論說報告

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SOME EXPERIMENTS RELATING TO THE THERMAL FLEXURE OF A THIN PLATE HEATED ON ONE SURFACE

By Noboru Yamaguti, C. E., Member
and O. Kazita.

Synopsis

For the purpose of verifying the mathematical results given in the previous papers (Vol. XIII No. 4 and Vol. XIV No. 3 of this Journal) series of experiments were carried out with a square ebonite plate of 13 cm. sides and of 3 & $7\frac{1}{2}$ mm. thickness. The temperature difference between upper and lower surfaces was raised up to 15°C. The experimental results show the validity of our calculations not only qualitatively but also quantitatively.

Introduction

For the purpose of verifying the mathematical results obtained in the previous papers a series of experiments were worked out with square plates having 13 cm. sides. As a specimen an ebonite plate was taken, as it is less conductive and well expansive under heating, these physical properties being no doubt favourable for our present purpose. The plate, supported horizontally, was heated uniformly on its lower surface and cooled on its upper surface. The deflection of the plate due to thermal flexure was measured with two different boundary conditions to compare the results with the theoretical ones.

Apparatus and Method

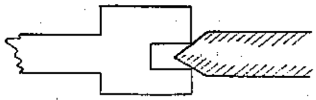
The whole apparatus specially designed for our present purpose is reproduced in **Pl. I** and **II.** **Fig. 1** and **2** show the skeleton drawings of it, in which E is the specimen of ebonite plate. It is supported on its four sides in independent steel frames having ⊐ section. The specimen will be supported simply by these frames or clamped down with fillets and screws in the grooves. The frames are facilitated to slide backwards in plane of the specimen, or can be clamped by means of set screws *S*. A heating box is arranged just beneath the specimen. It is a flat electric furnace covered with copper plate of about 2 mm. thick. The heating circuit consists of threo sndependent coils. Their imput can be controlled separately to maintain the iurface of the specimen at uniform temperature, which is measured with thin thermo-junction of copper-constantan cemented at various points of both surfaces of the specimen. These are shown in **Pl. III.** **Fig. 1** is the curves of E. M. F. of junctions at various points taken against the time coordinate and **Fig. 2** is the characteristic of junctions.

Just above the specimen placed another copper vessel, covering the same area with that of the heater, through which ice water is circulated constantly during the observation to keep the upper surface of the plate as cool and uniform as possible. With these devices we could obtain the temperature difference up to 15°C between upper and lower surfaces of the specimen and we could limit the local irregularity of the temperature distribution of each surface within a few percent of the highest temperature. To measure the deflection of the plate a dial indicator having 1/100 mm. division is arranged so that it can travel above the specimen on a bridge. The bridge itself is made to slide on the parallel supports. These are all carefully finished to give perfect straightness. Narrow slits are prepared in the cooling vessel through which the leg of indicator can reach the surface of specimen and the deflection of plate can be observable by traversing the indicator along the bridge.

Experimental Results

At first we compared the thermal deflections due to the different supported conditions with an ebonite plate of 3 mm. thickness. When its four sides are tightly clamped in the grooves with fillets and set screws we obtain no appreciable amount of deflection for the temperature difference up to 15°C between upper & lower surfaces. Whereas if the set screws are slightly loosened a remarkable amount of deflection sets in. The results of observation are shown in **Pl. IV.** Here our fundamental mathematical result is justified by the experiment.

To investigate general configuration of the thermal flexure of a square plate with supported edges, an ebonite plate of $7\frac{1}{2}$ mm. thickness was used.



Its four edges were finished in knife-edge form and were fitted to the grooves in frames as shown in the following figure. The set screws were, of course, left loose.

The deflections at various points of the upper surface were measured after it is bended by heating. These were plotted in a contour map shown in **Pl. V** which coincides finely well with our mathematical one.[1] The experimental results show that the above supports are suitable enough to satisfy the boundary condition of the supported edges.

In my previous paper[1]
we have

$$\xi = K_1 \frac{8l^2}{\pi^3} \frac{E\Phi}{2D\left(1-\frac{1}{m}\right)} \quad \cdots\cdots (1)$$

where K_1; Numerical constant depending on the measured position, the mode of supports of edges and Poisson's Number of material.

l; Size of the plate.

E; Young's Modulus.

m; Poisson's Number.

$\Phi = \alpha \int_{(a)} (\theta - \theta_m) x dx$; Term due to thermal gradient.

$= \alpha \Theta I$ for the linear temperature gradient.

$D = \frac{EI}{1-\frac{1}{m^2}}$; Flexural rigidity of plate.

Therefore, assuming the temperature gradient is linear, we have

$$\xi = K_1 \frac{4l^2}{\pi^3}\left(1+\frac{1}{m}\right)\alpha \frac{\theta_2-\theta_1}{a} \quad \cdots\cdots (2)$$

where α; Coefficient of expansion.

$\theta_2 - \theta_1$; The temperature difference between upper and lower surfaces.

a; Thickness of plate.

(1) Journ. Civil Eng. So. Japan. Vol. XIII, No. 4, Pl. I.

If we assume l, m, α, a are constants independent of temperature we have the following simple relation

$$\xi \propto \theta_2-\theta_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

This is fulfilled fairly well as shown in the following table.

Deflections at the center of an ebonite plate 4 sides free to rotate, $130\times130\times7\frac{1}{2}$ mm. (**Pl. VI.**)

$\theta_2-\theta_1$	8.°5	10.°5	14.°5
ξ	$0.65\times\frac{20}{100}$	$0.85\times\frac{20}{100}$	$1.30\times\frac{20}{100}$ mm.

Owing to lack of experimental data about the physical constants of our specimen at various temperature we are not able to compare these values of deflections exactly with those of calculated ones. We take, for instance, the following values of expansion coefficient and Poisson's Number

$$\alpha=.000077$$
$$\frac{1}{m}=.49 \qquad \text{(Landolt's Table)}$$

and we have $l=130$ mm., $a=7.5$ mm. For a plate with 4 sides free to rotate, we can avail directly our calculated table of K_1 (Vol. XIII, No. 4, p. 540), as K_1 is not dependent on the value of Poisson's Number in this case. At the center of plate with such supports we have $K_1=.5656$. Therefore if we calculate a center deflection for a temperature difference of 14.°5 C, we obtain

$$\xi=K_1\frac{4\times130\times130}{\pi^3}\times1.49\times.000077\times\frac{14.5}{7.5}$$
$$=.485\,K_1$$
$$=.275\text{ mm.}$$

This coincides pretty well with the above value obtained by experiment

$$\xi=.260\text{ mm.}$$

Conclusion

From the present experiments we see that our mathematical results are verified to be satisfactory at least with our experimental conditions. Naturally, the engineering structure such as walls of buildings and slabs of bridges may sometimes, set forth more complicated conditions than those of our present experiments, still we might conclude that we are able to apply our theory approximately to them.

We wish to express our hearty thanks to our Colleage Mr. Z. Tuzi M.E. for his various important suggestions in carrying out the above experiments. We must also not fail to record our cordial thanks to Viscount Dr. M. Okôchi, President of the Institute of the Physical and Chemical Research, Tokyo, Japan, who had the liberality to allow us to perform the present experiments in one of the laboratories of his Institute.

Tokyo, May, 1928.

Pl. I.

Fig. 1. Apparatus for Measuring Thermal Flexure

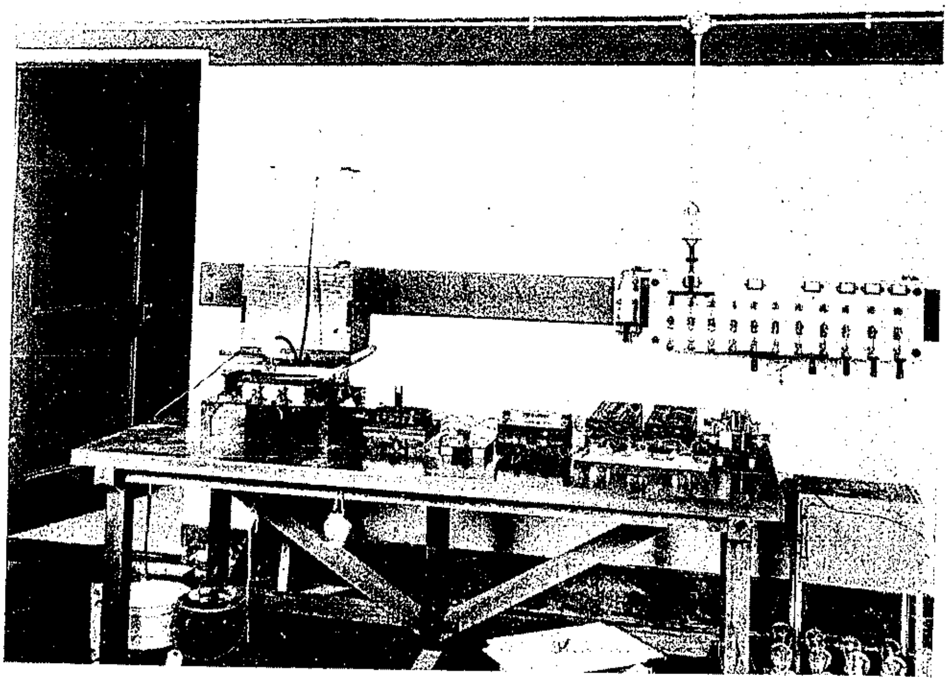
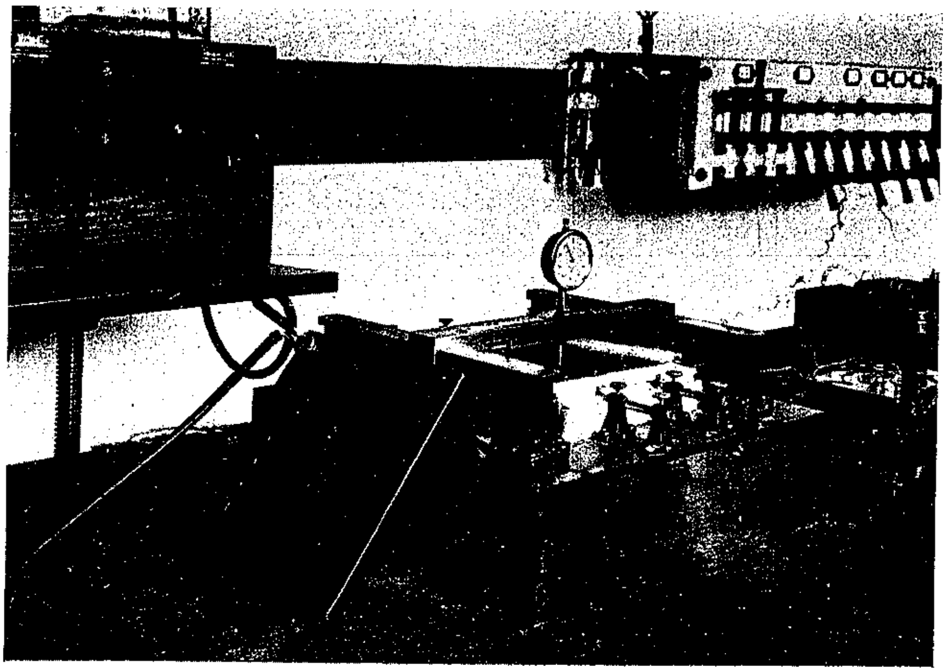


Fig. 2. Apparatus for Measuring Thermal Flexure



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Pl. II.

Fig. 1. Apparatus for Measuring Thermal Flexure

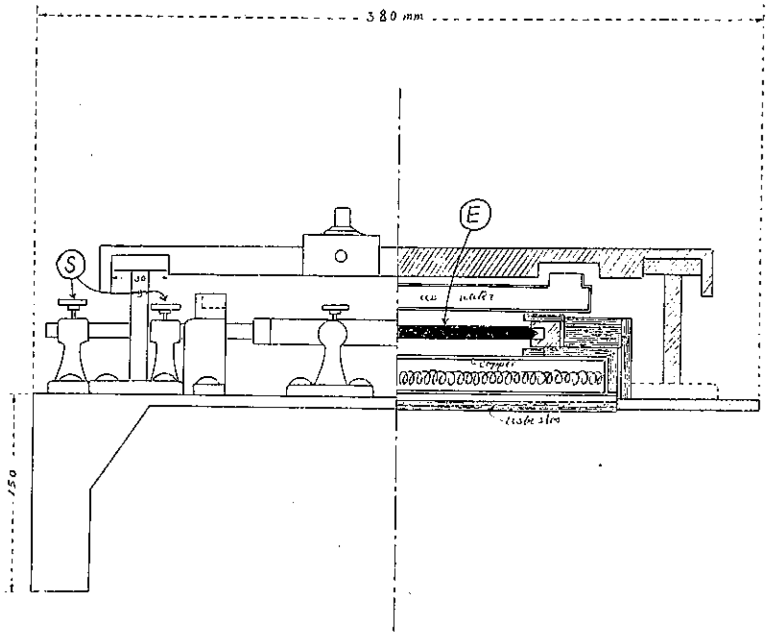
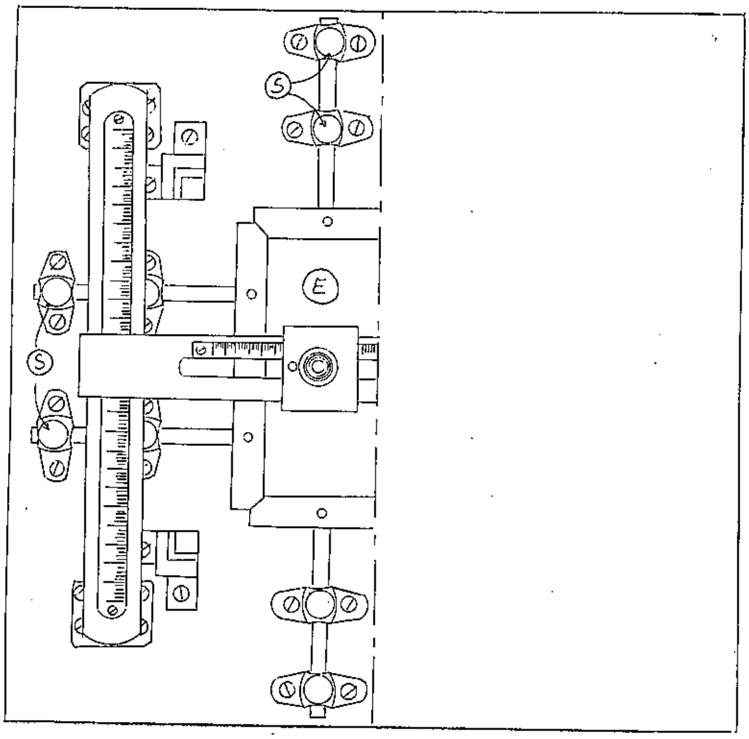


Fig. 2. Apparatus for Measuring Thermal Flexure



Pl. III.

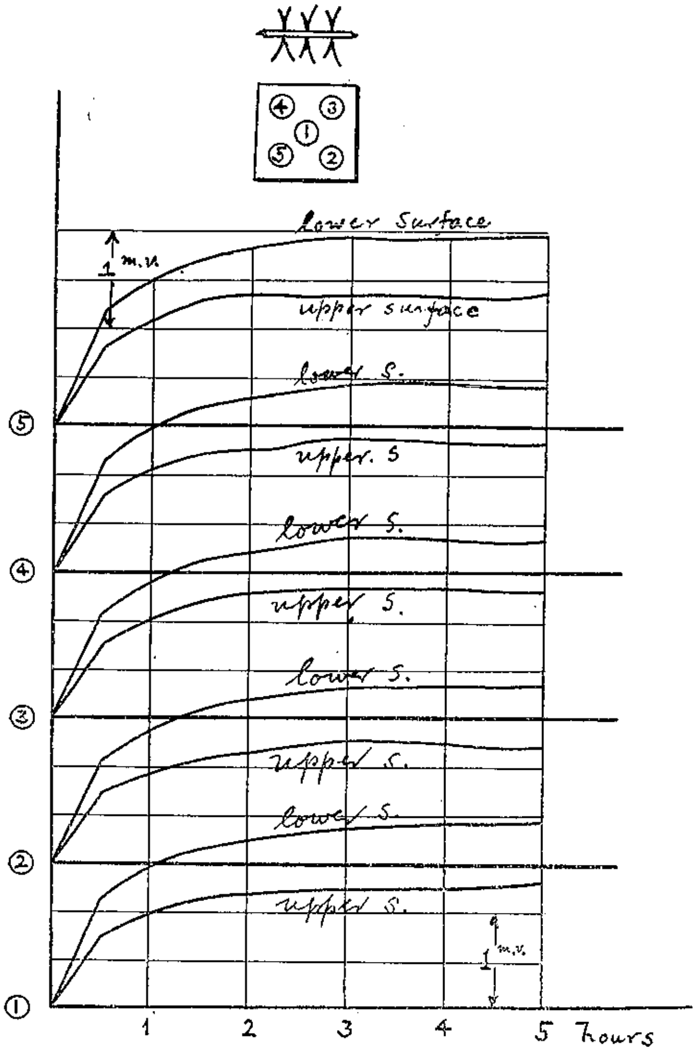


Fig. 1. Temperature-Time Curves (Ordinates show E. M. F. generated)

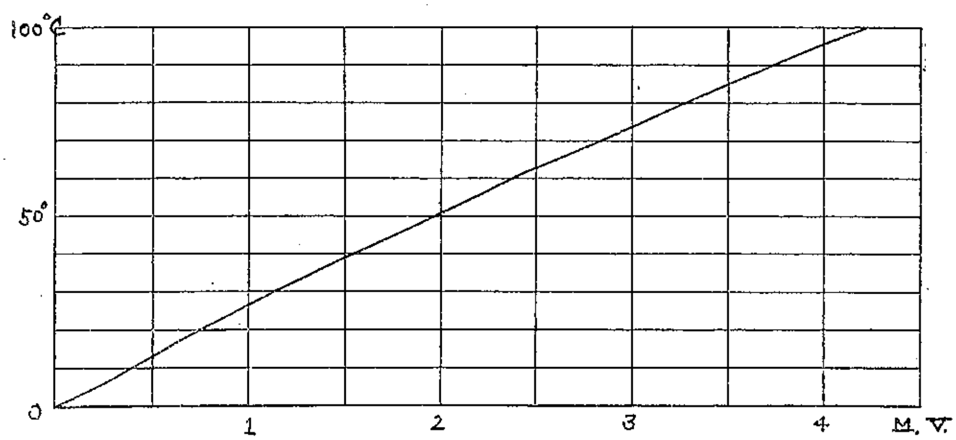


Fig. 2. Characteristic of Thermo-junction

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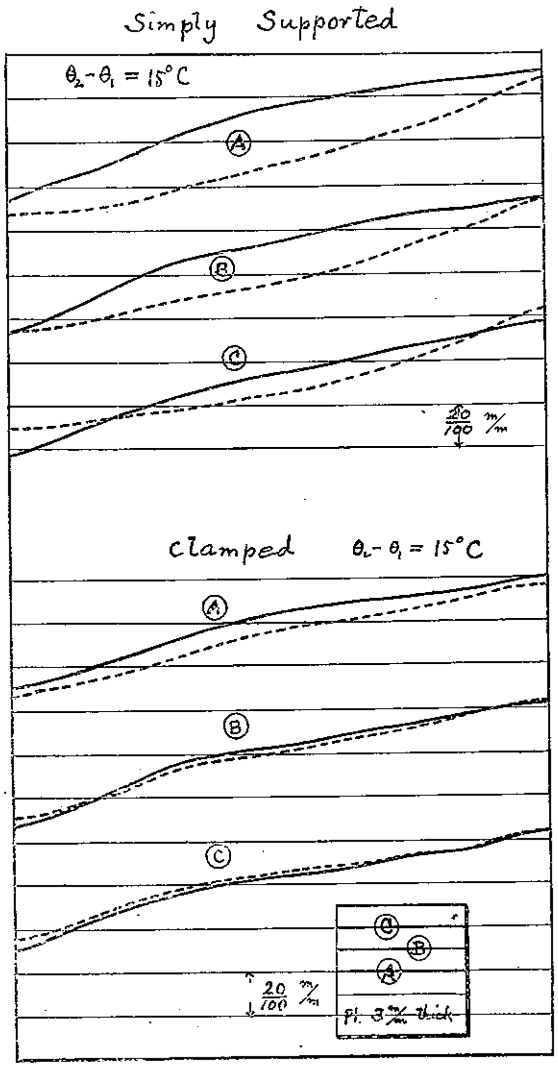
Pl. IV.

Comparison of Deflections of 2 different modes of Supports



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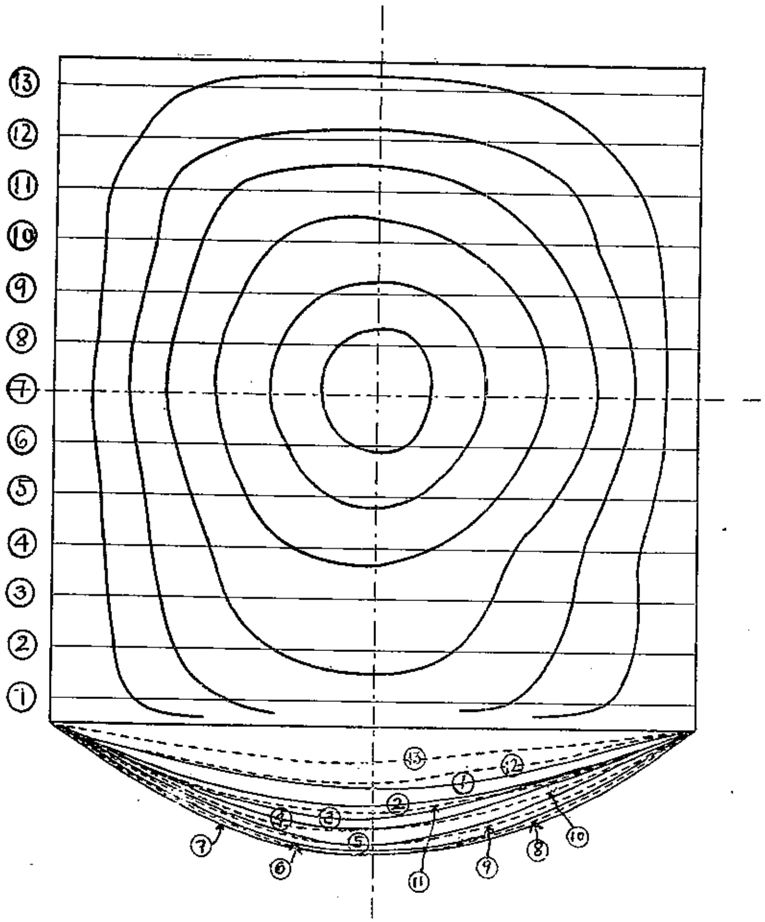
Pl. V.

A Contour Map of Thermal Deflection



Pl. VI.

Fig. 1. Comparative View of Thermal Deflection of C. L. of Plate at Various Stages

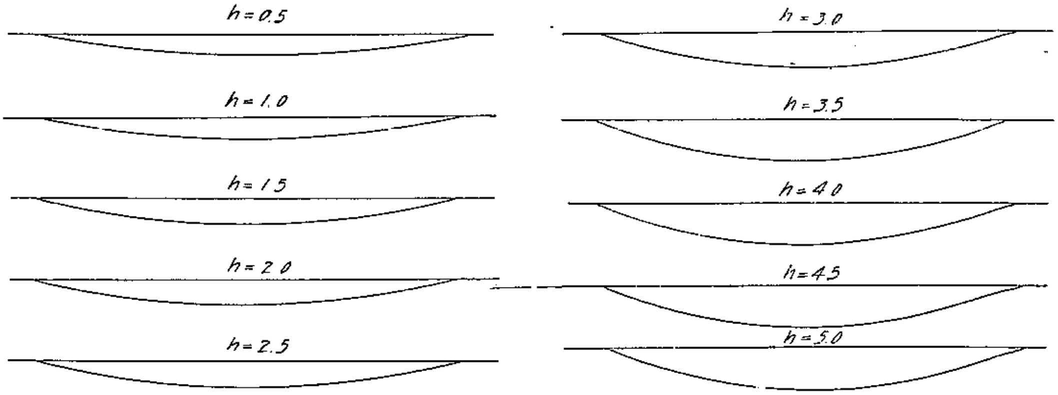
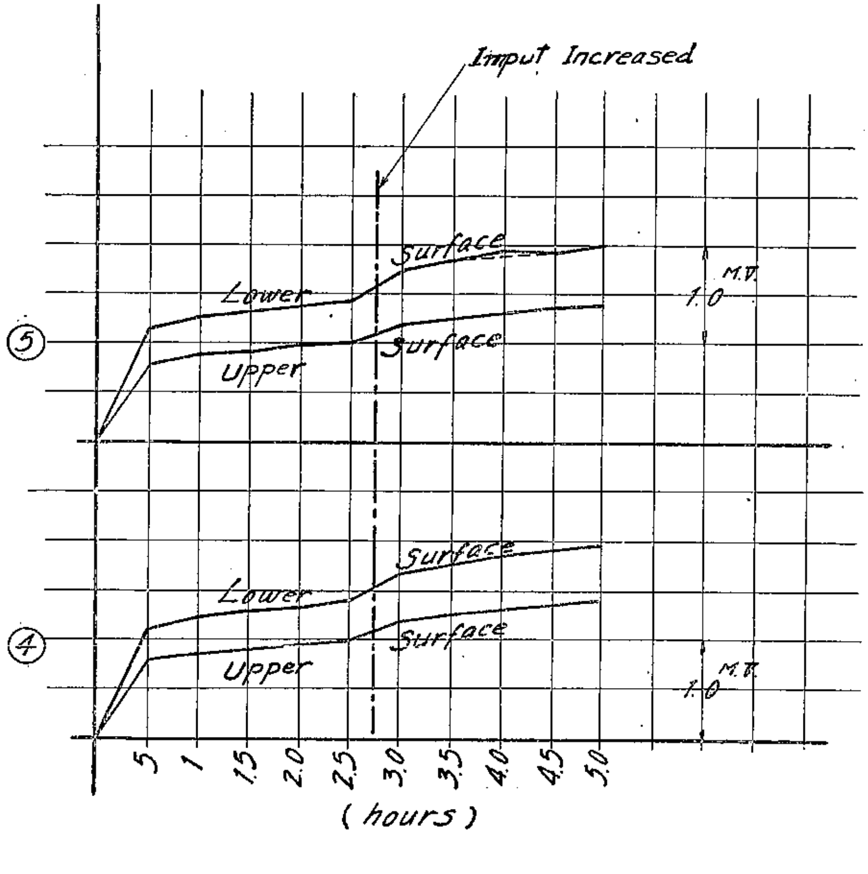


Fig. 2. Temperature-Time Curves (Ordinates show E. M. F.)



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