Balancing a Survey, when The Error of Closure is Due to Linear Measurement Only.

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It often occurs that in the survey of a closed traverse, there is an error of closure. In general, we have no reason to believe that the error is caused by either one of erroneous measurements of angles or of lengths, and, contrary to this, it is most probable that the error is due to both of them. In such a case, balancing the survey may be effected by the well known rule of Bowditch.* ● But when the angle condition of the closed traverse is satisfied, the errors of closure must be thought to have been caused by erroneous measurements of lengths only. If the Bowditch's rule is applied in this case, not only the bearing of each course is changed, but also the included angles between two successive courses are changed.▲ Some authors are too indifferent to correct the error in question by a quite arbitrary manner.⌒

The following might be a solution in such a case:—

Let

$a_1,\ a_2, \cdots\cdots a_n$ be the lengths of courses of a closed traverse,

$\delta a_1,\ \delta a_2, \cdots\cdots \delta a_n$ Corresponding errors of the lengths,

and

$\theta_1,\ \theta_2, \cdots\cdots \theta_n$ corresponding bearings of courses free from error;

Then, the observation equations are:

$$(a_1+\delta a_1)\cos\theta_1+(a_2+\delta a_2)\cos\theta_2+\cdots\cdots$$
$$+(a_n+\delta a_n)\cos\theta_n=o \cdots\cdots\cdots\cdots (1)$$

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* Prévot-Topographie, méthodes, 1900, P. 66-69.

● Wright & Hayford-Adjustment of observations, 19 6. P. 157-158.

▲ Johnson-The Theory & Practice of Surveying, 1904. P. 201.

⌒ Nugent-Plane Surveying, 1902. P. 47-53.

$$(a_1+\delta a_1)\sin\theta_1+(a_2+\delta a_2)\sin\theta_2+\cdots\cdots\cdots$$

$$+(a_n+\delta a_n)\sin\theta_n=o \cdots\cdots\cdots\cdots\cdots\cdots\cdots(2)$$

Or,

$$[\delta a\cos\theta]+l_1=o \cdots\cdots\cdots\cdots\cdots\cdots\cdots(1')$$

$$[\delta a\sin\theta]+l_2=o \cdots\cdots\cdots\cdots\cdots\cdots\cdots(2')$$

where [ ] denotes the Summation and

$$l_1=[a\cos\theta]$$

$$l_2=[a\sin\theta]$$

By the principle of the least squares, $[p(\delta a)^2]$ must be a minimum, when p expreses the weight of each course. Multiplying indeterminate constants $-2K_1$ and $-2K_2$ to (1′) and (2′) resp., and adding to $[p(\delta a)^2]$,

$$[p(\delta a^2)]-2K_1\{[\delta a\cos\theta]+l_1\}-2K_2\{[\delta a\sin\theta]+l_2\}$$

$$=\Omega,\text{ say}\cdots\cdots\cdots\cdots\cdots\cdots\cdots(3)$$

Ω must be also a minimum, so that its first differential coefficients with respect to δa_1, δa_2, $\cdots\cdots\delta a_n$ must be all equal to o; *i. e.*,

$$\frac{\partial\Omega}{\partial(\delta a_1)}=p_1\,\delta a_1-(K_1\cos\theta_1+K_2\sin\theta_1)=o$$

$$\frac{\partial\Omega}{\partial(\delta a_2)}=p_2\,\delta a_2-(K_1\cos\theta_2+K_2\sin\theta_2)=o$$

... ...

$$\frac{\partial\Omega}{\partial(\delta a_n)}=p_n\,\delta a_n-(K_1\cos\theta_n+K_2\sin\theta_n)=o.$$

Or,

$$\left.\begin{aligned}\delta a_1&=\frac{1}{p_1}(K_1\cos\theta_1+K_2\sin\theta_1)\\ \delta a_2&=\frac{1}{p_2}(K_1\cos\theta_2+K_2\sin\theta_2)\\ &\cdots\cdots\cdots\cdots\cdots\cdots\\ \delta a_n&=\frac{1}{p_n}(K_1\cos\theta_n+K_2\sin\theta_n)\end{aligned}\right\}\cdots\cdots\cdots\cdots(4)$$

Substituting these for (1′) and (2′), we obtain

$$\left[\frac{1}{p}\cos\theta\ (K_1\cos\theta+K_2\sin\theta\right]+l_1=o$$

$$\left[\frac{1}{p}\sin\theta(K_1\cos\theta+K_2\sin\theta)\right]+l_2=o.$$

Or, Correlate equations are:

$$K_1\left[\frac{\cos^2\theta}{p}\right]+K_2\left[\frac{\cos\theta\sin\theta}{p}\right]+l_1=o \cdots\cdots\cdots\cdots\cdots\cdots(5)$$

$$K_1\left[\frac{\cos\theta\sin\theta}{p}\right]+K_2\left[\frac{\sin^2\theta}{p}\right]+l_2=o \cdots\cdots\cdots\cdots\cdots\cdots(6)$$

Now we may consider the errors of linear measurements as compensating, so that the weight p inversely varies as a and, (5) and (6) become

$$K_1[a\cos^2\theta]+K_2[a\cos\theta\sin\theta]+l_1=o \cdots\cdots\cdots\cdots\cdots\cdots(5')$$

$$K_1[a\cos\theta\sin\theta]+K_2[a\sin^2\theta]+l_2=o \cdots\cdots\cdots\cdots\cdots\cdots(6')$$

which give

$$K_1=-\frac{[a\sin^2\theta]l_1-[a\cos\theta\sin\theta]l_2}{[a\cos^2\theta][a\sin^2\theta]-[a\cos\theta\sin\theta]^2} \cdots\cdots\cdots(7)$$

$$K_2=-\frac{[a\cos^2\theta]l_2-[a\cos\theta\sin\theta]l_1}{[a\cos^2\theta][a\sin^2\theta]-[a\cos\theta\sin\theta]^2} \cdots\cdots\cdots(8)$$

These are somewhat complicated compared to the general case of Bowditch's rule.

Having found the values of K_1 and K_2 from (7) and (8), we may easily find out the value of δa_1, δa_2, $\cdots\cdots\delta a_n$ which are to be applied to the corresponding lengths; *i. e.*,

$$\delta a_1=K_1\,a_1\cos\theta_1+K_2\,a_1\sin\theta_1$$

$$\delta a_2=K_1\,a_2\cos\theta_2+K_2\,a_2\sin\theta_2$$

$$\cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots$$

$$\delta a_n=K_1\,a_n\cos\theta_n+K_2\,a_n\sin\theta_n.$$