguity for plate-girders, than for lattice-girders, for which a more economical distribution of material is possible.

It is beyond the scope of our present work to enter into the discussion of

mathematical investigations of continuous web strains, nor will it be of much use when we consider in the light of experiments how different often is the actual state of affairs in a piece of material subjected to stresses in different directions from what the theory asserts. Under such circumstances, the less strained part tends to help the more strained part, and the stresses change from one direction to another in manners depending upon the intensities of stresses and molecular arrangements of the materials, and thus defy all correct determination of actual stresses. On this point the experiment of Mr. Baker* on the strength of beams is of great interest.

In order, however, to form some notion of the nature of stresses in a continuous web, and to derive formulas necessary for proportioning the same, we will view the matter in the simplest manner possible. In a beam supported at both ends, and subjected to the action of a vertical force, it is evident that at any of its ver-

* See Minutes of the Institution of Civil Engineers, 1880.

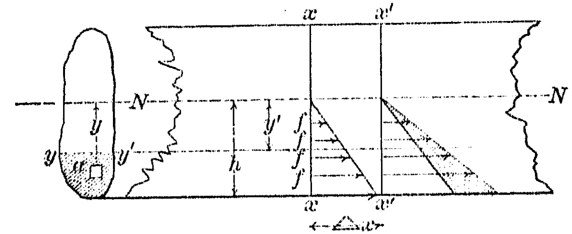
tical sections, by the virtue of the bending stress, that part of the beam above the neutral axis is subjected to compression, and that below to tension, whose intensities attain maximum values at the outermost fibers of the beam, and decrease to zero at the neutral axis. This intensity at any point is at once obtained from the well-known equation of flexure:

$$\frac{M}{I}y = f \qquad \text{(1)}$$

i. e., the bending moment M, divided by the moment of inertia I of the section of the beam, multiplied by the distance y of the point from the neutral axis, gives the intensity f of the stress at that point.

Now the bending moment M increases from the end toward the middle of the beam, and with it evidently the intensity f of the horizontal stress also; so that f varies not only in vertical direction on both sides of the neutral axis, but also in the direction of the length of the beam. Let x x and x' x' be two sections of the beam very close to each other, and N N

Fig. 1



the neutral axis. Then the variation of the value of f in both sections may be represented by triangles with apices in the neutral axis, while the variation in the longitudinal direction between these two sections, by the differences of the areas of two triangles, as shown shaded in the figure. This increase of horizontal stress from one section to another produces at every longitudinal layer force tending to slide it past the next above it, and is transmitted undiminished toward the neutral axis, where this shearing force, which has been increasing at every layer, attains its maximum intensity. This stress is called the *longitudinal shear*, and can be at once obtained from equation (1). Thus let f' be the corresponding value of f in section $x'\ x'$; and let M and M′ be the bending moment in the two sections $x\ x$ and $x'\ x'$ respectively, and a, an infinitely small cross area, distant y from the neutral axis.

The total horizontal stresses acting in that part of the section lying between the extreme fiber distant h from the neutral axis, and the layer $y'\ y'$ distant y' from the axis in $x\ x$ and $x'\ x'$ are respectively:

$$\sum_{y'}^{h} f\,a \quad , \qquad \sum_{y'}^{h} f'\,a$$

The longitudinal shear in the layer $y'\ y'$ between the two sections is therefore equal to

$$\sum_{y'}^{h} f'\,a - \sum_{y'}^{h} f\,a$$

Substituting in the expression the values of f and f' given by equation (1) we obtain

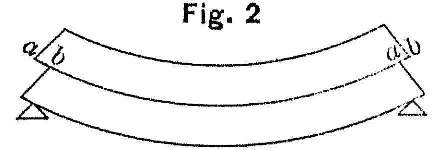
$$\sum_{y'}^{h} f'\,a - \sum_{y'}^{h} f\,a = \left(\frac{M' - M}{I}\right) \sum_{y'}^{h} y\,a$$

Since the area on which this horizontal shear is acting is equal to $b.\ \triangle\ x$ when b is the breadth of the cross section at the layer $y'\ y'$ and $\triangle\ x$ the distance between x and x', we obtain for the intensity of the shear

$$\left(\frac{M' - M}{I}\right) \sum_{y'}^{h} y\,a . \frac{1}{b\ \triangle\ x} \qquad (2)$$

The existence of such a stress becomes evident to any one, when, instead of a single beam, two beams be made to lie one upon another, which either by their own weight or specially applied forces will assume a shape as shown in Fig. (2).

Fig. 2



If they are now joined together by bolts, they will act as a single beam, and the shearing stress on these bolts is indicated by the force required to bring the two points *a* and *b* together.

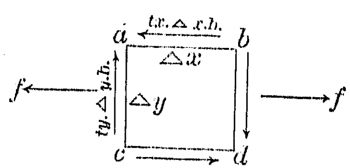
The *vertical shearing action* at any section of the beam is simply the reaction due to the load at one end minus that part of the load lying between that and the section, or

$$S_x = \left(R - \sum_{0}^{x} w\right) \qquad (3)$$

in which S_x is that vertical shear at a point distant x from one end at which the upward section is R.

Thus at every point in a beam there are two shearing actions taking place at the same time. Imagine *a b c d* (Fig. 3)

Fig. 3



to be an infinitely small portion of the

side of a beam at a point distant y' from the neutral axis. Suppose the sides of this area element be Δx and Δy and the breadth of the beam at the point to be b. There are then found two shearing stresses on this element, one vertical and the other horizontal. These two shears form two pairs of couples acting around the body, as shown by four arrows. Let t_x represent the intensity of the horizontal shear at this point and t_y that of the vertical. The amount of the horizontal shear is equal to

$$t_x . \Delta x . b.$$

that of the vertical shear is likewise

$$= t_y \Delta y b.$$

In order that the body be in equilibrium, the moments of these couples must be equal, *i. e.*,

$$t_x . \Delta x . b . \ \Delta y = t_y . \Delta y . b . \ \Delta x$$

Consequently

$$t_x = t_y \qquad (4)$$

showing that at every point in the beam the intensities of the vertical and horizontal shears are equal, and hereafter designate them with one common letter t. The value of t_x has already been deduced in equation (2) namely:

$$t = \frac{M'-M}{b.\Delta x.I} \sum_{y'}^{h} a\, y \qquad (5)$$

But, as will be explained in the next chapter,

$$M = Rx - \sum_{o}^{x} w(x'-d)$$

in which d represents the distance of loads w from the end at which the reaction is R. Likewise:

$$M' = Rx' - \sum_{o}^{x'} w(x'-d)$$

consequently

$$M'-M = R(x'-x) - \sum_{o}^{x} w(x'-x)$$

but

$$x'-x = \Delta x$$

hence

$$\frac{M'-M}{\Delta x} = R - \sum_{o}^{x} w$$

From eq. (3) we obtain

$$\frac{M'-M}{\Delta x} = S_x$$

Substituting this value of $\frac{M'-M}{\Delta x}$ in eq. (5) we get

$$t = \frac{S_x}{bI} \sum_{y'}^{h} a\, y \qquad (6)$$

This equation gives at once at any point of the beam the intensity of longitudinal and vertical shears.

At the neutral axis where $y' = o$ eq. (6) becomes

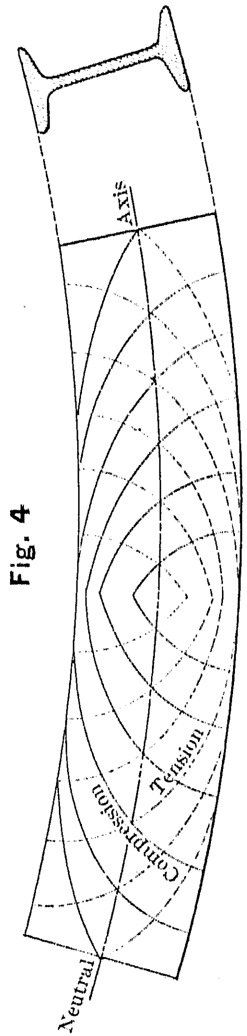
$$t = \frac{S_x}{bI} \sum_{o}^{h} a\, y \qquad (7)$$

There still remains to be considered stresses acting on the sides of the element (Fig. 3); of which one is the horizontal force f, whose value was already given in equation (1), tending either to compress together or pull asunder the two faces $a\ c$ and $b\ d$, according as it is on the upper or the lower side of the neutral axis; and the other is the direct compression on $a\ b$ due to the weight lying on it; but the latter is usually so small that it can entirely be taken out of consideration

At the neutral axis where $f{=}o$, tx and ty are then the only stresses, and we know from mechanics that the resultant action of two equal shears at right angles to each other, exactly as tx and ty are, is equivalent to that of two equal and opposite stresses at right angles to each other, called the principal stresses, and making an angle of 45° with the shearing stresses. But at a distance each side from the neutral axis, now comes in the third stress f, which evidently gives the new direction to the line of resultant stress, turning the axis of principal stresses toward itself more and more as its intensity increases.

Since for all directions and intensities of stresses there can always be found two planes at right angles to each other, and in each of which the resultant action of these stresses is normal to its surface, forming the planes of action of the prin-

Fig. 4



cipal stresses, and as soon as we know the intensities and directions of these stresses, we can determine, either graphically or analytically, the direction and intensities of the principal stresses; the values of t and f' at any point will give us the direction and intensities of maximum tension and compression at that point.

Fig. 4 represents the appearance which the lines of principal stresses thus obtained presents. The lines of maximum tension shown dotted, cut the lines of compression every time at right angle. Both lines cross the neutral axis at the inclination of 45° to the same, and run almost parallel to it in the middle of the beam in the neighborhood of extreme fibers. If we represent with p the intensities of stress along these curves, we have

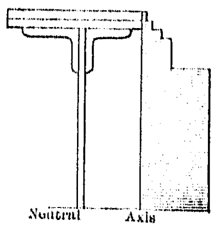
$$p = \tfrac{1}{2} f' \pm \sqrt{4t^2 + f'^2} \qquad (8)$$

giving us at every point two values of p, viz.: compression and tension, and their directions being at right angles to each other.

Now comes in the question how the *web should be proportioned* to resist such stresses. There are almost as many methods advanced as there have been authorities on the subject. Perhaps the rational, and, at the same time, the most practical way of proportioning the web is to make its section sufficient merely to resist the entire shearing stress, without, however, passing the limit below which the *oxidation* by the moisture of the air, and the excessive increase of *pressure* on the rivet become unfavorable elements; and wherever in the web thus proportioned be found section that is not strong enough to resist as a column in the line of maximum compression, the intensity of the same, to rivet stiffeners composed of angles or plates. In America the thickness of the web plate is hardly ever made less than ⅜ inch, and on the continent of Europe rarely less than one centimeter.

Although, as we have already seen, the shearing stress at any section is not uniformly distributed, yet when we solve eq. (7) for different points in a vertical section of a flanged beam with a very thin web, we find the values of t vary from the extreme fiber toward the neutral axis, as shown by the shaded area (Fig. 5).

Fig. 5



The increase throughout the web is almost inconsiderable, and we can on that account consider the *shearing stress* to be *entirely borne by the web*, and to be uniformly distributed over its section, *i.e.*,

$$\frac{Sx}{A} = t \text{ ———————— } (9)$$

in which A is the area of the cross section of the web plate. Or, since it is by the virtue of the bending moment that the unequal distribution of stress takes place in the section, if we now, under the supposition that flanges alone resist the entire bending moment, and the web only the shearing action, solve eq. (7), confining the summation to flanges only, we obtain

$$\frac{Sx}{b\, h_x} = t$$

in which h_x is the distance between the centers of gravity of top and bottom flanges; and b the thickness of the web, or what is practically the same as eq. (9.)

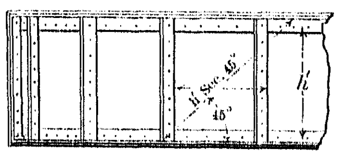
Consequently at any point of the web, by dividing the shearing force at that point by the distance between the centers of gravity of flanges we obtain the amount of shearing stress per unit of length in its direction, which we will designate by s, and we have

$$s = t\, b = \frac{Sx}{h_x} \text{ ———————— } (10)$$

The significance of these formulas will appear in the following chapters.

In order, therefore, to determine the thickness of a web plate, we first obtain the maximum shearing force, which, of course, will be found always at the end of the beam, then divide this by the shearing strength of iron per unit area, and the quotient is approximately the necessary section. But, as was already explained, the action of the shearing stresses at the neutral axis is equivalent to compression and tension at right angles to each other and of equal intensity, making an angle of 45° with the axis, the web is still in danger of failing by flexure under this compressive stress. Consequently the web, with its thickness as already proportioned for shearing, must now be examined for its strength as a column inclined at 45°, and of the length, therefore, of h' sec. 45° fixed at both, h' being the vertical

Fig. 6



distance between the upper and lower rows of rivets in the web. For this pur-

pose, Gordon's column formula may be used, which is

$$c = \frac{8000}{1 + \frac{l^2}{3000\, b^2}}$$

in which c is the allowable compressive stress per square inch; b the thickness of the web and $l = h'$ sec. 45°. It may be here remarked that the value of 8000 for the numerator of the second term of this equation may be in some cases found to be taken too low, but to avoid the tedious operation of conforming to its ever changing values (see the next chapter) we have taken it as constant, and we are so much on the safe side. In order further to save the trouble of working out the formula for every case, is given the following table, from which, knowing the values of h' and b, we at once obtain the value of c.

Now if we know the amount of shear at any point where we wish to determine the *strength of the web*, we divide that shear by the cross sectional area of the

TABLE.

$\frac{h'}{b}$	C	$\frac{h'}{b}$	C
40	3870	70	1880
42	3680	75	1690
44	3500	80	1520
46	3320	85	1380
48	3150	90	1250
50	3000	95	1140
52	2860	100	1040
54	2700	110	880
56	2590	120	760
58	2470	130	650
60	2350	140	570
65	2100		

latter, and then compare this shearing stress per square inch thus obtained with the value of c in the table, against the corresponding value of $\frac{h'}{b}$. If the shear per square inch is *less* than c, it shows that the web is stiff enough by itself, but if, on the contrary, c is less than the shear, then the web must be stiffened.

So far as this resistancy against compressive stress alone is concerned, stiffen-

ers inclined 45° may be more effective in strengthening the web than the vertical ones, but on account of the difficulty of construction of the former, and also for the action which the latter comes into when the load lies on the top of the girder direct from the flange, stiffeners are always made vertical.

The *spacing of stiffeners* is more a matter of experimental determination and judgment than of mere calculation—the fact which our foregoing discussion of the web stresses might suggest to any one. Mr. Jos. M. Wilson* has given a safe rule, which says that "stiffeners in girders over 3 feet in depth shall be placed at distances apart (center to center) generally not exceeding the depth of the full web plate with the maximum limit of 5 feet. In girders under 3 feet depth they may be 3 feet apart, and in some special cases, where there is little or no shearing stress, at greater distances." This refers, of course, to those parts of the girder where there are no concentrat-

*Compare Penna. R. R. Co. Bridge Specification.

Fig. 1

SCALE: 20 ft. — 1 inch.

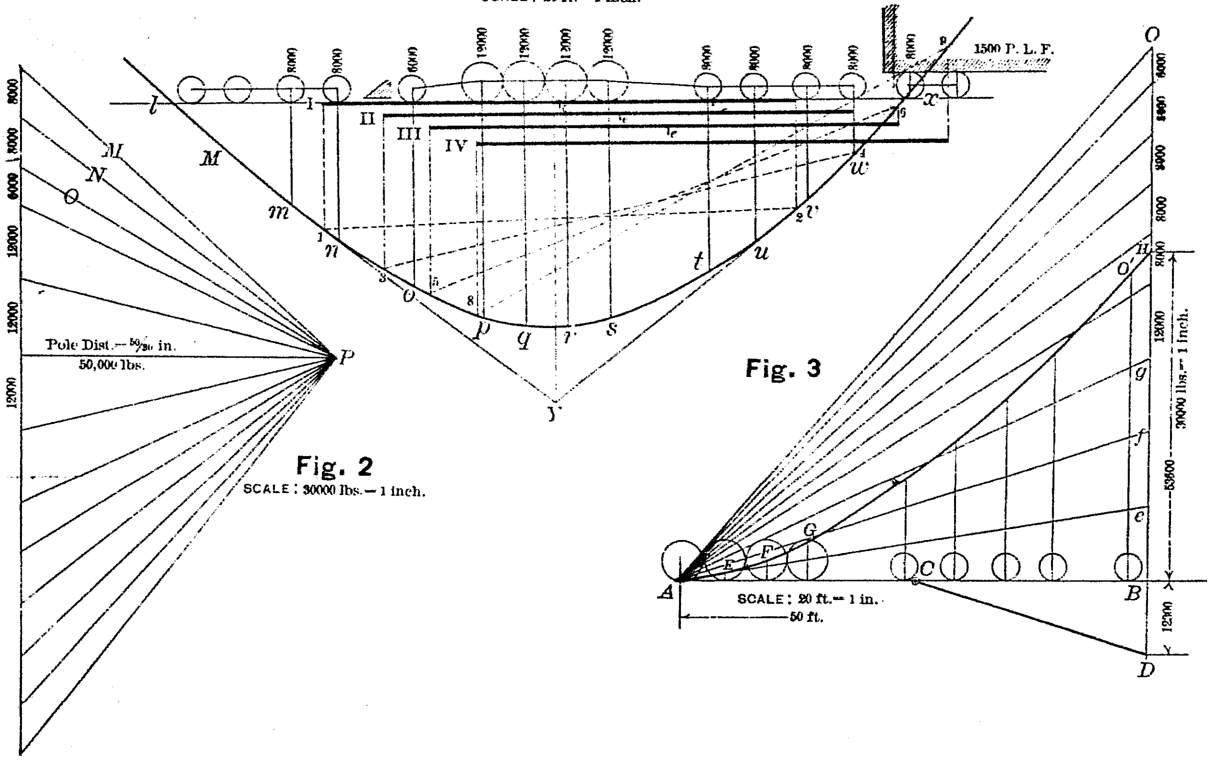
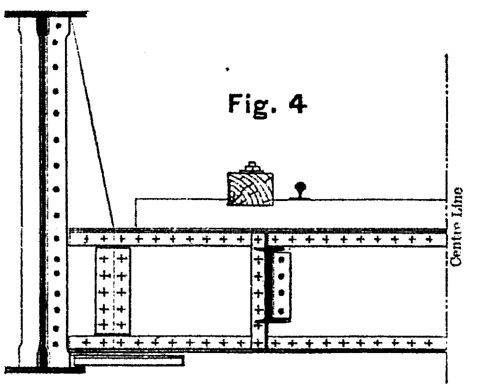


Fig. 2

SCALE: 30000 lbs. — 1 inch.

Fig. 4

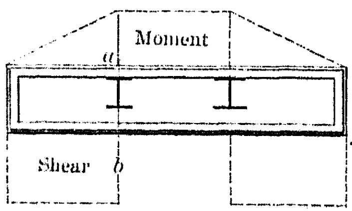


ed loadings, as in floor beams supporting stringers, or as in through plate-girders with floor beams, in which cases stiffeners are invariably to be riveted at these points of connections, as shown in Fig. 4, pl. I.

There are several other rules for spacing stiffeners given by different engineers.*

There is still another point to be con-

Fig. 7



sidered in the strength of webs, those girders in which the *maximum bending*

*As one of those worthy of notice, Mr. O. Chaunte, in the N. Y., Lake Erie and Western R. R. Specification, says that when the least thickness of web is less than $\frac{1}{80}$ of the depth of the girder the web shall be stiffened at intervals not over twice the depth of the girder.

moment and *great shearing stress* exist at the same point. Thus in a floor beam at the point of connection of the stringer with the web, there co-exist at the point, at infinitely small distance to the left of a b, maximum bending moment and shear, which also is almost maximum. Here, specially in the vicinity of the upper row of rivets, the valve of p given by equation (8) should be found out. If p is less than the allowable stress, the web is strong enough. If, however, p exceeds it, the moment of inertia of the girders must be increased.

Flange Stresses and Flanges.

The *dead load* on a plate-girder bridge consists of the weight of girders themselves and the floor system which it carries, and may, under most circumstances, be considered as a uniformly distributed load. For standard gauge single track railway bridge, the weight of floor system (consisting of cross-ties, guard rails, rails, splices, bolts and spikes) rarely exceed 450 *lbs. per lineal foot of the track.*

The *weight of girders* alone can be best determined by the use of a simple empirical formula given by Mr. G. H. Pegram in a paper read before the Am. Soc. of Civil Eng., in Feb., 1886.

$$W=\left(75+\frac{s}{a}\right)s\sqrt{s}\ *$$

in which s = span in feet (effective length), and W=weight in pounds of entire iron work of a single track deck plate-girder when s is between 20 and 80 feet, and a is the constant depending on the kind of moving load.

There are several other formulas, among which those given by Profs. Burr and DuBois give very good results.

As to the *rolling load*, the diagram given on page 30, showing a load system consist-

* In this formula, it is assumed that the allowable stress in tension is 10,000 lbs. per sq inch, in shearing 7,500 lbs, and in compression 8,000 lbs., reduced by ordinary column formula when necessary. Further that the live load stresses are increased from 80 to 8% for spans of from 25 to 80 ft., to allow for the effects of impact.

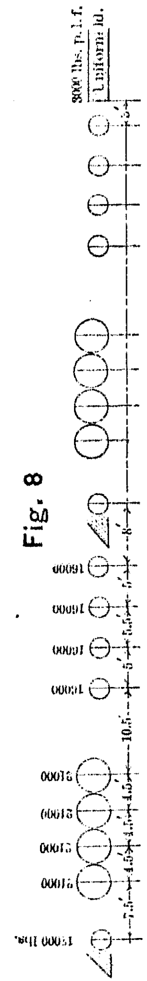


Fig. 8

ing of two 80 ton consolidation engines, coupled and followed by a uniform load of 3,000 lbs. per lineal foot, represents about the heaviest rolling load system in the United States, although these figures have in few cases been exceeded.

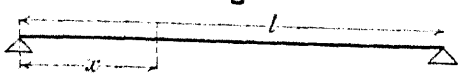
Some passenger engines may for small spans give more unfavorable loading than these, and every case must be investigated for itself; and a bridge engineer must have before him all possible sorts of rolling load of the road for which the bridge is to be designed.

The *bending moment* due to *uniformly distributed loading* is at once obtained by the well-known formula of simple beams:

$$M = \frac{1}{2} w x (l - x) \text{ ———— } (12)$$

in which M is the bending moment at any point distant x from one end of the beam; w, the uniform load per unit of length, and l the length of span.

Fig. 9



The word span will throughout be used to denote the effective length on the distance between centers of bearings.

For the bending moment at the center ($x = \frac{1}{2} l$) equation (12) becomes:

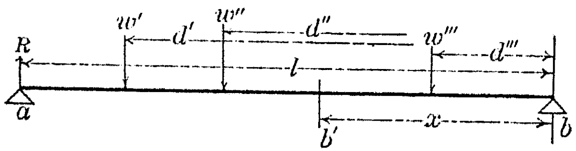
$$M = \frac{1}{8} w l^2 \text{ ———— } (13)$$

The graphical representation of the bending moment due to a uniform load is a parabola with apex at the middle of the span.

The *maximum bending moment due to the rolling load* can be determined either analytically or graphically, the latter method being, however, far more preferable, on account of the rapidity with which the work can be performed. With the *analytical* method one has to solve the equation of bending moment at different sections of the beam under the position of the moving load giving the maximum moment. Thus, with loading like Fig. 10, we obtain for the reaction "R" at one end:

$$R = \frac{w'''d'' + w''d'' + w'd'}{l}$$

Fig. 10



and consequently for the bending moment at any section, distant x from one end, we get

$$M' = R(l - x) - \{w''(d'' - x) + w'(d' - x)\}$$

Repetitions of such a work require, even when accurate results are not needed, considerable work.

On the other hand, with the *graphical method* one has merely to construct an equilibrium polygon, and by placing the given span in different positions under the rolling load (which amounts to the same thing as letting the rolling load to pass from one end of the girder to the other), the maximum moments at different sections are more readily obtained.

We know from statics that in a simple girder with loads distributed over it

either uniformly or arbitrarily, the *maximum bending moment at any section* is found when the loads on each side of this point are to each other as the parts into which the point divides the span. But for a single load traveling over the girder, it is evident that the maximum moment at any point is found when the load lies at that point. Consequently the maximum bending moment at any point of the girder will be found when one load lies at the point, and the rest so located as to fulfill the condition already mentioned; or, expressed by formula:

Moment will be maximum at any point x (Fig. 11) when

$$\frac{\sum_x^a w + \text{a part of } w^x}{a\,x} = \frac{\sum_b^x w + \text{the remaining part of } w^x}{x\,b}$$

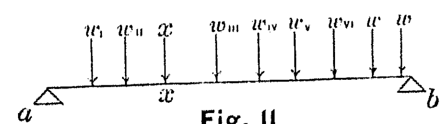


Fig. 11

To find the load w^x, which, when resting on a section, gives the maximum moment for that point, requires some work, as some trials must be made for every load and point; but it is usually sufficient for all practical purposes, for short spans, to place the second driver of the second locomotive over the section at which we want to determine the moment, and obtain the same by the method to be mentioned in the following pages. Of this one can be easily convinced by a few trials.

The following points will *suggest what position and what part* of the load system are to be looked for to find the maximum moments.

1. For *uniformly* distributed load over the entire span, the maximum bending moment is in the middle of the span, and therefore at the center of gravity of the total load.

2. When the load is *not uniformly* distributed, the center of gravity, and with it the point of maximum moment, will approach the heaviest load.

3. For a load system of the second class, the *absolute maximum* of moments, however, does not occur at the middle point of the span, but under the load which is nearest to the center of gravity of the system, when they (that load and the center of gravity of the system) are equidistant from the middle point of the span.*

Now we proceed to an actual example:

As *an example*, let us take a single track standard gauge deck girder of 50 ft. span, *i. e.*, measured between the centers of bed plates.

The weight of the girder, according to the formula already given, with $a=10$† will then be equal to

$$W=\left(75\times\frac{50}{10}\right)50\sqrt{50}=28,000 \text{ lbs.}$$

* For proof see DuBois' Element of Graph. Statics, chap. vii.

† Mr. Pegram has given for the rolling load we have taken, 7 for the value of a. But on account of differences in assumptions, as will be seen further on, from his, we make $a=10$.

The *weight of the floor system* assumed to be 420 lbs. per lineal ft.=

$420 \times 50 = 21{,}000$

Total dead load=49,000 lbs.

Since there are two girders, the load per lineal foot on one will be equal to $\frac{49{,}000}{2 \times 50} = 490$ *lbs.*=W.

We will divide the entire span into say 10 equal parts to determine at these points of division the maximum stresses(Fig. 12).

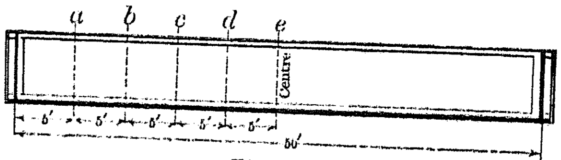


Fig. 12

Eq. (12) at once gives the moments at these points as follows:

At *a* $M = \frac{1}{2} \times 490 \times 5(50-5) = 55{,}100$ ft. lbs

b $M = \frac{1}{2} \times 490 \times 10(50-10) = 98{,}800$"

c $M = \frac{1}{2} \times 490 \times 15(50-15) = 128{,}600$"

d $M = \frac{1}{2} \times 490 \times 20(50-20) = 147{,}000$"

e $M = \frac{1}{2} \times 490 \times 25(50-25) = 153{,}100$"

These moments, when graphically represented, form the ordinates to the parabola with vertex at the middle of the span (Fig. 13.)

The *bending moment* due to the *moving load* will be obtained graphically,* and the whole process is nothing more than what is to be seen in Figs. 1 and 2 of Pl. I, and requires but little explanation.

In a horizontal line (Fig. 1, Pl. I,) lay down to any convenient scale; here taken at 20 feet to an inch (in practice a larger scale, say 5 feet to an inch, should be chosen), the system of load for length enough to cover the ranges of various positions of 50 feet girder, for which the bending moments are to be determined. As stresses need to be determined for one girder only, but half the weights are to be taken. Lay down in a vertical line to a proportionate scale, here 30,000 lbs. to an inch (in practice take say 10,000 lbs. to an inch); the weights of the

* Proofs for the steps of the method here used are found in any books on graphical statics. See DuBois' Graph. Statics.

load system beginning at one end in succession, as shown in Fig. 2. This is called the force diagram. In it choose any point P (called *pole*) at some convenient distance from the vertical line of weights, say $1\frac{2}{3}$ inch. This distance represents, according to the scale, the horizontal force of $1\frac{2}{3}\times30,000=50,000$ lbs.

From the pole P draw lines **PM**, **PN**, PO, etc., to the end of each weight (*Fig.* 2). Then starting from any point *l* in Fig. 1, draw a line *l* **M** parallel to **PM**, and produce it until it cuts in point *m*, the perpendicular let fall from the load corresponding to the first weight in the force diagram. From this point *m* draw a line parallel to PN (of the force diagram), cutting the next perpendicular in *n*. In this way complete the polygon *l m n o p* . . . *t u v w x*, observing carefully that each side of the polygon always occupies the corresponding place, as the line to which it is parallel does in the force diagram. Thus the side *n o* running between perpendiculars, let fall from the

loads 8,000 and 6,000, is parallel to the line PO (of force diagram) which is drawn to the point of junction of those weights.

The position of the locomotive, as shown in the figure, follows the rule already deduced.

The center of gravity, or more properly the *resultant* of the system of any number of loads, is at once found by prolonging the outermost sides of the polygon formed by so many loads as the number for which the resultant is required. Thus the resultant of the loads lying on the girder in the position I, viz.: 8,000, 6,000, 4 of 12,000 and 2 of 8,000, is found by producing the outermost sides *m n* and *u v* until they meet in Y, through which the resultant must pass.

According to what has already been said, the maximum moment is found when the center *e* of the girder lies midway between the perpendiculars through Y, and that through the load nearest to it, viz. *r*. I shows the girder in such a position. Let fall from the ends of the girder perpendiculars cutting the sides of the polygon in points 1 and 2; join 1, 2. The maximum *vertical* distance between the line 1, 2 (called the closing line), and the sides of the equilibrium polygon, multiplied by the horizontal force represented by the pole distance, gives the maximum bending moment. Thus we find the maximum vertical distance to be at r, and which measures 11.65 feet. But the horizontal force has already been taken at 50,000 lbs., hence the maximum moment is equal to

11.65×50,000=582,500 ft. lbs.

The maximum moments at the center and at e differ very little from this absolute maximum, and consequently they will be assumed to be alike.

Shift the span now to the right till the second driver comes over the point c, as shown in position II. Let fall the perpendiculars; join 3 and 4, and measure the maximum vertical distance between 3, 4, and the polygon. This we find to be 10.1 ft. Hence the maximum moment at c is equal to

10.1×50,000=505,000 ft. lbs.

In the same way, by bringing the second driver over points a and b, as shown by positions III and IV, we obtain the maximum moments at these points. These we find to be:

At b 7.8 ×50,000=390,000 ft. lbs.
a 4.26×50,000=213,000 " "

The total maximum moments at every point of the girder will evidently, when platted give a shape as shown in Fig. 13, in which the lower curve is a parabola,

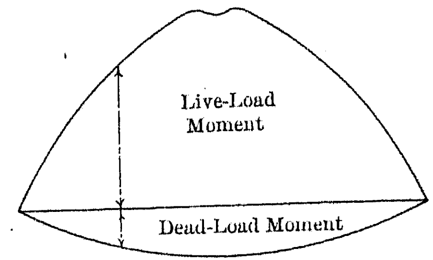


Fig. 13

and due to the dead load. The sum of the ordinates at any point on both sides of the horizontal line gives the total maximum moment at that point. Thus we have for the total moments at several points:

	Dead.		Live.		Total.	
At *a*	55,100	+	213,000	=	268,100	ft. lbs.
b	98,000	+	390.000	=	488,000	" "
c	128,600	+	505,000	=	633,600	" "
d	147,000	+	582,500	=	729,500	" "
e	153,100	+	582,500	=	735,600	" "

In determining the *sectional area* of *flanges* we will, as already said, take the web plate out of consideration, and suppose that the flanges alone resist the bending moment, mainly for the reason that the part which the web contributes to the flange section is inconsiderable, as seen in the following calculation:

Moment of inertia of a web $=\frac{bh^3}{12}$

$$\frac{I}{y}=A_1\,h_0 \quad \therefore \quad \frac{bh^3}{12}\quad\frac{1}{\frac{h}{2}}=A_1\,h_0$$

Assuming that $h=h_0$, we obtain $A_1=\frac{bh}{6}$

i. e., the sectional area which the web contributes to the flange is but $\frac{1}{6}$ of its own, which is certainly a very small amount, seeing the thickness of the web to be usually not more than $\frac{3}{8}$ inch.

The *depth of girder* is more or less a matter of judgment for each case. Such depths are usually given, as one can fit a plate from the rolling mill for web without the necessity of shearing. For a given case one can find the most economical depth by few trials, by bearing in mind that the increase of depth increases the weight of web and stiffeners, while it decreases the flange areas, and *vice versa*. The *depths* usually *vary from* $\frac{1}{6}$ to $\frac{1}{12}$ of the *span*, according to the lengths of the latter; the shorter the span the more nearly the ratio approaching to the first figures, and the greater the span is, the more to $\frac{1}{12}$.

Our attention will entirely be confined to the construction of parallel flanged girders, as the curved flanges are, on account of the much increased cost of construction, rarely used.

It may be here remarked that it is usually only between spans of 15 and 80 feet that plate girders can be advantageously used. For spans less than 15 feet solid rolled beams, and for greater spans than 80 feet, open girders are usually used.

We will fix the depth of our girder at 4 feet 6 inches. When we have one or more plates on the flanges we can, without much error, assume the distance between the *centers of gravity of the flanges* to be equal to the distance back to back of the flange angles, and take this as the effective depth of the girder.

As the flange section increases, the effective depth evidently varies more or less, but we can assume them to be constant throughout; at least we are on the safe side by doing so.

To find *flange stresses* we have now but to divide the maximum bending moment already obtained by the effective depth. Thus we obtain:

		adding 15 %
At $a\ \frac{268,100}{4.5} =$	59,600 lbs.	68,500 lbs.
$b\ \frac{488,000}{4.5} =$	108,500 "	124,800 "
$c\ \frac{633,600}{4.5} =$	140,800 "	161,900 "
$d\ \frac{729,500}{4.5} =$	162,100 "	186,400 "
$e\ \frac{735,600}{4.5} =$	163,500 "	188,000 "

This addition of 15 % is to provide to a certain extent for the impact which the moving load traveling over the girder brings about.

The *actual amount of stress* produced by the moving load like a locomotive is not possible of accurate determination. The imperfect condition of the track causes the moving load to produce shocks, and then again the centrifugal force of the unbalanced weight of driving wheels, which act like a hammer with repeated blows,* as well as the vertical

* Shocks on Railway Bridges. Mr. J. W. Cloud, Prof. Am. Inst. of Mining Engineers, 1881.

component of the thrust of the connecting rod of the locomotive, produces stresses not always possible of accurate determination.

Allowable stress:—There is a diversity of opinions as to the amount of allowable stress in a plate-girder, but all agree in this, that, on account of the great difference between the dead and live load, or more properly between minimum and maximum stresses, and the suddenness with which they change from one to the other proportionally as the spans decrease, the allowable stress should be much lower than in structures of longer spans than is usually spanned with plate-girders.

Laundhart's formula gives the allowable stresses conformably to the long series of experiment (of Wöhler), and is the one that is being more and more used by engineers, but not without some room for doubt whether it is applicable to all kinds of stresses under all circumstances. The following is that

well-known formula for stresses in the same direction: †

$$p = u\left(1 + \frac{t-u}{u}\,\frac{\textit{minimum stress.}}{\textit{maximum stress.}}\right)$$

in which t is the stress which by a single application produces rupture, which, according to Wöhler, can be accomplished by somewhat less stress than t if repeated sufficient number of times; u is the stress per square inch after indefinite number of repetitions of which the bar still returns to its original unstrained condition, and consequently its value becomes greater the less the number of repetitions to be endured is; p is the stress per square inch, which, after greatest number of repetitions possible, is still just capable of sustaining the load. Wöhler found for a certain axle-iron that:

$t=56,900$ lbs. $u=31,200$ by flexure

and $t=46,800$ " $u=31,200$ by tension.

and for a certain kind of steel, that:

$t=104,400$ lbs. $u=51,300$ by tension.

† Compare Weyrauch's Structure of Iron and Steel. Transl. by Prof. DuBois, chap. ii.

From these figures we can give the Laundhardt formula, the following shape to be used both for iron and steel.

$$a=c\left(1+\frac{min.}{max.}\right)\text{———}(14)$$

in which a is the allowable stress per square inch, and c is equal to u divided by the factor of safety, and is:

For wrought iron plate and shapes in tension,	8,000 lbs.
For wrought iron plate and shapes in compression,	7,000 "
For steel in tension,	10,000 "
" " " compression,	9,000 "

Further to obtain the actual allowable stress for the *compressed flanges*, the amount given by the above formula, whenever the unsupported distance is more than 12 times its width, must be reduced by Rankine's formula, deduced from Fairbarn experiment, viz:

$$K=\frac{a}{1+\frac{l^2}{5,000\,b}}$$

in which k is the actual allowable stress

per square inch, and b the width of the flange, and l the unsupported length; a the allowable stress already found.

In all the allowable stresses above given, it is supposed that *iron* has the elastic limit of not less than 26,000 lbs., and an ultimate strength of not less than 50,000 lbs. per square inch when tested in ordinary test piece, and that steel, to have not less than 36,000 lbs. and 60,000 lbs. for these values.

Returning now to our 50 ft. girder we find the ratio of the dead to total stress as already found or $\left(\frac{\text{min.}}{\text{max.}}\right)$ is about $=\frac{1}{5}$ throughout. Consequently

$a=8,000\left(1+\frac{1}{5}\right)=9,600$ lbs. for tension.

$a=7,000\left(1+\frac{1}{5}\right)=8,400$ lbs. for compression.

Hence we obtain the following sections:

Top flange:

At a $\frac{68,500}{8,400}=$ 8.2 square inches.

b $\frac{124,800}{8,400}=14.8$ " "

$c\ \dfrac{161,900}{8,400}=19.3$ square inches.

$d\ \dfrac{186,400}{8,400}=22.2$ " "

$e\ \dfrac{188,000}{8,400}=22.4$ " "

Bottom flange:

At $a\ \dfrac{68,500}{9,600}=\ 7.1$ square inches.

$b\ \dfrac{124,800}{9,600}=13.0$ " "

$c\ \dfrac{161,900}{9,600}=16.9$ " "

$d\ \dfrac{186,400}{9,600}=19.4$ " "

$e\ \dfrac{188,000}{9,600}=19.6$ " "

It now remains to find out the sizes of angles and plates to give these required sections. It is important, in making up flange areas, to keep in mind that there must always exist a *certain relation between the sectional areas of flange angles and plates.* No normal stress can exist in flange plates without first pass-

ing through angles which alone form connection with the web. Plate-girders are unlike rolled beams, in which every part is connected molecule to molecule, but have stresses transmitted only through rivets which are distributed only at certain distances apart. Consequently the flange angles are at every point more or less subjected to stress in addition to their own. This additional stress will evidently increase with the amount of plates on the back. It is a good practice, therefore, to make the girder so deep that the flanges do not require a large number of plates to be packed one upon another, and then to choose angles as heavy as possible, consistent with the total flange area required.

Everybody who does any design in iron should be provided with "Book of Shapes," issued by almost every iron company in the country. Carnegie's book is especially to be recommended for the purpose. In order to give the girder the greatest amount of moment of resistance, it is usual to use angles with unequal legs, with longer legs in horizontal position. It is important that the *top flange* of a plate-girder should always have a plate running from one end to the other, even when angles may be found which alone are sufficient to make up the required section. The addition of such a plate evidently requires more work of punching and riveting, but it gives great lateral stiffness to the flange, and also helps to distribute the stress more uniformly than with angles alone, while at the same time furnishing a cover to the small channel left between the angles, as the web is not usually flush with their backs and where water can stand.

In marking up the *bottom flange section*, rivet holes must be deducted to obtain the net section, and in so doing the diameter of the rivet hole should be taken at least as $\frac{1}{8}$ inch larger. This latter provides to a certain extent for the damage done to the strength of the metal in the process of either punching

or drilling. On the other hand, for the top flange, the gross sectional area may be taken as making up the same when the riveting is well done, *i. e.*, every rivet completely filling up its own hole.

Thus we obtain the following sections:
For top:

$$\left.\begin{array}{ll} 2\ L^s & 3\frac{1}{2}\times5\times\frac{1}{2}=8.0 \\ 1\ \text{pl.}\ 12 & \times\frac{1}{2}\quad=6. \end{array}\right\}\text{throughout}$$

$$\left.\begin{array}{ll} 1\ \text{pl.}\ 12 & \times\frac{3}{8}\quad=4.5 \\ 1\ \text{pl.}\ 12 & \times\frac{3}{8}\quad=4.5 \end{array}\right\}\text{partly.}$$

Section in the middle $=23.0$ sq. inches.

For bottom flange, supposing we use $\frac{7}{8}$ inch diameter rivets:

Rivet holes. Throughout

$2\ L^s\quad 3\frac{1}{2}\times5\times\frac{1}{2}=8.0-2\times(\frac{7}{8}+\frac{1}{8})\times\frac{1}{2}=7.0$

$1\ \text{pl.}\ 12\times\frac{1}{2}=6.\ -2\times(\frac{7}{8}+\frac{1}{8})\times\frac{1}{2}=5.0$

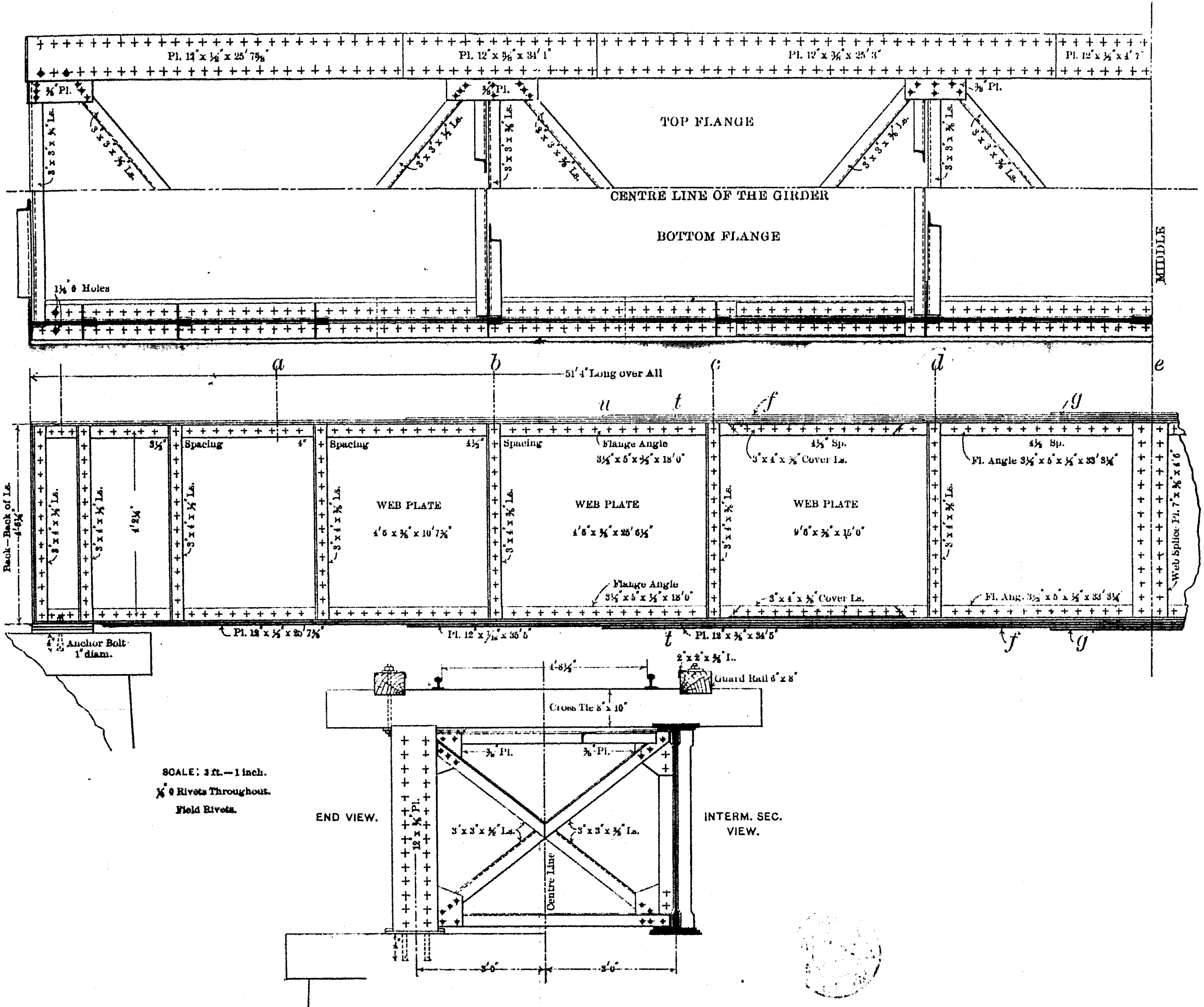
Partly.

$1\ \text{pl.}\ 12\times\frac{7}{16}=5.25-2\times(\frac{7}{8}+\frac{1}{8})\times\frac{7}{16}=4.35$

$1\ \text{pl.}\ 12\times\frac{3}{8}=4.5\ -2\times(\frac{7}{8}+\frac{1}{8})\times\frac{3}{8}=3.75$

Net sectional area in the middle $=20.1$ square inches.

It is a fact worthy of notice that the majority of plate-girders of iron as well as of steel, though of equal top and


TOP FLANGE
CENTRE LINE OF THE GIRDER
BOTTOM FLANGE
MIDDLE
51' 4" Long over All
WEB PLATE
WEB PLATE
WEB PLATE
Flange Angle
Anchor Bolt
1" diam.
Guard Rail 6" x 8"
Centre Line
END VIEW.
INTERM. SEC.
VIEW.
SCALE: 3 ft.—1 inch.
Field Rivets.

bottom flanges, fail by tension,* a fact which has been observed in the results of experiments made some years ago by the Dutch Government to determine the comparative strength of iron and steel riveted girders. This is evidently due to the considerably diminished effective section of the bottom flange for tension by rivet holes, and the metal still further weakened by punching or drilling. Rolled I beams usually fail by compression when their webs are strong enough.

In Plate II is to be seen the disposition of metals in flanges as well as in the whole girder. In order that the rough edges of the web plate be not found sticking out beyond the backs of flange angles, the depth of the girder back to back of the latter is made $\frac{1}{4}$ inch ($\frac{1}{8}$ inch may, however, do just as well,) more than the width of the web plate, so that the additional flange plate may have a flat surface to be riveted on.

In the figure, all dimensions are given

*See Charles Bender's Principles of Economy in Designs of Metallic Bridges, p. 18.

exact, but each piece should be *ordered* a little longer than is actually necessary, to provide for cutting and milling works in the shop. Thus, for pieces whose lengths are less than 10 feet, allowance of $\frac{1}{2}$ inch, and for more than 10 feet, $\frac{3}{4}$ to 1 inch, according to the length, should be given.

Those *flange plates which but partly cover the flange* should be run as many inches beyond the point where, according to the calculation we have made, the sectional area of that plate must form the required section, as to catch sufficient number of rivets on the flange in order to transmit the amount of stress which the plate is required to sustain. Thus for the plate $12'' \times \frac{3}{8}''$ on the top flange, the amount of stress which it is expected to receive is equal to

$$12 \times \tfrac{3}{8} \times 8{,}400 = 37{,}800 \text{ lbs.}$$

Now if we have two rows of vertical rivets on the flange, and supposing that each rivet has the shearing strength of 4,400 lbs., the plate must be extended to take in at least $\frac{37{,}800}{4{,}400} = 8$ rivets beyond the point at which it is calculated to form the flange section.

For the latter method, that part of Fig. 3 lying above the line A B is all that is necessary for our case.

If we look at equation (3) we see the shearing action of a moving load system is greatest at any section x when the system is in such a position that R shall be greatest possible, and $\sum_{o}^{x} w$ the least possible. Consequently for a single load it gives the maximum shear at any point when it lies over that point, and for a uniformly distributed load the shear is maximum at any point when the greater of the two parts into which the point divides the length of the girder is covered with the load.

To simplify the operation a little, we will take off the head load of the first locomotive and make the whole train advance from one end to the other. This gives evidently a somewhat greater amount of shear than when we take all

the wheels into consideration, but the amount is very small, being at the middle of the girder, where the excess is greatest, equal to

$$6{,}000\times\frac{25-7.5}{50}=2{,}100 \text{ lbs.}$$

In order now to determine the *maximum shear* at every point, lay off (Fig. 3) the locomotive weights along the line A B, with the first driver over the point A. In this position it gives the maximum shear at A. It is of course supposed that the first driver rests on the point at an infinitely small distance to the right of A. Beginning at the point B draw a vertical line and lay upward all the weights in succession, beginning at the first driver. Take the point A as the pole, and draw as before the radial lines from A, and complete the equilibrium curve A H. The process is exactly the same as in the case of Figs. 1 and 2. The line A E coincides with A e, E F parallel to A f, F G to A g, and lastly O′ H parallel to A O. The distance B H which, according to the scale, is equal to the force of 53,600 lbs., is the amount of shear at A, and to obtain maximum shear for every other point between C and A we have but to measure in like manner the ordinate to the equilibrium polygon A E F G— —H from the line A B.* At the point B shear is zero, as the locomotive is outside of the girder. For the total shear at any point we measure along the vertical line at that point between the equilibrium polygon and the line C D. Thus we obtain the maximum shears at several sections into which we have previously divided the girder, the following amounts:

	Dead load shears.	Live load shears.	Total.
At end	12,300	53,600	65,900
a	9,800	45,200	55,000
b	7,400	37,300	44,700
c	4,900	30,000	34,900
d	2,500	23,500	24,000
e	0	17,800	17,800

According to what we have said in

* For proof see DuBois' Graph. Statics, chap. vii.

Chapter I, we have but to divide the shear eq. (9) at the end where it is maximum, by the allowable stress per square inch for shear to obtain the *necessary web section*. Experiments have shown that the *shearing strength of wrought iron* across the fiber is equal to about $\frac{4}{5}$ the tensile strength. The shearing strength along the fiber varies sometimes considerably, being, however, between $\frac{1}{5}$ to $1\frac{1}{5}$ times that across the fiber, according to the kind of iron and mode of treatment in rolling mills. Consequently, where a piece of iron is subjected to shears in both directions, a somewhat smaller allowable stress should be given than merely across the fibers, say $\frac{2}{3}$ the allowed tensile stress. The ratio of minimum to maximum shear at the end of the girder is about $\frac{1}{5}$, and same as in the case of flange stress; consequently we obtain at once the allowable shearing stress:

$$9{,}600\times\tfrac{2}{3}=6{,}400 \text{ lbs.}$$

Web.

The equation of vertical shearing force has already been given by eq. (3) viz.:

$$S_x = R - \sum_0^x w$$

For a given position of load, and hence for a given value of R, therefore,

S_x will be maximum when $\sum_0^x w = o$.

For the *dead load* of girder we have

$$S_x = R - w'x$$

in which w' represents the load per unit of length. At the end of the girder where $x = o$

$$S_x = R = 490 \text{ lbs.} \times 50' \times \tfrac{1}{2} = 12{,}300 \text{ lbs.}$$

At the middle of the girder where $w'x = R$ the shearing force is zero. In Fig. 3, Plate I, A B represents the length of the girder, and C its middle point. If we now draw B D according to the scale of force, equal to 12,300 lbs., and connect C D, the amount of shearing action at every point between the end and the middle point of the girder will be equal to the ordinate from the line C B to this inclined line.

The maximum shearing action to the *moving load* can of course be determined by the use of equation (3) which is entirely general, but it requires too much work when the number of points for which the amount is required to be determined is large; and in such cases graphical method is far preferable. Adding 15 % for impact to the end shear, we obtain

$$65{,}900 \times 15\,\% = 75{,}800 \text{ lbs.}$$

The section required is therefore equal to:

$$\frac{75{,}800}{6{,}400} = 11.9 \text{ square inches.}$$

The depth of the web is 4′—6.″ Supposing the number of rivet holes in a vertical row to be 14, and taking the thickness of the web at the least allowable dimension of $\frac{3}{8}''$, we have the net section:

$$(54'' - 14'') \times \tfrac{3}{8}'' = 15 \text{ square inches,}$$

which is about 3 square inches still on the side of safety.

Having thus found out that our web

of $\frac{3}{8}$ inch is strong enough so far as *shearing stress* is concerned, it now remains to be seen whether it can resist the stress as a column (Chapter I, p. 25).

Taking the vertical distance between two rows of rivets at 4′—2″=50″ we have

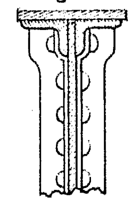
$$\frac{h'}{b}=\frac{50''}{\frac{3}{8}''}=133.$$

From the table on page 25 we see that the allowable stress per square is about 620 lbs., which is even less than the least shearing stress which we have at the middle of the web, and is equal to $\frac{17,800}{15}=1,190$ lbs. per square inch. Consequently the web must be *stiffened throughout.*

We have spaced stiffeners, as shown in Plate II. The spacing is, to a certain extent, governed by the positions of lateral and cross bracing connections; for at every point where we have cross-bracing we must have stiffeners to form connection, and also to give the stiffness at the section. Toward the end of the

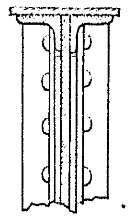
girder, stiffeners are spaced much closer than toward the middle, for the evident reason of greatly increased shear, and hence of compression at the end.

Fig. 14



Stiffeners must always be *tightly fitted* between the flange angles. In order to bring stiffeners in contact with web and vertical leg of angles, they are either bent, as shown in Fig. 14, or fillers may be used, as in Fig. 15. The first method requires less material than the second, but requires the work of bending the angles, for which particular dies must be had to give the required amount of bending, as will be economical when there are a large number of stiffeners to be bent in the same way. The second method is therefore more preferable for cases where girders to be made are but few in number.

Fig. 15



LATERAL FORCES.

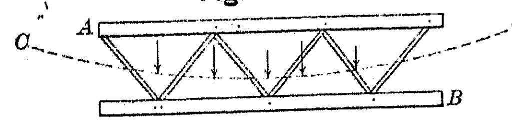
In case the track on the girder is curved, the *centrifugal force* of locomotive must be taken into consideration. If A and B represent two flanges of the girder (in case of deck girder, top flanges, and in case of through girders, bottom ones,) and c c the center lines of the track, then the centrifugal force acts radial to the curve, as indicated by the arrows, and its amount is expressed by the well-known formula:

$$\text{Center force} = \frac{w \times v^2}{r \times 32.2},$$

in which w is the moving load on the

girder, v its velocity in feet per second, and r the radius of curvature of the track in feet. We have thus to consider the girder flanges top or bottom, according as the girder is deck or through, as forming a truss lying on its side. The flange section should be increased according to the stress obtained as the chord of this lateral truss. The lateral bracing should also be proportioned to resist the maximum stress due to this force, in addition to the wind stress to be presently considered.

Fig. 16



Wind pressure on a railway bridge is usually taken at 50 lbs. per square foot when the bridge is unloaded, and 30 lbs. when loaded. It is calculated that, at the latter pressure, all empty cars will be blown away.

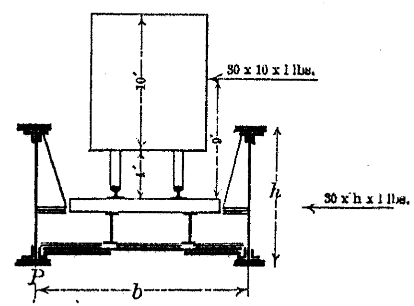
In plate girders of ordinary spans (say up to 65 ft.) only one system of lateral bracing is sufficient for deck spans with the system on the top, together with cross bracings, and for through spans on the bottom.

In *deck* span it is hardly necessary to consider the stress in the *flanges* due to the wind pressure, as the latter on the girder itself is for ordinary spans quite a small amount, and when the girder is loaded the stress due to the overturning moment of the wind pressure on the train will in the top flange be almost neutralized by the stress coming from the lateral system, and in the bottom flange, where the stress is due to this moment alone, can entirely be neglected, as being a very small amount. This is in many cases only true under the supposition that the total wind pressure is carried by the top lateral system to the abutment, and that the intermediate cross bracing merely serves to keep the two girders parallel to each other, and also to carry some wind pressure on the lower half of the girder up to the lateral system; and consequently the increase

of vertical loading on the leeward side girder is only due to the overturning moment above the rails.

But in *through* span, the stress on the lower flange, due to the wind pressure, has sometimes to be calculated, to see if the total stress per square inch, due to the rolling load and wind pressure, does not exceed the maximum allowable stress (*i. e.*, the value of a when $\frac{min.}{max.}=1$ in the Launhardt's formula. (See p. 49.) The flange stress due to the wind in this case can be easily calculated.

Fig. 17



The stress on the flange due to the overturning (Fig. 17.) moment of the wind pressure on the train is equal to

$$\frac{30\times10\times l\times9}{b}\times\tfrac{1}{8}\times l+h,$$

in which h is the effective depth of the girder. The stress coming from the lateral system is equal to

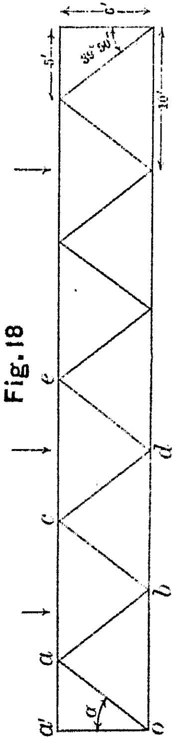
$$\{(30\times10\times l)+(30\times h\times l)2\}\times\tfrac{1}{8}\times l+b,$$

and the total stress is equal to

$$\frac{l^2\ (2{,}700+300\ h+60\ h^2)}{8\ h\ b}.$$

Now, as to the *stress in the lateral bracing* itself (Fig. 18), we have a traveling load due to the wind pressure on the train surface, and a dead load due to the pressure on the girder surface itself. We will, however, take both as a traveling load, as the pressure on the girder itself is after all quite a small amount.

Fig. 18



The total wind pressure on the system of the 50 foot girder, when the train covers the entire span, consists of:

On train surface $10 \times 50 \times 30 = 15{,}000$ lbs.
On girder* " $2 \times 5 \times 50 \times 30 = 15{,}000$ "

Total, $= 30{,}000$ lbs.

which gives for each point a load of $\frac{30{,}000}{10} = 3{,}000$ lbs. Consequently, we obtain the following stresses, under the supposition that the girder is in each case covered with the load from one end up to the diagonal, in which we then determine the stress. It is not, however, necessary to calculate stresses in more than one or two diagonals near the end of the girder, as stresses in the rest are inconsiderable.†

Stress in $a\ b = 3{,}000\ (8+7+6+5+4+3+2+1)\ \frac{1}{10}$ sec. $39^0 - 50^1 = \pm 14{,}000$.

*Here we have taken twice the surface of one girder, where but 1½ times is sufficient, as the leeward girder is more or less protected by the windward one.

†After a little practice in designing girders, one finds it hardly necessary to make any calculation for these braces for ordinary spans.

Stress in $a\ o = 3{,}000\ (9+8+7+$ — — — —$)\ \frac{1}{10}$ sec. $39^0 - 50^1 = \pm 17{,}600$.

Stress in $a'\ o = 3{,}000\ (\times 10 \times \frac{1}{2}) = \pm 15{,}000$

The sign $\pm$ indicates that each stress will be tension or compression, according as the wind is blowing from one side or from the other. Taking the two facts into consideration, that while the stress is alternating in each brace, and consequently requiring a low allowable stress, it is very seldom that such a wind as we have made calculation for blows, and still more seldom to be blowing at the very time when the train is passing over the bridge, we will take the allowable stress at 8,000 lbs. per square inch, and reduce it by the ordinary formula of columns.

If we use $3'' \times 3''$ angles for the brace, we have in the following formula for least radius of gyration (r^2), about .8 inch, the length of the brace (l) being about 7 feet.

$$\frac{8{,}000}{1 + \frac{(12\ l)^2}{40{,}000\ (r)^2}} = \frac{8{,}000}{1 + \frac{(84)^2}{40{,}000 \times .8}} = 6{,}500$$

lbs. per square inch.

This gives the following sections:

$$\text{For } a\ o\ \frac{17{,}600}{6{,}500} = 2.7^{\text{D}''}$$

$$\text{“}\ b\ a\ \frac{14{,}000}{6{,}500} = 2.1^{\text{D}''}$$

$$\text{“}\ a'\ o\ \frac{15{,}000}{6{,}600} = 2.3^{\text{D}''}$$

The lateral braces are, however, constantly subjected to stresses due to the vibration which the slight lateral motion of the passing train sets in the girder, and for which we have no means of correct determination. To provide for this we add 40 % to those sections we have just obtained, making:

For $a\ o\ 2.7^{\text{D}''} + 40\% = 3.8^{\text{D}''} = 3'' \times 3'' \times \frac{5}{8}''$ L
“ $b\ a\ 2.1^{\text{D}''} + 40\% = 2.9'' = 3 \times 3 \times \frac{1}{2}$ L
“ $a'\ o\ 2.3 + 40\% = 3.2^{\text{D}''} = 3 \times 3 \times \frac{3}{8}$ L
“ $b\ c$—cd—and $d\ e$ $3 \times 3 \times \frac{3}{8}$ L

The end cross bracing should be made strong enough to carry the entire wind pressure down to the abutment, where it is resisted by friction and anchoring. The amount of stress on the cross brace is evidently 11,300, multiplied by the

secant of the angle of inclination of the brace.

The connection of diagonals to the girder is effected by plates. The number of rivets connecting the plate with the angle should be so many as their total shearing strength shall be equal to, or greater than the full strength of the angle taken as a column.

Although it is not necessary in ordinary spans, yet it is well to keep in view to let the diagonals intersect as near the center line of the flange as can conveniently be done.

The intermediate cross bracing are put in merely to give stiffness to the entire girder, and to help to carry the wind pressure to the lateral bracing above, and may be $3'' \times 3'' \times \frac{3}{8}''$ L in our case.

RIVETS AND RIVET SPACING.

As already said, the shearing strength of iron is equal to about $\frac{4}{5}$ of its tensile strength. In order to cover all the changing values of unit stress, and thus to avoid tedious work of calculation, we will take the unit stress for shear—what we may assume to be the lowest for ordinary rivet iron. Taking, then, the unit tensile stress at 9,000 lbs. (for c in formula 14), we obtain the *allowable shearing stress* of

$$9{,}000 \times \tfrac{4}{5} = 7{,}200 \text{ lbs.}$$

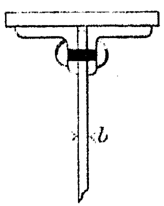
We will assume that the shearing stress is uniformly distributed over the cross-section of the rivet. The strength of a rivet $\frac{7}{8}$ inch in diameter for single shear will then be

$$(\tfrac{7}{8})^2 \times \tfrac{1}{4} \times 3.1416 \times 7{,}200 = 4{,}300 \text{ lbs.}$$

And for double shear $= 8{,}600$ lbs.

For a riveted joint as shown in section by Fig. (19) the amount of stress

Fig. 19



on the rivet while on the one hand must not exceed the double-shear strength, on the other hand must not be so great as to crush the rivet hole or the rivet itself. The latter is the first to fail. It is a well-established fact that the *allowable compressive* stress per square inch on *the projected area* (diameter of hole × thickness of the plate) *of the rivet* hole can be as much as and even more* than twice the shearing stress allowed, or $7{,}200 \times 2 = 14{,}400$ lbs.

If $b = \frac{3}{8}''$ we have for $\frac{7}{8}''$ diameter rivets the *bearing value* of

$$\tfrac{7}{8} \times \tfrac{3}{8} \times 14{,}400 = 4{,}700 \text{ lbs.}$$

Since we have no plate or angle in the girder whose thickness is less than $\frac{3}{8}$ inch, and as the single-shear value of the rivet is less than its bearing value on $\frac{3}{8}''$ plate, we need not take bearing value into consideration, except when the rivets are double-shear.

Rivet holes are made either by drilling or punching. The former does not

---

*Garber has shown by experiment that this unit stress may be twice the unit tensile stress, or, in this case, equal to $9{,}000 \times 2 = 18{,}000$ lbs.

weaken the plate as much as the latter does, but drilled holes produce a cutting action on rivets unless they are reamed. It is, however, useless to enter here into any discussion on the relative merits of these two processes, as, in the present state of bridge building in America, viz., of competition in designs and prices, no bridge builder will go into the extra work of drilling and reaming rivet holes, when in a few minutes a longest plate or angle that comes out of a rolling mill can be punched from one end to the other, with all the accuracy and correctness of templet works. The diameter of rivet holes should not be more than $\frac{1}{16}$ inch greater than that of the rivet. Every rivet should fill up the hole, and in case any signs of looseness are detected, should be cut and riveted anew.

Rivet Spacing.—In order that a piece be punched with a machine it is evident that rivet holes should be spaced in *straight lines*, in directions both of width and length. With most machines it is necessary that no rivet spacing in a row

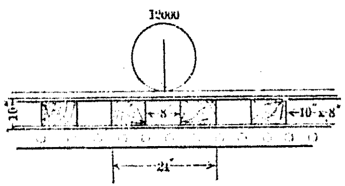
should contain fraction less than $\frac{1}{4}$ inch.

Rivets in a row are spaced rarely closer than three times the diameter of rivets. Thus with $\frac{7}{8}''$ rivets, we take $2\frac{3}{4}''$ as the minimum distance allowable between the centers of rivets in a row.

The *maximum distance allowable* between the rivets is determined from the consideration that, in the part of the girder subjected to compression, the thinnest plate composing that part shall be able to sustain the maximum stress, acting on its section without buckling between the rivets, by which they are held fast to other plates and angles. Generally, if spacing does not exceed *twelve times* the *thickness of the thinnest plate* on the flange, we are on the safe side. Thus, in our girder the least thickness of the plate being $\frac{3}{8}''$, we have for the maximum distance, center to center of rivets, $12\times\frac{3}{8}=4\frac{1}{2}$ inches.

Rivets connecting Web and Flanges. The stress which the rivets connecting the web and flanges sustain is evidently due to the stress which is transmitted from one to the other. We have here two stresses to consider, and for whose resultant we proportion the number of rivets. The one is *vertical*, being due directly to the load resting on the flange of the girder, and whence, through the rivets, transmitted to the web. It is difficult to find out what really will the amount of this stress be, as it depends upon the stiffness of the material intervening the load

Fig. 20



and rivets. We will not be very far from the maximum effect if we take the weight of one driver, as distributed over the space covered by two cross-ties, making the vertical stress to be resisted by the rivet $\frac{12,000}{24''}=500$ lbs. per inch

run of the girder. The weight of the track is inconsiderable.

The other stress is horizontal, and was already found in the first chapter of the book given by the eq. (10), viz.:

$$s=\frac{S_x}{h_c}.$$

This stress s, as already explained, is nothing but the maximum movement of flange stress at every section of the girder. On page 59 we have given the maximum values of S_x at the end, and sections a, b, c, d, e of the girder, from which we can now obtain the values of s at these points.

At end $s=\frac{65,900}{4.5\times 12}=1,200$ lbs. per inch run.

a " $=\frac{55,000}{4.5\times 12}=1,000$ " " " "

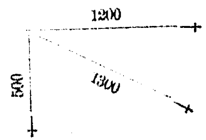
b " $=\frac{44,700}{4.5\times 12}=$ 810 " " " "

c " $=\frac{34,900}{4.5\times 12}=$ 640 " " " "

d " $=\frac{24,000}{4.5\times 12}=$ 440 " " " "

$e \quad s = \frac{17,800}{4.5 \times 12} = 320$ lbs. per inch run.

The resultant of s thus obtained with the vertical stress of 500 lbs. is therefore the stress on the rivets, and we have:



At end $\sqrt{1,200^2 + 500^2} = 1,300$ lbs. per inch run

$a \quad \sqrt{1,000^2 + 500^2} = 1,120$ " " " "

$b \quad \sqrt{810^2 + 500^2} = 950$ " " " "

$c \quad \sqrt{640^2 + 500^2} = 810$ " " " "

$d \quad \sqrt{440^2 + 500^2} = 670$ " " " "

$e \quad \sqrt{320^2 + 500^2} = 600$ " " " "

The shear of rivet is double, consequently we take its bearing value, viz.: 4,700 lbs.

At the end where we have the stress of 1,300 lbs. per inch run, we must therefore space rivets,

$$\frac{4,700}{1,300} = 3\tfrac{1}{2} \text{ inches apart.}$$

At $a \quad \frac{4,700}{1,120} = 4$ inches apart.

At $b \quad \frac{4,700}{950} = 5$ " "

It is not necessary to calculate any further, as this last spacing already exceeds the distance which we have already taken as the maximum. It is true that the spacing, $4\frac{1}{2}$ inches, which we have taken as the maximum allowable, is for the flange; yet, in order that rivets in the two legs of the angle should stagger each other, both legs must have the same spacing. This staggering of rivets, so long as there is room enough for the riveter, after one rivet is in place, to rivet the other, is not necessary in compression flange. But, as already said, in order that the rivet holes may lie in straight lines, vertical and horizontal, the top and bottom rows should be spaced alike. In the bottom flange angles the rivets must stagger completely, since we have deducted only one rivet hole out of each angle in calculating the net sectional area. In fact, the spacing in top and bottom flanges are made alike.

Rivets in Flanges.—Having determined the spacing of rivets connecting the web with flanges, it remains for rivets in the latter alone merely to be so spaced as to stagger with them. The distance between the rows of rivets on the flange plate should not exceed 30 times the thickness of the thinnest plate; making in one girder the maximum allowable distance $30 \times \frac{3}{8} = 11$ inches. For flange plates more than 12 inches wide it is the usual practice to put in 4 rows of rivets.

Except at the ends of the girder, no rivets in the bottom flanges should be countersunk, unless special allowance is made for the amount of section taken away by the reaming of holes, and where the leg of the stiffener angle fouls the rivet head, the former may be cut off enough to clear the latter.

As already said, every flange plate, except of course the one that covers the full length of the girder, should be extended beyond the point at which it be-

gins to form the necessary section of the flange, far enough to take in so many rivets whose total shearing strength equals the strength of the plate itself. Thus the uppermost plate f (Pl. II), 12″ $\times \frac{3}{8}$″, which, according to our calculation, begins to form the necessary part of the flange area at about the point t is extended to take in $(12 \times \frac{3}{8} \times 8{,}400) \div 4{,}400 = 8$ rivets to the point u. In the same way, in the bottom flange, the plate f', which is required to form the section at about the point t', is extended beyond that, to take in $(10 \times \frac{3}{8} \times 9{,}600) \div 4{,}400 = 8$ rivets.

The rest of the plates are proportioned in the same way.

SPLICING.

Particular attention should be given to splices in plate girders. Both flanges should be fully spliced, and no contact joint should be depended upon, although in all possible cases as good a contact should be given to pieces cut in the top flanges.

Girders not much longer than 50 feet

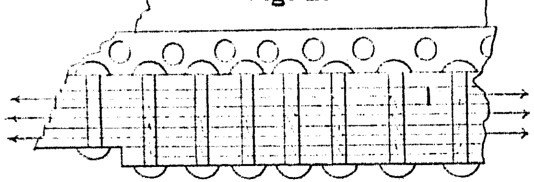
can usually be shipped in single pieces, so that the work of erection consists merely in putting them in place, and riveting lateral connections.

Girders whose lengths are less than 30 feet hardly require any splicing, as all the pieces can be obtained in full length.

In our girders, all the full-length pieces are to be spliced. Plates $g \times g'$ (Pl. II), are splices for the long flange plates which are joined in the middle. These splices must, of course, be long enough to take in on each side of the joint as many rivets or more as their total shearing strength be equal to the strength of the plate. The stress is supposed to be transmitted in a manner something like as shown by dotted lines in the figure.

Angles are spliced between *c* and *d*.

Fig. 21



The splices may consist of two bars riveted on both legs of the angle, or, better, of cover angles, as we have done. Cover angles are always given such a shape as to fit closely into the inside of ordinary angles. Since the flange angles are $3\frac{1}{2}\times 5''\times\frac{1}{2}''$, each having the sectional area of $4.0^{\square}''$, we take for splice the cover angle $3''\times 4''\times\frac{5}{8}''$.

The web is spliced in the middle and at *b*. The maximum shear at *b*, with proper allowance for impact, is already found equal to 51,400 lbs. The number of rivets required is, therefore, equal to $\frac{51,400}{4,700}=11$, against which we find we have a few more than is necessary. The splice plates in the middle are made to fit tight between flanges, to serve as stiffeners there.

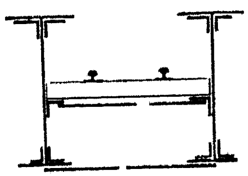
THROUGH GIRDERS.

Nearly all that has been said thus far about the deck girders applies equally well to through ones. Since it is not possible in a through girder to have top lateral bracing, particular attention should be paid to give lateral stiffness to the top flanges. This is done by connecting the top flanges with floor beams by stiff plates, as shown in Fig. 4, Pl. I. In case floor beams are so spaced that the ratio of the width of the top flange plate to the distance between them exceeds 12, the allowable unit stress in the flange should, as already explained on page 49, be reduced by the given formula.

Since the load now travels between the girders, and is transmitted to the web through floor beams, the rivets connecting the flanges with the web need be proportioned only for the horizontal stress.

Through girders are sometimes constructed without floor beams or stringers, the cross-tie resting directly on the angles and riveted to the web, as shown in Fig. (22).

Fig. 22



BED PLATES.

Bed plates should be so proportioned that the maximum pressure on the masonry shall not exceed a certain amount, depending upon the kind of masonry. Pressure of 250–300 lbs. per square inch can be allowed usually on good coping.

Bed plates should be made so thick that the pressure will be uniformly distributed under its entire area.

The maximum end reaction, with proper allowance for impact, we have already found (p. 62) to be 75,800 lbs. Taking the allowable pressure on the masonry to be 300 lbs., we have for the required area of our bed plate $\frac{75,800}{300} =$ 253 square inches. Hence we have a bed plate $16'' \times 16'' = 256$ square inches.

In girders longer than about 70 feet, it is necessary to provide one end with a nest of rollers. But in smaller spans a mere sliding arrangement is sufficient. This is done by slotting anchor-bolt holes in the girder flange, enough to accommodate the forward and backward motion of that end of the girder, due to the extreme changes of temperature, which may be usually taken at 150°. The co-efficient of expansion of iron is about .000007 for 1° Fahrenheit. Consequently, for 50 feet girders we slot the bolt hole $50' \times .000007 \times 150° \times 12'' = \frac{5}{8}''$. The diameter of the bolt being 1″, the dimensions of the hole become $1\frac{5}{8}'' \times 1''$.

OTHER FORCES THAN THOSE ALREADY CONSIDERED.

In the chapter on Flange Stress we have mentioned the fact of the existence of certain forces for the stresses caused, by which, however, we have no exact way of determination. To these may be added those due to the changes of temperature, to the longitudinal thrust of the moving load, to vibration, and to the application of brakes on wheels over the girder. The amount of some of these forces can be easily determined, and their effects calculated. Thus the stress due to the changes of temperature is simply

the amount of friction at the sliding end of the girder, and is either tension or compression, according as the girder is contracting or expanding. The amount of force due to the application of brakes on wheels is simply that of friction existing between the rail and those wheels.

But it is doubtful that even but once in the life of a bridge the combined action of all these forces will take place. Even if it did, we are quite certain that the large factor of safety we have assumed at the beginning will cover every possible condition of loading.

WEIGHT.

The actual weight of the girder can now be determined. It is to be remembered that a *bar of iron of* 1 *square inch cross-section, and* 1 *yard long, weighs very nearly* 10 *lbs.* Steel weighs about 2 per cent. more than iron. The total weight of rivets in a girder is about 6 per cent. of the total weight of the girder, but in determining the actual or shipping weight of a girder, only their heads are to be taken, as their bodies merely fill up thin holes. A pair of heads of rivets, as usually made, weighs:

For $\frac{7}{8}''$ diam. rivets, about 0.5 lbs.
" $\frac{3}{4}''$ " " " 0.4 "
" $\frac{5}{8}''$ " " " 0.3 "

Weight of 50 feet span girder:

				Weight Lbs.
4	Web	plates	$54'' \times \frac{3}{8}'' \times 25'$—$7\frac{1}{2}''$	.. 6,912
8	Flange	"	$12'' \times \frac{1}{2}'' \times 25'$—$7\frac{3}{8}''$	.. 4,102
2	"	"	$12'' \times \frac{3}{8}'' \times 34'$—1 "	.. 1,022
2	"	"	$12 \times \frac{7}{16} \times 35'$—5 "	.. 1,239
2	"	"	$12 \times \frac{3}{8} \times 25'$—3 "	.. 758
2	"	"	$12 \times \frac{3}{8} \times 24'$—5 "	.. 732
4	"	splice	$12 \times \frac{1}{2} \times 4'$—7 "	.. 367
4	Web	"	$8 \times \frac{3}{8} \times 4'$—6 "	.. 180
4	End cover plate		$12 \times \frac{3}{8} \times 4'$—$7\frac{1}{4}''$	.. 276
5	Connection	"	$10 \times \frac{3}{8} \times 1'$—6 "	.. 94
4	"	"	$12'' \times \frac{3}{8}'' \times 2'$—1 "	.. 125
2	"	"	$12'' \times \frac{5}{8}'' \times 1'$—6 "	.. 45
16	"	"	$10 \times \frac{3}{8} \times 1'$—0 "	.. 200
4	"	"	$12 \times \frac{3}{8} \times 1'$—0 "	.. 60
4	Bed	"	$16 \times \frac{3}{4} \times 1'$—4 "	.. 213
8	Bars	"	$2 \times \frac{1}{2} \times 1'$—4 "	.. 36

8 Flange *angles* $3\frac{1}{2}'' \times 5'' \times \frac{1}{2}''$(40 lbs p. y.) $\times 18'$—0″ weigh, 1,920 lbs.

8 Flange *angles* $3\frac{1}{2} \times 5 \times \frac{1}{2}$ (40 lbs. p. y.) $\times 33'$—$3\frac{1}{4}$ weigh, 3,549 lbs.

24 Stiffener *angles* $3 \times 4 \times \frac{1}{2}$ (32 lbs. p. y.) $\times 4'$—6″ weigh, 1,152 lbs.

32 Stiffener *angles* $3 \times 4 \times \frac{3}{8}$ (24 lbs. p. y.) $\times 4' \times 6''$ weigh, 1,152 lbs.

6 Lateral brace *angles* $3 \times 3 \times \frac{3}{8}$ (21 lbs. p. y.) $\times 6'$—8″ weigh, 280 lbs.

2 Lateral brace *angles* $3 \times 3 \times \frac{1}{2}$ (28 lbs. p. y.) $\times 6'$—8″ weigh, 123 lbs.

2 Lateral brace *angles* $3 \times 3 \times \frac{5}{8}$ (38 lbs p. y.) $\times 7'$—5″ weigh, 195 lbs.

8 Cross brace *angles* $3 \times 3 \times \frac{3}{8}$ (21 lbs. p. y.) $\times 7'$—2″ weigh, 403 lbs.

8 Cross brace *angles* $3 \times 3 \times \frac{3}{8}$ (21 lbs. p. y.) $\times 5'$—0″ weigh, 280 lbs.

4 End cross brace *angles* $3 \times 3 \times \frac{5}{8}$ (38 lbs. p. y.) $\times 6'$—10″ weigh, 348 lbs.

4 End cross brace *angles* $3 \times 3 \times \frac{5}{8}$ (38 lbs. p. y.) $\times 5'$—0″ weigh, 253 lbs.

4 Cover *angles* $3 \times 4 \times \frac{5}{8} \times 4'$—0″ weigh, 215 lbs.

2,530 pairs of rivet heads weigh, 1,265 lbs.

4 anchor bolts 1″ diam. 9″ long, nuts, etc., weigh, 10 lbs.

Making the *total weight* = 27,504 "

ESTIMATE OF COST.

The cost of a girder depends upon the market price of the material of which it is made, the distance of transportation, and the nature of the site of the crossing. Under ordinary circumstances, with the following prices of material, our 50 foot girder, free on board the car at the place of fabrication, will cost about as estimated below:

6,912 lbs.	Web iron	@ 3 cts. =	$207.36
9,449 "	Plate "	@ 2.5 " =	236.23
9,870 "	Angle "	@ 2.5 " =	246.75
1,800 "	Rivets	@ 3.75 " =	70.50
Total wt.	Cost of Fabrication	@ 0.75 " =	206.28
"	Erection	@ 0.5 " =	137.52
Painting		=	12.00
		Total,	$1,116.64

GENERAL REMARKS.

Girders much larger than 50 feet have usually to be shipped in more than one piece. Since the riveting done in the field is more or less imperfect, as compared with that done in the shop, some allowance should be made in the number

of those rivets in the design. The use of bolts instead of field rivets should be avoided, unless they are turned and made conical, and holes drilled to fit them, thus insuring perfect contact.

In our 50 foot girder we have riveted connection plates to the girder, but when the distance of transportation is very great, and reloading necessary, they may become bent or broken; consequently, under such circumstances, they should be shipped separately.