# 論 說 報 告

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# ON THE SPIRALLED AND RODDED CONCRETE COLUMN

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## Synopsis.

In this paper the subject is divided into five sections.

In section I, the brief descriptions are given on the construction of the spiralled concrete column, spiralled and rodded concrete column, and also on the execution of these columns.

In section II, the deformations within the limit of working stresses are fully treated of.

The correct methods and formulae for the calculations of strength of the column are given. They will enable us to find the exact stresses and strains in concrete and in steel.

Questions about the deformation in each material are clearly solved in the section.

The merit and efficiency of the spiral and rod in the column are fully discussed in this section.

In section III, the right and correct methods for the calculation of the ultimate strength and ultimate load—these being the foundamental solution for strength and load on the column—are proposed for the column from the mathematical and mechanical point of view.

Out of the experimental results given by Prof. Mörsch and the writer's equations, the factors determining the increase of strength in concrete due to spiral are obtained.

The merit and efficiency of the spiral in the column for the ultimate strength and ultimate load are fully explained for various degrees of strength of concrete.

The relations between the increase of strength of concrete and its increase due to spiral are clearly shown in the section.

Lastly the writer proceeds to the calculations of the working load by the ultimate load formulae thus obtained.

In section IV, the writer shows many formulae which have been used or are now employed in various countries. These formulae are compared with those obtained in section III, and critical judgment is given upon the result.

Section V is entirely the résumé of the discussions in this paper. Here some important new facts and correct formulae are pointed out for the observations and calculations of strength in the said column.

## Contents.



## Notations used.

$A$ =total cross section area.

$A_c$ =area of concrete in cross section.

$A_k$ =core concrete area.

$A_s$ =cross section area of rods$=A-A_c$

or core area$-A_k$; $p=\frac{A_s}{A_k}$ or $p=\frac{A_s}{A_c}$

$A'_s$ =area of equivalent longitudinal rod which has same volume or weight with spiral in unit length of column.

$$=\frac{\pi d^2}{4}\pi D\frac{1}{s}=\frac{\pi^2 d^2 D}{4s}=\frac{\pi a D}{s}$$

$E_s$ =modulus of elasticity of steel.

$E_c$ =modulus of elasticity of concrete in compression.

$n$ =modular ratio $=\frac{E_s}{E_c}$

$f_u$ =ultimate strength of plain concrete (per unit area).

$f_s$ =stress in steel rod (per unit area).

$f_{s.t.}$=tensile stress in spiral (per unit area).

$f_c$ =working stress in concrete (per unit area).

$f$ =longitudinal stress in general (per unit area).

$f_o$ =increased strength in concrete due to spiral (per unit area).

$f_z$ =shearing strength of concrete (per unit area).

$P$ =total ultimate load.

$P_{safe}$=total safe or working load.

$V'$ =volume of spiral per unit length of column.

$V$ =volume of concrete core per unit length of column.

$v = \frac{V^s}{V} = \frac{A'_s}{A_k}$ = volume ratio of spiral and core concrete per unit length of column.

$= \frac{4a}{Ds}$

$s$ = pitch of spiral.

$D$ = diameter of core.

$d$ = diameter of spiral wire.

$a$ = cross section area of spiral wire.

$= \frac{\pi d^2}{4}$

$q$ = radial stress produced by the spiral (per unit area).
or radial stress acting to the spiral ring due to the axial pressure $f$ (per unit area).

$\frac{1}{\sigma}$ = Poisson's ratio.

$\phi$ = frictional angle of granular non-coherent material.

## Section I. General construction of the Column.

Spiralled concrete column is made by inclosing the cylindrical concrete core with helical steel wires or coils. But practical considerations lead to the addition of longitudinal rods, and which is named here spiralled and rodded concrete column. The pitch between the wires should not exceed 12～16 times the diameter of the least longitudinal rod and should be small enough to resist the lateral expansion of the core.

The least diameter of the helical coil $= 4^{mm.}$ and there should be at least six longitudinal rods of diameter of $1.3 \sim 5.^{cm.}$

The ends of the column are said to be fixed when the axis at the ends remains in its original position and direction under all loads, and this would appear when the ends are sufficiently secured to other parts of the construction. Fixing the ends will produce a stiffer and consequently stronger column.

Concrete should be deposited in such a manner as will permit the most thorough homogeneity. The placing of concrete should be continuous and not suspended.

Before depositing concrete, the reinforcement should be carefully placed in accordance with the designs. It is essential that moderate means be provided to hold it in its proper position until the concrete has been deposited.

The forms should be substantial and the inside space free from debris or dust.

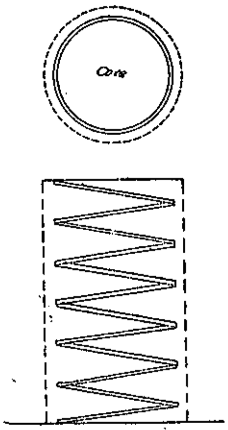


Fig. 1.
Spiralled Concrete column

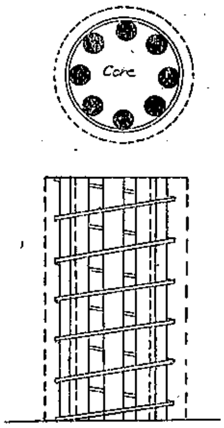



Fig. 2. Spiralled and rodded Concrete column

## Section II. Formulae for working stress.

Since the spiral prevents the lateral expansion of the concrete core,

$$\frac{f-f_o}{E_c}=\frac{f}{E_c}-\frac{2q}{\sigma E_c} \quad \cdots\cdots\cdots\cdots (1)$$

$$\therefore \quad f_o=\frac{2q}{\sigma} \quad \cdots\cdots\cdots\cdots (2)$$

which is a longitudinal stress increased by the spiral.

The strength of the spiralled concrete core is, therefore,

$$f=f_c+\frac{2q}{\sigma} \quad \cdots\cdots\cdots\cdots (3)$$

Let $\Delta l_1$ = longitudinal strain of plain concrete column.

$\Delta l_2$ = longitudinal strain of rodded concrete column.

Then we have $\Delta l_1 = k\Delta l_2$

where $k=(1+np)$

The strength of the spiralled and rodded concrete core is therefore,

$$f=k\left(f_c+\frac{2q}{\sigma}\right) \quad \cdots\cdots\cdots\cdots (4)$$

and applying the two relations, $\left(v=\frac{4a}{Ds}\text{ and }q=\frac{2f_{s.t.}a}{Ds}\right)$, we have

$$q=\frac{f_{s.t.}}{2}v \quad \cdots\cdots (5)$$

Therefore equations (3) and (4) are respectively

$$\left.\begin{aligned} f&=f_c+\frac{f_{s.t.}}{\sigma}v \\ f&=k\left(f_c+\frac{f_{s.t.}}{\sigma}v\right) \end{aligned}\right\} \quad \cdots\cdots (6)$$

From the potenzgesetz $\lambda=\frac{f_c^x}{E_c}$, we may put $x=1$ within the limit of working stress and therefore $\lambda=\frac{f_c}{E_c}$.

Since the shortening of the concrete and the rod is equal, we have $f_s=nf_c$; and

$$\frac{\text{lateral strain}}{\text{longitudinal strain}}=\frac{1}{\sigma}$$

Therefore from the longitudinal strain, the stress in the rod = longitudinal strain $\times E_s$, the stress in the spiral = lateral strain $\times E_s$. Poisson's ratio $\frac{1}{\sigma}$ is constant for the perfect fluid and its maximum possible value is $\frac{1}{2}$. The ratio can not be greater than $\frac{1}{2}$, otherwise a material could increase in volume due to compression, which is not the case. For ordinary metals, such as cast iron, wrought iron and steel, the ratio is $\frac{1}{3}\sim\frac{1}{4}$.

In the case of the concrete, the ratio has a dependance on the strength of the concrete.

In the discussion of the concrete, it was often taken as $\frac{1}{4}$, but this would evidently be in error. Greater brittleness and less elasticity seldom appear in the better concrete; 1 : 2 : 4 or better proportions are used for the concrete with reinforcement. The first effect of applying a pressure to the concrete column would probably be to decrease the volume in the direction of the pressure with little or no lateral expansion. Compared with steel, concrete lacks in density having small voids amounting sometimes to 20% and is granular in nature with probably a state of tension existing in the mass of cement, this being due to the shrinkage in setting.

If there is little or no lateral expansion in the first stage of applying the pressure, the ratio $\frac{1}{\sigma}$ would be expected to be zero or $\sigma=\infty$ and to increase as the pressure increases. This does not occur regularly for all concretes.

Concrete is deformed irregularly by the pressure until the certain pressure is reached. Up to this limit, concrete adjusts itself to the pressure, the small voids being filled up and its compactness being increased, and all the initial stresses are relieved. At about $10kg/cm^2 \sim 30kg/cm^2$, the irregularity of defomation is largely eliminated and from this point up to a certain higher stress, the limit of which lies well beyond the ordinary working stresses, the mass is thoroughly compacted and the deformation obeys a definite law ranging to $\frac{1}{\sigma}=\frac{1}{5}\sim\frac{1}{9}$ For the greater load, $\frac{1}{\sigma}$ is greater and would be equal ultimately to about $\frac{1}{5}$.

For the stress within the limit of working load or for one-fourth of the ultimate, $\frac{1}{\sigma}=\frac{1}{7}\sim\frac{1}{9}$ will be correct (E. Probst;—Vorlesungen über Eisenbeton).

In the better concrete which is less brittle and more elastic, the deformation obeys the definite law more quickly than in the inferior kinds. At the certain higher limit of stress the destruction of the bond in the concrete begins and the deformation would become irregular again.

It is noteworthy that the quantity is made greater by the smaller pitch of the spirals. In other words, the smaller the pitch the smaller the deformation of the concrete.

When the pitch is equal to zero, the limiting case happens, and the greater the strength of the mantel or the cylinder, the less the lateral strain of the concrete core.

In this paper, Poisson's ratio for working stress or one-fourth the ultimate is taken to be $\frac{1}{7}\sim\frac{1}{9}$

Since diametrical and circumferential strains are equal, the equation of condition

$$\frac{2q}{v}+qn-\frac{1}{\sigma}qn-\frac{1}{\sigma}fn=0$$

will hold for spiralled concrete core, from which we have

$$q=\frac{\frac{1}{\sigma}fnv}{2+nv\frac{\sigma-1}{\sigma}} \qquad \text{(7)}$$

and

$$f_{s.t.}=\frac{\frac{2}{\sigma}fn}{2+nv\frac{\sigma-1}{\sigma}} \qquad \text{(8)}$$

Equations (7) and (8) indicate the theoretical relation between the longitudinal stress $f$ and the radial stress $q$ or the hoop stress $f_{s.t.}$ respectively,

If $n=15,\ \frac{1}{\sigma}=\frac{1}{7},$

then $q=\frac{2.14fv}{2+12.86v}$

or if $n=10$, then $q=\frac{1.43fv}{2+8.57v}$

Further if $v=0.02$, then

for $n=15,\quad q=\frac{f}{50}$ or $f_{s.t.}=2f$

strain in spiral $=\frac{2f}{2,100,000}$

for $n=10,\quad q=\frac{f}{76}$ or $f_{s.t.}=1.4f$

strain in spiral $=\frac{1.4f}{2,100,000}$

Combining equation (3) and (7), we have

$$f=\frac{f_c}{1-\frac{2\left(\frac{1}{\sigma}nv\right)}{\sigma\left(2+nv\frac{\sigma-1}{\sigma}\right)}} \qquad \text{(9)}$$

This is the theoretical formula for the strength of the spiralled concrete core. The maximum possible value of $f$ is,

when $\frac{1}{\sigma}=\frac{1}{2}$ and $n=15$,

$$f=\frac{f_c}{1-\frac{15v}{4+15v}} \qquad (10)$$

Also when $n=10$,

$$f=\frac{f_c}{1-\frac{10v}{4+10v}} \qquad (10\,a,)$$

Since

$$\frac{15v}{4+15v}>\frac{10v}{4+10v},$$

$$f \text{ equation } (10) > f \text{ equation } (10,\ a)$$

The values of

$$\frac{2\left(\frac{1}{\sigma}-nv\right)}{\sigma\left(2+nv\frac{\sigma-1}{\sigma}\right)}$$

$n=15$ (maximum $f$)

| $v$ | $\frac{1}{\sigma}=0.2$ | $\frac{1}{\sigma}=0.1$ |
|---|---|---|
| 0.005 | 0.003 | 0.001 |
| 0.010 | 0.006 | 0.001 |
| 0.015 | 0.008 | 0.002 |
| 0.020 | 0.011 | 0.003 |
| 0.025 | 0.013 | 0.003 |
| 0.030 | 0.015 | 0.004 |
| 0.035 | 0.017 | 0.004 |
| 0.040 | 0.019 | 0.005 |

Combining again equations (3) and (7), we have

$$q=\frac{\sigma n f_c v}{\sigma^2\left(2+nv\frac{\sigma-1}{\sigma}\right)-2nv}$$

and again from equation (3), we have

$$f=f_c\left(1+\frac{2nv}{2\sigma^2+nv(\sigma-2)(\sigma+1)}\right) \qquad (11)$$

This is another form of the formula for the strength of the spiralled concrete core. The maximum possible value of $f$ is,

when $\frac{1}{\sigma}=\frac{1}{2}$ and $n=15$,

$$f=f_c(1+3.75v) \qquad (12)$$

also when $n=10$,

$$f=f_c(1+2.5v) \qquad (12\,a,)$$

Thus two forms of equation (9) are (11) are obtained.

The values of $f$ for various values of $\frac{1}{\sigma}$.

| $\frac{1}{\sigma}$ | $n=10$ Values of $f$. | $n=15$. Values of $f$. |
|---|---|---|
| $\frac{1}{5}$ | $f=f_c\left(1+\frac{20v}{50+180v}\right)$ | $f=f_c\left(1+\frac{30v}{50+270v}\right)$ |
| $\frac{1}{6}$ | $f=f_c\left(1+\frac{20v}{72+280v}\right)$ | $f=f_c\left(1+\frac{30v}{72+420v}\right)$ |
| $\frac{1}{7}$ | $f=f_c\left(1+\frac{20v}{98+400v}\right)$ | $f=f_c\left(1+\frac{30v}{98+600v}\right)$ |
| $\frac{1}{8}$ | $f=f_c\left(1+\frac{20v}{128+540v}\right)$ | $f=f_c\left(1+\frac{30v}{128+810v}\right)$ |
| $\frac{1}{9}$ | $f=f_c\left(1+\frac{20v}{162+700v}\right)$ | $f=f_c\left(1+\frac{30v}{162+1050v}\right)$ |
| $\frac{1}{10}$ | $f=f_c\left(1+\frac{20v}{200+880v}\right)$ | $f=f_c\left(1+\frac{30v}{200+1320v}\right)$ |

Average from $\frac{1}{\sigma}=\frac{1}{5}\sim\frac{1}{10}$

$$f=f_c\left(1+\frac{20v}{118+497v}\right) \quad \text{when} \quad n=10. \qquad (13)$$

$$f=f_c\left(1+\frac{30v}{118+745v}\right) \quad \text{when} \quad n=15. \qquad (13a,)$$

From equation (12), the maximum possible increase of strength due to spiral is, when $v=0.02$, 7.5% while for ordinary cases $\frac{1}{\sigma}=\frac{1}{7}$ and therefore it is only 0.5%. When $n=10$ this is only 0.4%. The total working load for the spiralled and rodded concrete core is:—

$$\left.\begin{aligned} P_{safe}&=\frac{f_c}{1-\dfrac{2\left(\dfrac{1}{\sigma}nv\right)}{\sigma\left(2+nv\dfrac{\sigma-1}{\sigma}\right)}}kA_k \\ \text{or} \quad P_{safe}&=\left(1+\frac{2nv}{2\sigma^2+nv(\sigma-2)(\sigma+1)}\right)f_ckA_k \end{aligned}\right\} \qquad (14)$$

$$K=(1+nq)$$

As has been shown the spiral practically does not increase the strength of the concrete core within the limit of working stresses. Since this is concerned, the total concrete area may be used in computing the working load with the above formulae. Take, as an example, equation (13*a*), then

$$P_{safe}=f_c\left(1+\frac{30v}{118+745v}\right)(A_c+nA_s) \quad \cdots\cdots (15)$$

The term $\frac{30v}{118+745v}$ is negligible for the working load and we have

$$P_{safe}=f_c(A_c+nA_s)$$

The above equations serve to prove mathematically that the spiral practically does not increase the strength of the concrete core within the limit of working stresses.

The equation $f-f_c=0$ is practically true within these limits of loading.

If $\frac{1}{N}$ of the lateral expansion of the concrete core is restrained by the spiral, the equation of codition is

$$\frac{f}{N\sigma E_c}-\frac{q}{E_c}+\frac{q}{\sigma E_c}=0$$

$$\therefore \quad \left.\begin{aligned} q&=\frac{f}{N(\sigma-1)} \text{ for the spiralled core.} \\ q&=\frac{f}{Nk(\sigma-1)} \text{ for the spiralled and rodded core.}\end{aligned}\right\} \cdots\cdots (16)$$

Substituting one of these two in the equation

(1)., we have $$S=\frac{f}{E_c}\,\frac{N\sigma(\sigma-1)-2}{N\sigma(\sigma-1)} \cdots\cdots (17)$$

$S$ is the longitudinal strain when the lateral strain is restrained by the spiral by an amount equal to $\frac{1}{N}$.

The increase of the modulus of elasticity of concrete is equal to

$$\frac{N\sigma(\sigma-1)}{N\sigma(\sigma-1)-2} \cdots\cdots (18)$$

for the spiralled concrete core. From equations (16) and (7) we have

$$N=\frac{2\sigma+nv(\sigma-1)}{nv(\sigma-1)} \cdots\cdots (19)$$

$N$ depends on $\sigma$ and $v$. If $N=1$, then $2\sigma=0$ which is impossible, and the lateral strain cannot be all restrained by the spiral. For the spiralled and rodded concrete core, the increase of modulus of elasticity is

$$\frac{kN\sigma(\sigma-1)}{N\sigma(\sigma-1)-2} \quad \ldots\ldots\ldots\ldots (20)$$

and for $N$ it is the same as equation (19). If $n=15$, $v=0.02$ and $\sigma=7$, then $N=8.8$ and the lateral strain is restrained by an amount equal to $\frac{1}{8.8}$ and the corresponding increase of modulus of elasticity of concrete is, by equation (18), 0.5%. When $n=10$, this is respectively $\frac{1}{12.7}$ and 0.4%.

For the spiralled and rodded concrete core, the increase of the latter is, when $p=0.01\sim0.04$, $15\%\sim60\%$.

Spirals have little or no effect within the limit of working stresses while the rods are very effective within these stresses.

The increase in the strength and the modulus of elasticity due to the spiral is very small within the limit of the working stresses. These facts are more or less known from experiments; but owing the experiments relating only to the ultimate loading, the above facts have not been well explained for the working stresses.

The above discussions would be quite enough to solve the problems by mathematical and rational means.

All problems occurring within the limit of working stresses should be calculated by the above formulae.

The writer has given here the mathematical basis for the calculations of the working stresses.

## Section III. Formulae for ultimate strength.

In the ultimate load, the internal stress in the concrete exceeds the endurance limit and the assumptions made in section II cannot be applied.

### 1. Ultimate strength and ultimate load.

When the granular materials, such as, concrete is stressed to the ultimate, its failure occurs through the shear along the plane which makes a certain definite angle $\theta$ with the horizontal. The resistance to the failure is the sum of the strength of the concrete to resist shearing and of the frictional resistance to motion along this plane. At the instant of motion, the sum of these two resistances must equal to the shearing component of the pressure imposed when resolved along the shearing plane.

Let $f$ be the nominal stress and considering the plane making an angle $\theta$ with the horizontal, we have

$$\text{normal component} = f\cos^2\theta$$
$$\text{tangential component} = f\sin\theta\cos\theta$$

The tangential component is greatest when $\theta=45°$ and its maximum value is $\frac{f}{2}$. It has been considered that the plane making an angle $\theta=45°$ with the horizontal is hasthe plane of rupture. Coulomb, Rankine and others are responsible for such an assumption.

The actual plane of rupture, when the specimens have sufficient height, is the plane making a greater angle than 45° due to the normal component.

Consider a concrete core enclosed by the spirals and also the plane making an angle $\theta$ with horizontal, and assume $f>q$, then we have ;—

Resultant normal stress

$$= f\cos^2\theta + q\sin^2\theta$$

Resultant tangential stress

$$=(f-q)\cos\theta\sin\theta.$$

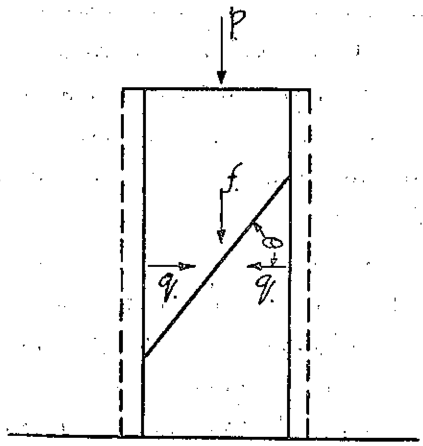



Fig. 3.

At the instant of failure, the condition

$$f=\frac{q\sin\theta(\mu\sin\theta+\cos\theta)}{\cos\theta(\sin\theta-\mu\cos\theta)}+\frac{f_z}{\cos\theta(\sin\theta-\mu\cos\theta)} \quad \cdots (21)$$

will exist. The second term is evidently equal to the ultimate stress $f_u$ of concrete.

$$\therefore \quad f=\frac{q\sin\theta(\mu\sin\theta+\cos\theta)}{\cos\theta(\sin\theta-\mu\cos\theta)}+f_u \quad \cdots\cdots (22)$$

and

$$f_0=\frac{q\sin\theta(\mu\sin\theta+\cos\theta)}{\cos\theta(\sin\theta-\mu\cos\theta)} \quad \cdots\cdots (23)$$

Since $q=\frac{f_{s.y.}}{2}v$ holds at the instant when the failure occurs, we have

$$f_0=\frac{f_{s.y.}}{2}v\frac{\sin\theta(\mu\sin\theta+\cos\theta)}{\cos\theta(\sin\theta-\mu\cos\theta)} \quad \cdots\cdots (24)$$

Thus the limit of ultimate strength is raised and is perceptible for all percentages of spiral. From equation (23), we have

$$\mu=\tan\phi=\frac{(f_o-q)\cos\theta\sin\theta}{f_o\cos^2\theta+q\sin^2\theta} \quad \cdots\cdots\cdots (25)$$

The plane on which the tendency to fail is greatest is obtained from the condition

$$\frac{d(\tan\phi)}{d\theta}=0.$$

Therefore, differentiating and eliminating the common factor, we have

$$\theta=45°+\frac{\phi}{2}.$$

Applying $\theta=45°+\frac{\phi}{2}$ in equation (25), we have

$$\frac{f_o}{q}=\frac{1+\sin\phi}{1-\sin\phi} \quad \cdots\cdots\cdots (26)$$

or

$$f_o=q\frac{1+\sin\phi}{1-\sin\phi}=\frac{f_{s.t.}}{2}v\frac{1+\sin\phi}{1-\sin\phi} \quad \cdots\cdots\cdots (27)$$

and

$$\sin\phi=\frac{f_o-q}{f_o+q}=\frac{2f_o-f_{s.t.}v}{2f_o+f_{s.t.}v} \quad \cdots\cdots\cdots (28)$$

Applying the condition $f_{s.t.}=n'f_{c.t.}$, we have also

$$q=\frac{n'f_{c.t.}}{2}v \quad \cdots\cdots\cdots (29)$$

where $f_{c.t.}$ = tension in concrete on contact face with spiral resulting from the expansion of the concrete core.

$n'$ = modular ratio of steel and concrete in tension.

As has been explained already by equation (2), $f_o=\frac{2q}{\sigma}$ is the increased strength due to the spiral within the limit of working stresses. While for the instant of column failure, the increase is $f_o=q\frac{1+\sin\phi}{1-\sin\phi}$. This indicates that the strength of the concrete core is greatly increased by the spiral for the ultimate loading compared with that of the working stresses.

The theoretical angle of rupture is greater than 45°, and for concrete $\theta=66°30'\sim67°30'$ and its fair average will be about $\theta=66°49'$ when $f_{s.t.}=2{,}800$ kg/cm²

Since the theoretical angle of rupture $\theta$ is greater than 45° and approaches 70° according to the value of $f_{s.t.}$ it is evident that the height of the specimens should be at least 1.5 times the least lateral dimension in order to offer an opportunity for failure on the theoretical angle.

This is a very important matter as Prof. J. Bauschinger has already studied this question.

From the above theoretical treatment the ultimate strength of the spiralled concrete core is, in the granular non-coherent condition,

$$\left.\begin{aligned} f&=f_u+q\frac{1+\sin\phi}{1-\sin\phi} \\ &=f_u+\frac{f_{s.t.}}{2}v\frac{1+\sin\phi}{1-\sin\phi} \end{aligned}\right\} \quad \cdots\cdots (30)$$

Prof. E. Mörsch states that the increased strength of the concrete core due to the spiral is $Mf_uA_s'$ and he found that $M$ varies as the strength of concrete $f_u$. His formula was

$$P=f_eA_s+A_k(1+Mv)f_u \quad \cdots\cdots (31)$$

and for $M$,

$$M=\frac{1}{v}\left\{\frac{P-f_eA_s}{f_uA_k}-1\right\} \quad \cdots\cdots (32)$$

where $f_e$=limit of stress in steel.

From equation (32), Prof. Mörsch found the factor $M$ for various cases of the spiralled and rodded column from the experiments.

When hoops are used, then $M=0$ and we have

$$P=f_eA_s+f_uA_c$$

Here the total concrete area $A_c$ is used instead of the core area $A_k$.

For the spiralled round core, the factor $M=53\sim57$ and for the spiralled square shaped core $M=19\sim62$. (E. Mörsch :— Die Eisenbetonbau). Thus there is a wide range of $M$ and when $M$ is assumed from the Mörsch experiment, the general exactness of the above equation cannot be ascertained. But from various experiments he concluded that the greater the unit strength of the concrete the less the value of $M$ and gave the following numbers.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $f_u kg/cm^2 1=20$ | | 140 | 160 | 180 | 200 | 220 | 240 |
| $M=$ | 71 | 59 | 50 | 43 | 38 | 34 | 31 |
| $Mf_u kg/cm^2$ | =8,520 | 8,260 | 8,000 | 7,740 | 7,600 | 7,480 | 7,440 |

He also states that the average value of $M$ is 45, and the greater the $f_u$ the smaller the $M$. The German regulation of 1916 will be obtained if $M=45$ is taken as further seen.

Thus Prof. E. Mörsch gave the experimental results. But no rational treatment has been propounded by him or by other writers.

As the writer has shown, the theoretical increase of strength due to spiral being given by equation (30), we have

$$(1+Mv)=\left(1+\frac{f_{s.t.}}{2f_u}v\frac{1+\sin\phi}{1-\sin\phi}\right)$$

and this is the fundamental equation showing the increased strength of concrete due to spiral. Also we have

$$M=\frac{f_{s.t.}}{2f_u}\cdot\frac{1+\sin\phi}{1-\sin\phi} \quad \ldots\ldots\ldots\ldots (33)$$

or

$$\sin\phi=\frac{2f_uM-f_{s.t.}}{f_{s.t.}+2f_uM} \quad \ldots\ldots\ldots\ldots (34)$$

By Prof. Mörsch's experiment, $\phi$ can be traced for each strength $f_u$ of concrete from writer's equation (34), which gives in turn the corresponding values for $\phi$ and $f_u$ ; $f_{s.t.}$ being the hoop stress at the instant of failure of concrete is taken here to be the yield point of steel, that is, $f_{s.t.}=2{,}400\ kg/\overline{cm}^2 \sim 2{,}800\ kg/\overline{cm}^2$. Professors Withey and Talbot give values of the ultimate lateral strain for a spirally reinforcedconcrete column, that lie between the limits of 0.0006 and 0.003. The greatest lateral strain is well before or at the end of the yield point elongation of a steel wire. The lateral strains due to the failure of the concrete core are equal to the strains in the steel wire. This shows that the spiral is stressed most to the yielding point when column failure or ultimate loading has occurred. The failure of the core is due to the failure of the concrete and not to the failure of thes spirals. When the failure of the concrete occurs, the spiral does not suffice to carry the load with its lateral support. If there were a greater quantity of steel, the failure of concrete does not produce the failure of the column because the spiral acting on the concrete core suffices to support the load; the ultimate strength is simply the amount which can be carried by the granular non-coherent core supported by the spiral.

However if one of the two materials, concrete or spiral, fails that is evidently the failure of the column and the column is useless. Whether the ultimate strength of the column is determined by the ultimate strength of steel or its yield point, is debatable, but latter appears to be more rational.

The writer recommends the modular ratio $n=11$ for concrete of an age of 70 days or more; and $n=10$ will be moderate for ordinary working stresses. But with the increasing stress in concrete $n$ becomes greater. $n=15$ when $E_c=14,0000$ $kg/cm^2$ and $n=20$ when $E_c=105,000$ $kg/cm^2$

The last examples are in the zone of ultimate loading. The modular ratio $n=15$ is assumed for ultimate loading as usual.

From equation (34) and $n=15$, we have the following numbers.

| $f_u kg/cm^2$ | $f_{s.t.}=2,800\ kg/cm^2$ | | $f_{s.t.}=2,400 kg/cm^2$ | |
|---|---|---|---|---|
| | $\phi$ equation 34. | $f_o$ equation 27. | $\phi$ equation 34. | $f_o$ equation 27. |
| 120. | 45°.48′ | 6.1 $q$ | 49°.10′ | 7.2 $q$ |
| 140. | 45°.15′ | 5.9 $q$ | 48°.30′ | 6.9 $q$ |
| 160. | 44°.36′ | 5.7 $q$ | 47°.40′ | 6.7 $q$ |
| 180. | 43°.45′ | 5.5 $q$ | 47°.05′ | 6.5 $q$ |
| 200. | 43°.31′ | 5.4 $q$ | 46°.40′ | 6,3 $q$ |
| 220. | 43°.13′ | 5.3 $q$ | 46°.25′ | 6.2 $q$ |
| 240. | 43°.06′ | 5.3 $q$ | 46°.15′ | 6.2 $q$ |

From these numbers, the writer draws, for

$$f_{s.t.}=2,800\ kg/cm^2,\quad \frac{1+\sin\phi}{1-\sin\phi}=5.3$$

with consideration of safety and of simplicty of formula to be obtained. Then we have

$$\text{minimum}\quad f_o=5.3\,q=5.3\frac{f_{s.t.}}{2}v=7,400v.$$

$$\therefore\qquad f=f_u+f_o=f_u+7,400v=f_u\left(1+\frac{7,400}{f_u}v\right)\cdots\cdots(35)$$

and the minimum ultimate load is, until

$f_u=240\ kg/cm^2$ is reached,

$$\text{(minimum)}\qquad P=f_cA_s+A_k\left(1+\frac{7,400}{f_u}v\right)f_u\cdots\cdots(36)$$

where $f_c=nf_u$

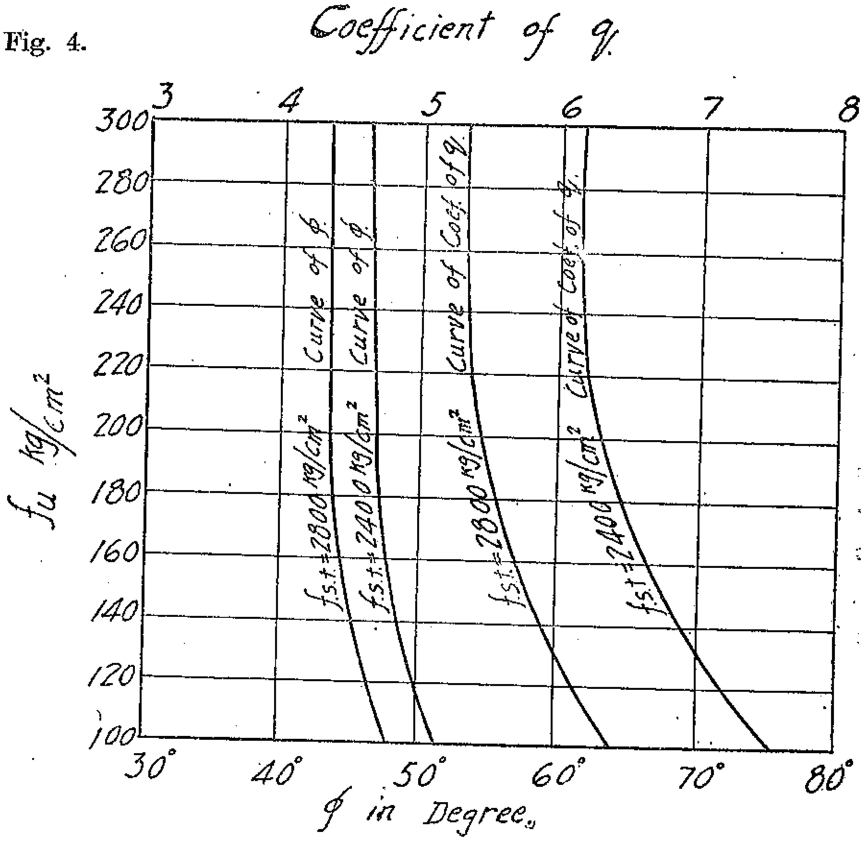
Since equation (34) is assumed to hold good, when $f_{s.t.}$ is greater, the angle $\phi$ is smaller and consequently $\dfrac{1+\sin\phi}{1-\sin\phi}$ is small, or vice versa; so small a variation in the angle of friction corresponds to so large a range of stress in the steel. As been shown above, the product

$$\frac{f_{s.t.}}{2} \cdot \frac{1+\sin\phi}{1-\sin\phi}$$

is constant for each strength of concrete and is independent whether $f_{s.t.} = 2{,}400$ $kg/\overline{cm}^2$

or $f_{s.t.} = 2{,}800 kg/\overline{cm}^2$ is taken.

Fig. 4.




The above table shows that the increase of strength in concrete due to spiral is greater for the weaker concrete with a given quantity of spiral, which indicates that the greater the ultimate strength of the concrete the less the merit of the spirals, Prof. Withy concludes from his experiments that concrete having a strength of 343 $kg/\overline{cm}^2$ or more will not have its strength noticeably increased by the spirals.

This means that $\frac{f_{s.t.}}{2} \cdot \frac{1+\sin\phi}{1-\sin\phi}$ is all constant with the greater strength of concrete. As the figure shows, $\phi$ is canstant and equal to about 43° for the strength greater than $f_u=240\ kg/\overline{cm}^2$ when $f_{s.t.}=2,800.\ kg/\overline{cm}^2$

Then
$$\frac{1+\sin 43^\circ}{1-\sin 43^\circ}=\frac{1+0.682}{1-0.682}=5.29$$

Therefore, for concrete having an ultimate strength greater than $240\ kg/\overline{cm}^2$, we have

$$f=f_u+f_o=f_u+7,400v$$

and the maximum ultimate load is, for $f_u>240\ kg/\overline{cm}^2$,

(maximum)
$$\left.\begin{aligned} P&=f_eA_s+f_uA_k+7,400A_s' \\ &=f_u(A_k+nA_s)+7,400A_s' \end{aligned}\right\} \qquad (37)$$

As equation (36) or (37) shows, the ultimate strength of the concrete core is increased with the greater quantity of spiral. But since the core is protected by the shell concrete, it should not be sheared off by the working load. From this point of view there is a certain limitation as is afterwards explained.

From the above the following conclusion is obtained :—

For a concrete having the ultimate strength less than about $240\ kg/\overline{cm}^2$, the minimum ultimate load formula should be equation (36). The strength to be increased by the spiral is the function of the ultimate strength of the concrete, and greater the strength of the concrete the less the merit of the spiral. However for the concrete having an ultimate strength greater than about $240\ kg/\overline{cm}^2$, the maximum ultimate load formula should be equation (37). Here the strength to be increased by the spiral is nearly independent of the ultimate strength of the concrete and is constant being equal to about 7,400 $A_s'$.

The ultimate strength is the function of age in the concrete. In an ordinary design, one month or three months of age is chosen, and the strength is assumed. The writer recommends a three months age and the values of $f_u$ in this paper are all referred to the concrete of an age of three months.

From equation (36), we have

$$\text{(minimum)}\ M=\frac{7{,}400}{f_u} \quad \cdots\cdots (38)$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $f_u kg/\overline{cm}^2=$ | 120 | 140 | 160 | 180 | 200 | 220 | 240 |
| minimum $M=$ | 62 | 53 | 46 | 41 | 37 | 34 | 31 |

Further from the condition that spiralled and rodded concrete core should have greater strength than rodded concrete column at the ultimate,

we have $$nf_uA_s+A_k(1+Mv)f_u \geqq f_u(A_c+nA_s)$$

$$\therefore \quad Mv \geqq \frac{1}{3} \quad \text{when} \quad A_c=\frac{4}{3}A_k$$

or $$\text{minimum} \quad v=\frac{f_u}{22{,}200} \quad \cdots\cdots (39)$$

for all percentages of rod. Therefore the minimum $M\ v$ is 0.33 for the ultimate loading.

No exact laws showing the relations between the increase of the ultimate strength of the concrete and the increase of strength due to spirals and rods are propounded. The formulae which have been proposed in various countries are all empirical. Investigators have assumed that the spiral adds a certain definite strengt to that of the concrete and all formulae relating to the strength of the core have been made without taking the strength of the concrete. In other words, investigators have assumed that spirals add a certain strength to the core irrespective of the strength of concrete. As shown by equation (30) the increases in strength of the concrete due to spial is

$$\frac{f_{s.t.}}{2}v\frac{1+\sin\phi}{1-\sin\phi}$$

and a small change in the angle of friction results in a change in the strength of the concrete.

The ultimate strength to be increased by spirals does not increase proportionately to the quantity of $v$ since the strength of the concrete is concerned.

From equation (33), we have

$$\left.\begin{aligned} f_{s.t.}&=2Mf_u\frac{1-\sin\phi}{1+\sin\phi} \\ \text{or} \quad q&=\frac{f_{s.t.}}{2}v=Mvf_u\frac{1-\sin\phi}{1+\sin\phi} \end{aligned}\right\} \cdots\cdots (40)$$

As already mentioned, consider the plane making an angle $\theta$ with the horizontal. The normal component$=f\cos^2\theta$ and the tangential component$=f\sin\theta$

$\cos\theta$. At the instant of failure, we have

$$f=\frac{f_z}{\cos\theta(\sin\theta-\mu\cos\theta)}$$

and the plane on which the tendency to fail is greatest is the plane making an angle $\theta=45°+\frac{\phi}{2}$ with the horizontal. Therefore the relations between the ultimate stress and the shearing strength in terms of $\theta$ and $\phi$ are respectively

$$\left.\begin{aligned} f&=2f_z\tan\theta \\ \text{and}\qquad f&=2f_z\frac{1+\sin\phi}{\cos\phi}\end{aligned}\right\}\cdots\cdots\cdots\cdots(41)$$

For the spiralled concrete core, we have

$$f_u\left(1+\frac{f_{s.t.}}{2f_u}v\frac{1+\sin\phi}{1-\sin\phi}\right)=2f_z\tan\theta\quad\cdots\cdots\cdots\cdots(42)$$

The increase of shearing strength of the concrete due to spiral can be found from the last equation.

## 2. Working load.

Concrete has no yield point. The concept of a yield point originated from steel. The properties of materials used in a monolithic structure may make it essential that a certain stress is not exceeded, which would place a limit on the working stress. There is a stress (as in steel), which is a certain definite percentage of the ultimate stress, an infinite number of repetitions of which will not cause failure, nor too large an increase in strain. These, then are definite criteria to be observed in concrete, instead of the so called yield point.

As to the concrete, Prof. Van Ornum states that, from his experiments, the endurance limit of concrete is 50% the ultimate stress. According to Van Ornum, 50% the ultimate stress is therefore the limit of stress in the working load calculations. If the strength of a concrete cube be 100, the strength of prism is $100\times0.8$ according to Prof. C. Bach.

The variation of strength of concrete is say 30%. From these considerations we have

$$\text{Factor of safety}\ \ F=\frac{100\times0.8-30}{2}=25\%.$$

Since the outside shell concrete supports the load within the limit of working conditions, we have

$$f_c(1+Mv)(A_k+nA_s)\leqq0.5f_uA_c\cdots\cdots\cdots\cdots(43)$$

Taking $4f_c=f_u$ and $A_c=\frac{4}{3}A_k$, we have

$$Mv\leqq\frac{8}{3(1+np)}-1$$

$$\therefore \qquad \text{maximum} \quad \left.\begin{aligned} v&=\frac{1}{M}\left(\frac{8}{3(1+np)}-1\right) \\ &=\frac{f_u}{7{,}400}\left(\frac{8}{3(1+np)}-1\right) \end{aligned}\right\} \quad \cdots\cdots (44)$$

Ordinary $f_u$ is less than $240 kg/cm^2$, therefore equation (44) is quite applicable to the conditions met in practice. From equation (38), factor $M$ has a definite constant value for each strength of concrete, and therefore from equation (44), $v$ should have a certain constant value for each strength of concrete for a given quantity of $p$. From the above considerations, the maximum safe load is

$$\text{maximum} \quad P_{safe}=\frac{8}{3}f_cA_k=2.7f_cA_k \quad \cdots\cdots (45)$$

The maximum value of $M\ v$ has a dependance on $P$ as shown in equation (44) and the greater the quantity of $P$ the less the quantity of $v$ in the given concrete. The quantity of $v$ should be selected, with accordance of $p$.

The strength of concrete is increased by the spiral by an amount equal to $(1+Mv)$, and from the point of view that the outside shell concrete should not fail under an endurance loading, the maximum $M\ v$ should be 1. 7.

From equation (35), we have

$$\frac{f}{f_u}-1=\frac{7{,}400}{f_u}v=Mv \quad \cdots\cdots (46)$$

and from equation (44), we have

$$\text{maximum} \quad f_{safe}=f_c\left\{1+\frac{1}{1+np}\left(\frac{5}{3}-np\right)\right\}=f_c(1+Mv) \quad \cdots (47)$$

The values of maximum $f_{safe} kg/cm^2$

| | | | | |
|---|---|---|---|---|
| $p=0.00$ | 0.01 | 0.02 | 0.03 | 0.04 |
| equation(47)$=2.7f_c$ | $2.32f_c$ | $2.05f_c$ | $1.84f_c$ | $1.7f_c$ |
| $Mv=1.7$ | 1.32 | 1.05 | 0.84 | 0.7 |

From the condition $nf=nf_c(1+Mv)\leqq 2{,}800\ kg/cm^2$, we have

$$\text{maximum } f_c = \frac{2,800}{n(1+Mv)} \quad \cdots\cdots\cdots (47,a)$$

$$\therefore \quad f_{safe} = \frac{2,800}{n(1+Mv)}\left\{1+\frac{1}{1+np}\left(\frac{5}{3}-np\right)\right\} = \frac{2,800}{n} \quad \cdots (48)$$

The values of maximum $f_c kg/cm^2$.

| | | | | | |
|---|---|---|---|---|---|
| $p=0.00$ | | 0.01 | 0.02 | 0.03 | 0.04 |
| equation (47 a) | = 70. | 80 | 91 | 101 | 112 |
| max $f_{safe}$ equation (48) | =187. | 187 | 187 | 187 | 187 |

Therefore beyond these stresses, the rod suffers in carrying the load and by the failure of the bond of cement the equation $p=f_c(1+Mv)\ (A_k+nA_s)$ does not hold true. Hence the ultimate load formula of the form of $p=f_u(1+Mv)\ (A_k+nA_s)$ is not correct and it does not show the true ultimate loading. Some observe $M=\frac{1}{v}\left(\frac{f}{f_u}-1\right)$, and although the formula $p=f_u(1+Mv)\ (A_k+nA_s)$ is stated, it does not well apply to the ultimate loading so long as

$$nf_u(1+Mv) > 2,800\ kg/cm^2.$$

$P_{safe}=f_c(1+Mv)(A_k+nA_s)$ is only the measure of the maximum safe load which lies under the endurance limit. As has been mentioned in Section II, the stress in concrete or in steel cannot be calculated by the last formula since $M$ is not so great as already denoted; $M=3.75$ is the greatest possible value for working conditions.

The maximum value of $M\ v$ should have a constant and equal quantity for all concretes, and only changes or varies with percentage of rod. Having dependance on $M\ v$ and $p$, the load may be raised from $f_cA_c=f_c\frac{4}{3}A_k=1.33f_cA_k$ to 2.7 $f_cA_k$ and the latter is the maximum safe load to prevent the shearing off of the outside shell. If $4f_c=f_u$, then 2.7 $f_cA_k=0.67f_uA_k$. But 2.7 $f_uA_k$ does not show thet heoretical ultimate load since $P=f_u(1+Mv)(A_k+nA_s)$ is not the true form of the formula for ultimate loading; 2.7 $f_uA_k$ is only true when $p=0$.

The stress in the steel rod should not be exceeded by the yield point. If the stress in the rod exceeds the yield point,

the equation $P=f(A_k+nA_s)$ does not hold good.

In the formula,

$$P = f_u(1+Mv)(A_k+nA_s),$$

$nf_u(1+Mv)$ exceeds the yield point of steel rod and the bond of concrete is not perfect and $nf=f_s$ does not exist. Therefore the formula is irrational and incorrect for finding the ultimate load.

While the formula $P=f_cA_s+A_k(1+Mv)f_u=nf_uA_s+A_k(1+Mv)f_u$ should be correct for the ultimate load since the concrete carries the load $=A_k(1+Mv)f_u$ and the rod carries the load $=f_uA_s$ in ultimate conditions. But the safe load $=nf_cA_s+A_k(1+Mv)f_c$ does not the correct form, and since the bond is perfect the safe load formula should be in the form

$$P_{safe}=nf_c(1+Mv)A_s+A_k(1+Mv)f_c$$

and this condition lasts until $nf_c(1+Mv)=2{,}800 kg/\overline{cm}^2$ is reached. The greater the strength of concrete the better the quality of steel rod recommended from the last equation. The ultimate load formula being. $P=nf_uA_s+A_k(1+Mv)f_u$, the safe load formula seems to be $P_{safe}=nf_cA_s+A_k(1+Mv)f_c$. But $Mnf_cvA_s$ should be added to the latter and this is not negligible, since for example,

$$f_c=50 kg/\overline{cm}^2, \quad n=15, \quad v=0.02, \quad A_s=0.02A_k$$

and $$M=\frac{7{,}400}{200}=37, \quad \text{then} \quad Mnf_cvA_s=11A_k$$

Therefore, $$P_{safe}=nf_cA_s+A_k(1+Mv)f_c$$

is only the measure for a safe load, any error being on the safe side.

From the above considerations, the writer concludes that, for a working load, the formula $P_{safe}=f_c(1+Mv)(A_k+nA_s)$ should be used and this is the form of a working load formula; however the ultimate load cannot be obtained by the same form of $P=f_u(1+Mv)(A_k+nA_s)$.

For the ultimate load, the formula $P=nf_uA_s+A_k(1+Mv)f_u$ should be used and the working load formula of the same type $P_{safe}=nf_cA_s+A_k(1+Mv)f_c$ gives only the safe measure for the calculation of the ultimate load. If $f_u=200 kg/\overline{cm}^2$, $f_c=50 kg/\overline{cm}^2$, $M=37$ and $n=15$, then stress in longitudinal rod $=15\times50=750$ $kg/\overline{cm}^2$. But stress in longitudinal rod equivalent to spiral is not equal to $37\times50=$

1,850 $kg/cm^2$, and this is independent of the compressive stress itself. Care must be taken that the last formula gives only a rough measure for the safe load. The theoretical formula under working stresses should be as shown in Section II.

From equation (44), spirals depend on the rods and the greater the percentage of the rod the less the quantity of the spirals. Therefore both the spiral and the rod should be considered in a discussion of the strength of a column. An increase of strength in the concrete core depends on the spirals which also depend on the rod.

It is always necessary to have $v$ and $p$ under a certain limit since the shearing off of the outside shell should be avoided.

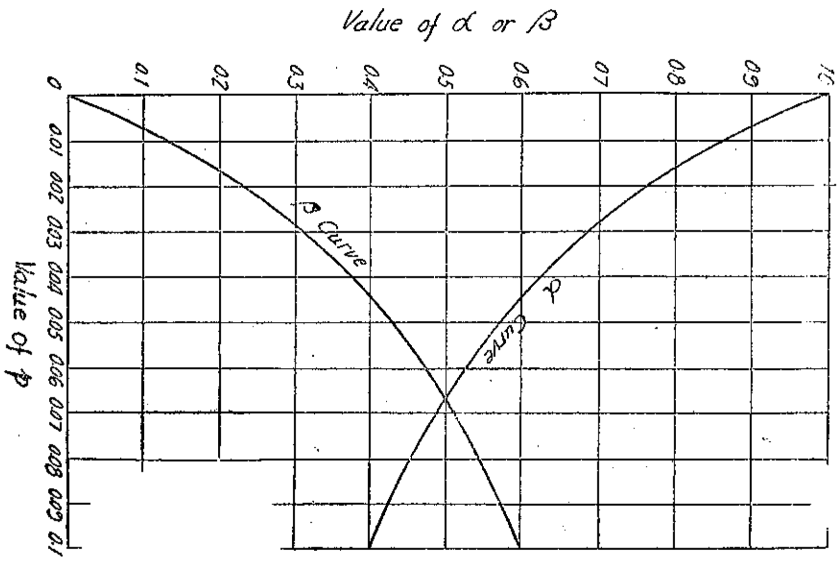



Fig. 5

Now $$P_{safe}=\alpha P_{safe}+\beta P_{safe}$$

where $$\alpha P_{safe}=f_c(1+Mv)A_k$$

$$\beta P_{safe}=nf_c(1+Mv)A_s$$

$$\frac{\beta}{\alpha}=\frac{nA_s}{A_k}=np$$

or $$\beta=15p\alpha$$

$$\alpha=1-\beta$$

$$\therefore \quad \left.\begin{aligned} \beta &= \frac{1}{1+\dfrac{1}{15p}} \\ \alpha &= \frac{1}{1+15p} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (49)$$

When $p=1/15=0.066$, then $\alpha=\beta$.

Therefore if a $p>0.066$ rod carries a greater load than the concrete and when a $p=0.066$ $\alpha$ curve intersects the $\beta$ curve and the concrete and rod carry the load in the same proportion;—

$f_c=50\ kg/\overline{cm}^2$, $M=37$, then from equation (44)

$$v=0.009,$$

$$\therefore \quad P_{safe}=2.7f_cA=135\ A_k$$

Then rod and concrete carry a load

$=67.5\ A_k$ respectively.

From equation (44), when

$$\frac{8}{3(1+np)}-1=0,$$

then

$$p=\frac{5}{45}=0.111$$

Therefore when $p=0.111$, the merit of the spirals disappears. For a spiralled and rodded concrete column, the safe load should lie within the certain limit, as already mentioned, to prevent the disintegration of the outside shell; the rod should always be used as a pitch holder in the spiralled column and the greater the percentage of rod the less the quantity of the spiral. When $p=0.111$, this means that

$$P_{safe}=f_c(A_c+nA_s).$$

In general, concrete column may be classed into two groups.

1. Concrete columns reinforced with rods.
2. Steel column reinforced with concrete.

In these two types of columns, the load to be supported is the sum of two materials concrete and steel. In the first type the rod alone cannot carry an appreciable load, but in the second type the rod can do so.

The boundary of these two types is often taken in practice to be $0.03\ A_c$ ro $0,04A_k$; but as far as the strength is concerned this boundary is not necessary.

The load to be supported by the two types is calculated by a single formula $P=f(A_c+nA_s)$ in both cases. In the spiralled and rodded concrete column, rod should be proportioned to the quantity of $v$ since the greater the quantity of the rod the less the percentage of the spiral. The quantity of the rod and spiral should be determined according to the quality of the concrete used and for the required strength of the column. For the better concrete, a less quantity of spiral and a greater quantity of rod should be used; on the other hand, for the weaker concrete a greater quantity of spiral and a less of rod must be used. Rich concrete is very desirable for the greater strength of a column. The failure of a concrete core is independent of the percentage of the rod, but spiral percentage, not only for the ultimate strength, but also for the ultimate lateral strain, is of the first important one. The rod increases the ultimate load by an amount equal to $nf_uA_s$ and also the stiffness of the spiralled concrete core.

A plain concrete column does not show even warning cracks until the ultimate stress is reached; all tends to prove that plain concrete does not shear off or even crack until the ultimate stress is reached.

It has been found that the outside shell concrete of the spiralled concrete column begins to disintegrate at $0.6f_u$ and the column having spirals and rods begins to disrupt the outside shell at $0.75f_u$ due to the action of embedded steel (Bulletin, No. 300. University of Wisconsin). There are therefore two stages. In the initial stages, the column and the outside shell are integral, but in the final stage, the shell shears off in part or in whole. If the shell is destroyed, the core supports the load which was supported partly by the shell. The core is now supporting the load that previously had been supported partly by the shell. Therefore the area and percentage based upon the core area should be used in computing the ultimate strength since the ultimate strength is independent of the shattered outside shell.

At the instant of the failure of the outside shell concrete, the load on the core is

$$0.6f_uA_c=0.6f_u\frac{4}{3}A_k=0.8f_uA_k$$

In the spiralled and rodded concrete column, the load on the core is

$$0.75f_uA_c(1+np)=0.75f_u\frac{4}{3}A_k(1+np)=f_uA_k(1+np)$$

In the spiralled and rodded column, to prevent the disruption of the outside shell concrete and also for the endurance limit, Prof. E. Mörsch recommends

$$f_c(1+Mv)=0.5f_u$$

The French regulation of 1,906 is $f_c\ (1+Mv)=0.6f_u$

The London Joint Committee's recommendation on reinforced concrete is $f_c(1+Mv)=0.66f_u$.

These are the maximum limits, the writer's value being $f_c(1+Mv)=0.67f_u$, as already mentioned, which coincides with that of the London Joint Committee recommendation.

## Section IV. Various formulae used.

M. Considère is the first contributor to our knowledge of the spiralled and rodded concrete column. From experiments M. Considère concluded that the strength of such a concrete column is the sum of the following three factors.

1. Strength of plain concrete.
2. Strength of rod.
3. Strength due to spiral.

M. Considère gave the coefficient 2.4 from the experimental result that if the same volume or same weight of metal were used as the spiral it gives a strength 2.4 times greater than that which is given when a longitudinal rod is used. His formula is

$$\begin{aligned}P&=f_uA_k+nf_uA_s+2.4nf_uA_s'\\&=nf_uA_s+A_k(1+36v)f_u\end{aligned}$$

As the writer has shown, the general equation of the increase of strength due to spiral being equation (30), take $f_{s}=2,800\,kg/\overline{cm}^2$, $\phi=43°20'$ and $f_u=210\,kg/\overline{cm}^2$,

then
$$f=f_u+\frac{2,800}{2}v\frac{1+\sin 43°20'}{1-\sin 43°20'}=f_u+7,560v$$

$$\therefore\quad P=nf_uA_s+A_k\left(1+\frac{7,560}{210}v\right)f_u$$

$$=nf_uA_s+A_k(1+36v)f_u$$

Thus the same formula is obtained and it is only applicable to his concrete.

The German regulation (1916) specifies that

$$P=f_u(A_k+nA_s+3nA_s')=f_u(A_k+15A_s+45A_s')$$

Prof. Foerster explains the equation (Foerster;—Grundzüge des Eisenbetonbaues) from Mörsch's experiment taking M=45 as the average. From Mörsch's equation

$$P=f_eA_s+A_k(1+Mv)f_u=f_eA_s+f_uA_k+Mf_uA_s',$$

if $\quad f_u=190,\ kg/\overline{cm}^2\ f_e=15\times190=2,850\ kg/\overline{cm}^2,$

and $\quad M=45 \quad \therefore\ P=2,850A_s+190A_k+45\times190A_s'$

and factor of safety $F=5.5$ then

$$P_{safe}=35(A_k+15A_s+45A_s')$$

Again if,

$$f_u=180\ kg/\overline{cm}^2,\ nf_u=f_e=15\times180=2,700\ kg/\overline{cm}^2$$

and $F=5.0$

then

$$P_{safe}=\frac{180}{5}A_k+\frac{2700}{5}A_s+45\frac{180}{5}A_s'$$

$$=35(A_k+15A_s+45A_s')$$

But from the writer's general equation, take

$$f_u=175\ kg/\overline{cm}^2 \quad \text{and} \quad \phi=43°43',$$

then

$$M=\frac{f_{r.t.}}{2f_u}\cdot\frac{1+\sin\phi}{1-\sin\phi}=\frac{2,800}{2\times175}\cdot\frac{1+\sin 43°43'}{1-\sin 43°43'}=45$$

$$\therefore \quad P=f_u(A_k+15A_s+45A_t')$$

is obtained and is only applicable to that concrete.

The American Concrete Institute recommedation is

$$P=f_u(A_k+nA_s+4nA_s')=f_u(A_k+15A_s+60A_s')$$

from the experiment. In this case, from equation (33), we have when

$$f_u=140 kg/\overline{cm}^2 \quad \text{and} \quad \phi=45°15',$$

$$M=\frac{2,800}{2\times140}\cdot\frac{1+\sin 45°15'}{1-\sin 45°15'}=60$$

and this is only applicable to this concrete.

The New York City building code, the Prussian (1907) and Austrian (1911) regulations are,

$$P=f_u(A_k+nA_s+2nA_s')=f_u(A_k+15A_s+30A_s')$$

But in this case taking $f_u=250\ kg/\overline{cm}^2$ and $\phi=43°$, we have

$$M=\frac{2,800}{2\times250}\cdot\frac{1+\sin 43°}{1-\sin 43°}=30$$

This is again only applicable to this concrete. Many other formulae of the considère type can be similarly treated; the coefficient $M$ varies from $30 \sim 60$.

As already noted, the above form of the formulae is the ultimate load equation and although a moderate calculation of the factor of safety is taken it does not show true or theoretical working load. Such a working load is only an approximate indication of the ultimate load. The above formulae do not show the relations between the increase in the strength of the concrete and its increase in strength due to the spiral.

Another type of formula is that recommended or laid down by the Joint Committee on reinforced concrete (London) and the French regulation of 1906.

The second report of the Joint Committee in London on reinforced concrete specifies that the increase in strength of the concrete due to the spiral at the instant of failure is equal to $f_u\ F_1\ S_1\ v$.

$F_1$ = form factor or form coefficient.

$S_1$ = spacing ratio; the ratio of the pitch of the laterals to the diameter of the core.

$$\therefore \quad f = f_u + f_u F_1 S_1 v = f_u(1 + F_1 S_1 v) = f_u(1 + Mv)$$

From this, $P = f_u(1 + Mv)(A_k + nA_s)$ is deduced and also $P_{safe} = \frac{f_u}{F}(1 + Mv)(A_k + nA_s)$ is obtained. The factor $M = F_1 S_1$ has been found by experiment since $M = \frac{1}{v}\left(\frac{f}{f_u} - 1\right)$; $M = 32$ is given as the maximum in the French regulation and also in the London recommendation. Therefore, if $v = 0.02$ and $M = 32$, then $f_{safe} = 1.64 f_c = 0.41 f_u$.

As the writer has already noted, a formula of this type is only applicable in the case of $n f_c(1 + Mv) \leqq$ yield point of steel. In order to find the working load this form of formula isco rrect, but not for the ultimate loading. Since 1. 64 $nf_u$ is greater than the yield point of steel, it is not the measure of the ultimate strength. The factor $M = 32$ is only applicable to a concrete of greater strength and cannot be used for concrete of lower strengths.

The spiral raises the ultimate strength of concrete and increases the security against sudden failure, by preventing the lateral expansion of the core. Although the spiral raises the strength of concrete, it depends to some extent on the rod. The rod increases the stiffness or rigidity of concrete and the increase in strength of concrete is modified by the employment of the rod; the greater the quantity of rods the less the longitudinal shortening of the concrete for the given loading. Therefore the spiral is affected by the rod when supporting the load.

As has been mentioned by equation (44), the quantity $v$ is the function of $M$ and $p$.

When $p=0$, then the spiral alone being used, the maximum $(1+Mv)f_c$ is $2.67f_c$ and this is the endurance limit of loading.

## Section V. Conclusion.

As shown, many formulae are applied in the calculation of the strength of spiralled and rodded concrete columns. All of them are found from experimental results and are empirical. They have no real rational basis being only applicable to a concrete column under the same conditions as those of the test specimens. These formulae give different results with the same data. They do not show the exact rules between the increase of the strength of concrete and increase of strength due to the spiral. Or they do not give exact rules for determining the increase of strength due to the spiral and rod. It is difficult to justify the designs thus made by known principles of mechanics.

The writer gives, in this paper, the rules relating to the working stress and the ultimate strength separately. Since the conditions of the concrete core in working and in ultimate loading differ, the formula should vary and the variations may be divided into two clasess as shown in Section II and in Section III. All deformations in the limit of working stresses should be calcuated by the formulae shown in Section. II. For the ultimate loading, the formulae (27), (30), (33) and (34) in Section III are important for the solution of the problems.

The relations between the increase in the strength of concrete and its increase in strength due to the spiral are clearly shown by these formulae.

The writer belives that the formulae, discussions and methods of calculation in this paper are logical and rational including new facts. The writer hopes that the unsoundness of the present day formulae would have been clearly solved by the treatment applied by the writer.

In this paper the bending of a column is not considered and therefore the discussion is only applicable to axial loading on the short column.

—(THE END)—