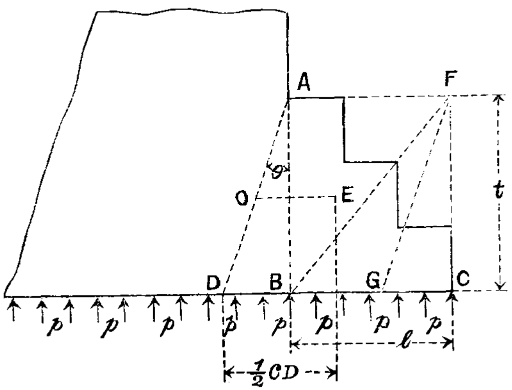
STRENGTH OF FOUNDATION FOOTING FOR A HEAVILY LOADED STRUCTURE.

By Prof. A. Inokuty, *Kogakuhakushi.*

Foundation courses of walls or chimneys have their width increased by a series of steps, with the object of distributing the pressure over a greater area than that of any bed joint in the body of the wall or chimney. The projecting portions of foundation footings thus formed have to resist considerable bending action when the structure is of great height. In the Figure let CDA represent an axial vertical section of a chimney foundation supporting a very large weight. It may be assumed that the vertical downward pressure



exerted by the bottom surface of the foundation upon the excavated and suitably prepared bed is uniform all over the surface and of intensity p lbs. per square inch. If the base of the foundation is large compared with the projecting portions BC, the latter may safely be regarded as a cantilever of length l, loaded uniformly with vertical upward reactions of the bed, of intensity p lbs. per square inch.

Considering a vertical layer C D A, whose thickness is unity, and taking the vertical cross section A B of the canti-lever, the bending moment at the section due to the uniform load p is $\frac{1}{2}pl^2$ and the moment of resistance of the section to bending is equal to $\frac{1}{6}ft^2$. Hence we have

$$\tfrac{1}{2}pl^2=\tfrac{1}{6}ft^2,$$

from which we obtain

$$f=\frac{3pl^2}{t^2} \qquad \text{......(1)}$$

and

$$t=l\sqrt{\frac{3p}{f}} \qquad \text{......(2)}$$

Now the bending stress f at the top or bottom edge of an oblque section such as A O D is greater than the f given by (1) and it will be found that there exists some particular direction of A O D for which the stress f is maximum. The bending moment at the section A O D is equal to the load on C D multiplied by the perpendicular let fall from the centre O of the section to the line of action of the resultant load, that is,

$$\begin{aligned} M &= p\times CD\times OE \\ &= p(l+t\tan\theta)(\tfrac{1}{2}CD-\tfrac{1}{2}t\tan\theta) \\ &= \tfrac{1}{2}pl(l+t\tan\theta). \end{aligned}$$

The moment of resistance of the section A O D is $\frac{1}{6}f(t\sec\theta)^2$. We have therefore $\frac{1}{2}pl(l+t\tan\theta)=\frac{1}{6}ft^2\sec^2\theta$,
from which we have

$$f=\frac{3pl^2}{t^2}\times\left(\cos^2\theta+\frac{t}{l}\cos\theta\sin\theta\right)$$

$$=\frac{3pl^2}{t^2}\times\tfrac{1}{2}(1+\cos 2\theta+m\sin 2\theta) \qquad \text{......(3)}$$

in which m stands for t/l. For the particular direction of the section AOD determined by $\tan\theta=m$, the above value of the bending stress f is just equal to that given by the previous equation (1). For all smaller values of θ, the above value of f is greater than that given by (1) and become maximum when

$$\tan 2\theta=m \qquad \text{......(4)}$$

Substituting this value of θ in (3) we obtain the greatest bending stress

STRENGTH OF FOUNDATION FOOTING FOR A HEAVILY LOADED STRUCTURE.

$$f = \frac{3pl^2}{t^2} \times \tfrac{1}{2}(1 + \sqrt{1 + m^2}) \quad \text{..........................} (5)$$

If, as is sometimes the case in practice, the length of the projecting portion l is equal to the depth t, then the section A O D of the greatest stress makes an angle $\theta = 22\tfrac{1}{2}°$ with the vertical section A B and the value of that greatest stress is

$$f = \frac{3pl^2}{t^2} \times \tfrac{1}{2}(1 + \sqrt{2}) = 1.207 \times \frac{3pl^2}{t^2},$$

which is greater by 21 per cent. than that given by the usual formula (1). Again if the depth t is $1\tfrac{1}{4}$ times the length l, the greatest bending stress f is 1.30 times the f given by (1).

The equation (4) gives us the following very simple geometrical construction for finding the direction of the section A O D for which the stress is greatest. Completing the rectangle A B C F, draw the diagonal F B. Bisect the angle B F C by the line F G. Then A O D drawn paralell to F G gives the required direction. The accompanying Table gives the ratio of the greatest bending stress occurring at the section A O D to the bending stress at the vertical section A B.

$m = \frac{t}{l}$	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$\frac{1}{2}(1 + \sqrt{1 + m^2})$	1.11	1.14	1.17	1.21	1.24	1.28	1.32	1.36	1.40

Solving the equation (5) for t we obtain

$$t = l\sqrt{\frac{3p}{f} + \frac{1}{4}\left(\frac{3p}{f}\right)^2} \quad \text{..........................} (6)$$

Suppose for example that the safe pressure p upon the foundation bed is one ton per square foot and the safe tensile stress f of the material of the foundation footing is three tons per square foot, that is, 46.7 lbs. per square inch, then according to the formula (2) the depth t would be equal to the length l; a more accurate value obtained by the above formula (6) is $t = 1.118\ l$.