On the calculation of the thin circular pipes under a uniformly distributed load.

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Case I. Bending moment due to external load in thin circular pipe uniformly distributed as shown in the figure (1).

Let $\omega =$ upper load intensity in pound per unit length of horizontal distance.
$\omega' =$ lower load intensity in pound per unit length of horizontal distance.
$C =$ distance from A measured along the circumference.
$E =$ modulus of Elasticity of the material.
$I =$ moment of inertia of the normal section of the pipe.
$A =$ cross-sectional area of the pipe.

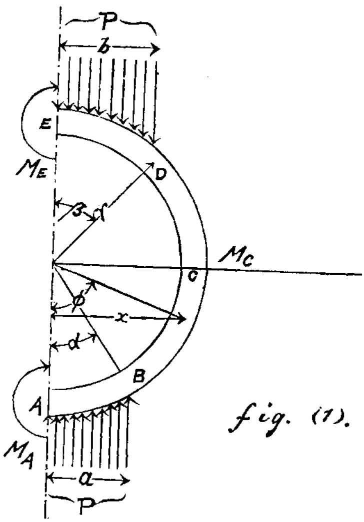


fig. (1).

put $\omega b = \omega' a = P$

$and\ M_A = X.$

then

Bending moment (M) at any point in AB

$$= X + \frac{\omega' x^2}{2}$$

axial force (N) at the same point

$$= -\omega' x \sin\phi$$

$$\therefore \quad \frac{dM}{dX} = 1 \quad and \quad \frac{dN}{dX} = 0$$

M at any point in BD

$$= X + P\left(x - \frac{a}{2}\right)$$

N at the same point

$$= -P \sin\phi$$

$$\therefore \quad \frac{dM}{dX} = 1 \quad and \quad \frac{dN}{dX} = 0$$

M at any point in DE

$$= X + P\left(x - \frac{a}{2}\right) + \frac{\omega(b-x)^2}{2}$$

$$= X - \frac{Pa}{2} + \frac{Pb}{2} + \frac{\omega x^2}{2}$$

N at the same point

$$= -\{P - \omega(b-x)\}\sin\phi = -\omega x \sin\phi.$$

$$\therefore \quad \frac{dM}{dX} = 1 \quad \text{and} \quad \frac{dN}{dX} = 0$$

By the principle of least works;

$$\int \frac{M}{EI} \frac{dM}{dX} dc + \int \frac{N}{EA} \frac{dN}{dX} dc = 0$$

Assuming E, I and A constant, we get

$$\int_A^B \left(X+\frac{\omega' x^2}{2}\right)dc+\int_B^D\left(X+Px-\frac{Pa}{2}\right)dc+\int_D^E\left(X-\frac{Pa}{2}+\frac{Pb}{2}+\frac{\omega x^2}{2}\right)dc=0$$

$$X\int_A^E dc+\frac{\omega'}{2}\int_A^B x^2dc+P\int_B^D xdc-\frac{Pa}{2}\int_B^E dc+\frac{Pb}{2}\int_D^E dc+\frac{\omega}{2}\int_D^E x^2dc=0$$

or $$X=-\frac{\omega'}{2}\frac{\int_A^B x^2dc}{\int_A^E dc}-P\frac{\int_B^D xdc}{\int_A^E dc}+\frac{Pa}{2}\frac{\int_B^E dc}{\int_A^E dc}-\frac{Pb}{2}\frac{\int_D^E dc}{\int_A^E dc}-\frac{\omega}{2}\frac{\int_D^E x^2dc}{\int_A^E dc}$$

Referring to the figure (1), we have

$$x=r\sin\phi$$
$$a=r\sin\alpha$$
$$b=r\sin\beta$$
$$dc=r\,d\phi$$

ϕ denoting the inclination of the tangent to the horizontal at any point (xy).
Introducing these values and integrating the terms respectively,

$$\int_A^E dc=r\int_0^\pi d\phi=\pi r$$

$$\int_B^E dc=r\int_\alpha^\pi d\phi=(\pi-\alpha)r$$

$$\int_D^E dc=r\int_{\pi-\beta}^\pi d\phi=\beta r$$

$$\int_B^D xdc=r^2\int_\alpha^{\pi-\beta}\sin\phi\,d\phi=r^2(-\cos\phi)_\alpha^{\pi-\beta}$$

$$\int_A^B x^2dc=r^3\int_0^\alpha \sin^2\phi\,d\phi=r^3\left\{-\frac{\sin 2\phi}{4}+\frac{\phi}{2}\right\}_0^\alpha$$

$$\int_D^E x^2 dc = r^3 \int_{\pi-\beta}^{\pi} \sin^2\phi \, d\phi = r^3 \left\{ -\frac{\sin 2\phi}{4} + \frac{\phi}{2} \right\}_{\pi-\beta}^{\pi}$$

whence we get finally,

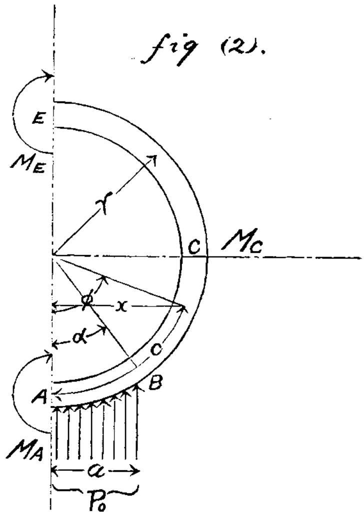
$$X(_{or} M_A) = -\frac{Pr}{2\pi \sin a} \left\{ -\frac{\sin 2\phi}{4} + \frac{\phi}{2} \right\}_0^{\alpha} - \frac{Pr}{\pi} (-\cos\phi)_{\alpha}^{\pi-\beta}$$

$$+ \frac{Pr(\pi - a)\sin a}{2\pi} - \frac{Pr\beta \sin\beta}{2\pi} - \frac{Pr}{2\pi \sin\beta} \left\{ -\frac{\sin 2\phi}{4} + \frac{\phi}{2} \right\}_{\pi-\beta}^{\pi}$$

Case II. Bending moment of thin circular pipe due to its own weight.

(E, I & A all constant)

fig (2).



Let ω = weight in pound per unit length of the circumference.
ω' = Reaction intensity in pound per unit length of horizontal distance.
C = distance from A measured along the circumference.

put $\omega \pi r = \omega' a = P_0$
and $M_A = X$

Then

Bending Moment (M) at any point in AB

$$= X + \frac{\omega' x^2}{2} - C\omega\left(x - r\sin\frac{\phi}{2}\right)$$

axial force (N) at the same point

$$= -(\omega' x - C\omega)\sin\phi$$

$$\therefore \quad \frac{dM}{dX} = 1 \quad and \quad \frac{dN}{dX} = 0$$

M at any point in BC

$$= X + P_0\left(x - \frac{a}{2}\right) - C\omega\left(x - r\sin\frac{\phi}{2}\right)$$

N at the same point

$$= -(P_0 - C\omega)\sin\phi$$

$$\therefore \quad \frac{dM}{dX} = 1 \quad and \quad \frac{dN}{dX} = 0$$

M at any point in CE

$$= X + P_0\left(x - \frac{a}{2}\right) - (\pi r - C)\omega\left\{x - r\sin\frac{\pi - \phi}{2}\right\}$$

$$+ 2\left(C - \frac{\pi r}{2}\right)\omega\left\{r\cos\left(\frac{\phi}{2} - \frac{\pi}{4}\right) - x\right\}$$

$$= X + P_0 x - \frac{P_0 a}{2} - C\omega x + P_0 r\left(1 - \frac{1}{\sqrt{2}}\right)\cos\frac{\phi}{2} + C\omega r(\sqrt{2} - 1)\cos\frac{\phi}{2}$$

$$+ \sqrt{2}\, C\omega r\sin\frac{\phi}{2} - \frac{P_0 r}{\sqrt{2}}\sin\frac{\phi}{2}$$

N at the same point

$$= -(P_0 - C\omega)\sin\phi$$

$$\therefore \quad \frac{dM}{dX} = 1 \quad \text{and} \quad \frac{dN}{dX} = 0$$

By putting $$\int \frac{M}{EI}\frac{dM}{dX}dc + \int \frac{N}{EA}\frac{dN}{dX}dc = 0$$

we obtain

$$\int_A^B \left(X + \frac{\omega' x^2}{2} - C\omega x + C\omega r \sin\frac{\phi}{2}\right)dc + \int_B^C \left(X + P_0 x - \frac{P_0 a}{2} - C\omega x + C\omega r \sin\frac{\phi}{2}\right)dc$$

$$+ \int_C^E \left\{X + P_0 x - \frac{P_0 a}{2} - C\omega x + P_0 r\left(1 - \frac{1}{\sqrt{2}}\right)\cos\frac{\phi}{2} + C\omega r(\sqrt{2} - 1)\cos\frac{\phi}{2}\right.$$

$$\left. + \sqrt{2}C\omega r \sin\frac{\phi}{2} - \frac{P_0 r}{\sqrt{2}}\sin\frac{\phi}{2}\right\}dc = 0$$

$$X\int_A^E dc + \frac{\omega'}{2}\int_A^B x^2 dc + P_0\int_B^E x dc - \frac{P_0 a}{2}\int_B^E dc - \omega\int_A^E Cx dc$$

$$+ \omega r\int_A^C C\sin\frac{\phi}{2}dc + P_0 r\left(1 - \frac{1}{\sqrt{2}}\right)\int_C^E \cos\frac{\phi}{2}dc + \omega r(\sqrt{2} - 1)\int_C^E C\cos\frac{\phi}{2}dc$$

$$+ \sqrt{2}\omega r\int_C^E C\sin\frac{\phi}{2}dc - \frac{P_0 r}{\sqrt{2}}\int_C^E \sin\frac{\phi}{2}dc = 0$$

or

$$X = -\frac{\omega'}{2}\frac{\int_A^B x^2 dc}{\int_A^E dc} - P_0\frac{\int_B^E x dc}{\int_A^E dc} + \frac{P_0 a}{2}\frac{\int_B^E dc}{\int_A^E dc}$$

$$+ \omega\frac{\int_A^E Cx dc}{\int_A^E dc} - \omega r\frac{\int_A^C C\sin\frac{\phi}{2}dc}{\int_A^E dc} - P_0 r\left(1 - \frac{1}{\sqrt{2}}\right)\frac{\int_C^E \cos\frac{\phi}{2}dc}{\int_A^E dc}$$

$$- \omega r(\sqrt{2} - 1)\frac{\int_C^E C\cos\frac{\phi}{2}dc}{\int_A^E dc} - \sqrt{2}\omega r\frac{\int_C^E C\sin\frac{\phi}{2}dc}{\int_A^E dc}$$

$$+\frac{P_0 r}{\sqrt{2}}\frac{\int_C^E \sin\frac{\phi}{2}\,dc}{\int_A^E dc}$$

But, referring to the figure (2), we have

$$x = r\sin\phi$$

$$a = r\sin\alpha$$

$$C = r\phi$$

$$dc = r d\phi$$

where ϕ denotes the inclination of the tangent at any point (x,y) to the horizontal.

Therefore, we can find, by carrying out the integrations as before;

$$X(_{or}M_A) = -\frac{P_0 r}{2\pi\sin\alpha}\left\{-\frac{\sin 2\phi}{4}+\frac{\phi}{2}\right\}_0^\alpha + \frac{P_0 r(\pi-\alpha)\sin\alpha}{2\pi}$$

$$-\frac{P_0 r}{\pi}(-\cos\phi)_\alpha^\pi + \frac{P_0 r}{\pi} - \frac{8P_0 r}{\pi^2}(\sqrt{2}-1)$$