The Stresses in Viaduct-Bents.

By I. Hiroi.

The writer having neglected to consider the effects of temperature changes in the discussion of stresses in viaduct-bents in his " The Statically-Indeterminate Stresses in Frames commonly used for Bridges ", has made it the subject of an article, supposing that it would be of interest to some of the members of the Society.

The temperature stresses in ordinary viaduct bents are often not possible of combination with stresses arising from other causes, unless the temperature at the time of erection is exactly known. The following discussion concerns the stresses produced by any given change in temperature alone.

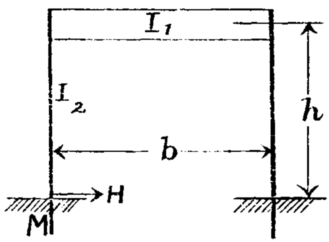


Fig. 1

Let t = the range of a temperature change.

θ = coefficient of expansion and contraction.

Then in the bent of Fig. 1 a rise of t above an initial temperature would increase the distance b

between the posts by $t\theta b$, were the latter free to move. The lower ends of the posts are, however, being assumed to be firmly fixed, moment and horizontal reaction would be produced at each end: each horizontal reaction H acting, as it were, through a distance of

$$\frac{t\theta b}{2}$$

Calling the moments producing compression on the outside fibre of the frame positive, we have for the bending moment at any point in the post

$$M - Hx$$

and in the cross-girder

$$M - Hh$$

Then, neglecting the effect of direct stresses, we get for the internal work in the frame

$$\omega = \frac{1}{EI_2}\int_0^h (M - Hx)^2 dx + \frac{1}{2EI_1}\int_0^b (M - Hh)^2 dx$$

$$= \frac{h}{EI_2}\left(M^2 - HMh + \frac{H^2h^2}{3}\right) + \frac{b}{2EI_1}(M - Hh)^2$$

in which I_1 and I_2 denote the moments of inertia of the cross-girder and posts respectively and E the modulus of elasticity of the material.

Then according to the principles of work

$$\frac{d\omega}{dM} = 0 \qquad\qquad \frac{d\omega}{dH} = t\theta b$$

So that we at once get

$$\frac{h}{I_2}(2M - Hh) + \frac{b}{I_1}(M - Hh) = 0$$

$$\frac{h}{3I_2}(3M - 2Hh) + \frac{b}{I_1}(M - Hh) = -\frac{t\theta b}{h}$$

from which

$$H = \frac{2hI_1 + bI_2}{h(hI_1 + bI_2)} M$$

$$M = \left(\frac{hI_1 + bI_2}{hI_1 + 2bI_2}\right)\frac{3EI_2 t\theta b}{h^2}$$

In case the lower ends of the posts are *hinged* $M = 0$ so that we get

$$H = \left(\frac{3I_1I_2}{2hI_1 + 3bI_2}\right)\frac{Et\theta b}{h^2}$$

Refering to the numerical example given on page 27 of the above cited work, for

$t =$ 50° Fah.
$\theta =$ .000007
$E = 30{,}000{,}000.$ lbs per sq. in.

we get

$$M = 263400 \text{ in.-lbs.}$$
$$H = 2840 \text{ lbs.}$$
$$M - Hh = 247,800. \text{ in.-lbs.}$$

and for the same with lower ends of posts hinged

$$H = 707 \text{ lbs}$$
$$-Hh = -127260 \text{ in.-lbs.}$$

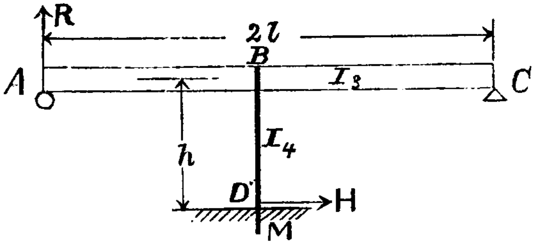
Showing that the moment in the post due to temperature change is very much augmented by rigid connections, while against horizontal forces the reverse would be true—a fact perhaps so well known as to hardly require any further comment.

Longitudinally, the temperature stresses produced in the posts also differ considerably according to the modes of connection made between the girder and the posts and the number of spans so connected.

Suppose that in Fig. 2. two continuous spans are fixed longitudinally at C and movable at A and the posts firmly fixed at D and rigidly riveted to the girder. Then a rise t in temperature will tend to displace the point B with respect to D by $t\theta l$, so that H would be acting, as it were, through that distance in the direction as shown in the figure.

Calling moments producing compression in the upper fiber of the girder and on the left side one of the post *positive*, we have for moment at any point of: —

Fig. 2



AB	Rx	with origin of x at A
BD	$M - Hx$	" " " " " D
BC	$\frac{Rl + M - Hh}{l} X$	" " " " " C

Again neglecting the effect of direct stresses, we get for the internal work in the frame

$$\omega = \frac{1}{2EI_3}\left\{\int_0^l (Rx)^2 dx + \int_0^l \left(\frac{Rl + M - Hh}{l} x\right)^2 dx\right\} + \frac{1}{2EI_4}\int_0^h (M - Hx)^2 dx$$

in which I_3 and I_4 represent the moments of inertia of the girder and posts respectively, both considered as being constant throughout.

Then since according to the principles of work

$$\frac{d\omega}{dH} = t\theta l \qquad \frac{d\omega}{dM} = 0 \qquad \frac{d\omega}{dR} = 0$$

we get

$$\frac{2l}{I_3}(Hh-M-Rl)+\frac{h}{I_4}(2Hh-3M)=\frac{6Et\theta l}{h}$$

$$\frac{2l}{I_3}(Hh-M-Rl)+\frac{3h}{I_4}(Hh-2M)=0$$

$$Hh-M-2Rl=0$$

from which

$$M=\left(\frac{lI_4+3hI_3}{2lI_4+3hI_3}\right)\frac{6EI_4t\theta l}{h^2}$$

$$H=\left(\frac{lI_4+6hI_3}{2lI_4+3hI_3}\right)\frac{6EI_4t\theta l}{h^3}$$

If the lower ends of the posts were hinged at D, M would disappear, so that we get

$$H=\left(\frac{I_3I_4}{lI_4+2hI_3}\right)\frac{6Et\theta l}{h^2}$$

Again refering to the previous example and assuming

$I_3=30000$ in^4. $l=50$ ft
$I_4=1000$ " $h=15$ ft

we get for the case of fixed posts

M = 1,126,400. in.-lbs.
H = 12,290 lbs.

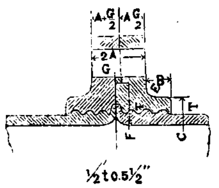
$$M - Hh = -1{,}085{,}800 \text{ in.-lbs.}$$

A comparison of such figures will show that in most cases occuring in practice, the max. moment in the post when the latter is firmly connected to the girder and to the foundation may, without material error, be assumed to be twice that produced when either end is hinged.

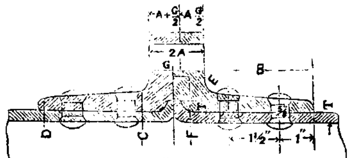
The combined action of lateral and longitudinal moments is to throw the greatest compression, in the above case, on the outermost corner on the left side of the post base.

By applying the foregoing computations to any form of post-sections it will at once be seen that a considerable stress is produced in certain portion of the post, in the kind of bents discussed, under changes of temperature which are by no means uncommon, and that such stress is greatest when the post is fixed at both ends, has been the principal reason of the adoption of hinged posts in older structures. Modern practice in this kind of construction is however to secure, at the sacrifice of material and work, the greatest possible amount of rigidity, which is so essential to structures intended to serve the purpose of rapid transit. Even expansion-joints are no longer provided for every span, but are placed at intervals as long as possible. It is for this reason sincerely hoped, that on the elevated lines of Tokyo, now in course of construction, more substantial structures will be constructed for street-crossings than those originally designed for the purpose; and thus not only to bring the first structures of the kind in this country, in line of modern improvements, but at the same time to make them safe and lasting ones to meet the demands of rapidly increasing speed and number of trains on the railways centering in the City.

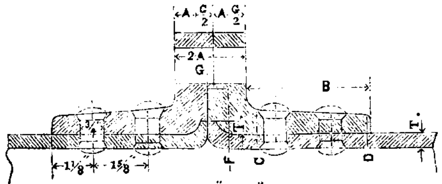
船用トシテ多管罐使用ノ増加ト蒸汽壓力ノ増大トハ鋼製管ノ必要ヲ生ジタリ下ノ表ハ此等ノ鋼管ニ於ケル鍔ノ寸法ヲ示ス



½ to 5½"



6" to 10"



10½" to 15"

Pipes to be rolled into Flanges
Thres scores 1/32" deep struck with
1/23 Rad. in each Flange.

Holes for Bolts to be Drilled 1/16" larger than Diameter of Bolts.
Pipes to be Beaded over Edge of Flange a distance. T = Thickness of pipe.
Entering Part of Flange to be Turned 1/32" less in Dia. than Receiving Part

STANDARD DIMENSIONS OF ROLLED AND RIVETED STEEL FLANGES FOR STEEL PIPE.

PRESSURES FROM 101 TO 300 LBS. PER SQ. INCH.

INTERNAL DIAM. OF PIPE.	DIMENSIONS OF FLANGE.								BOLTS				RIVETS		
	DIAMETER	MEAN THICKNESS A	DEPTH OF BOSS B	DIAM. OF BOSS, C	DIAM. OF BOSS D	RAD. OF FILLET E	DIAM. OF FEMALE PART F	DEPTH OF FEMALE PART G	NUMBER	DIAMETER	DIAM. OF PITCH CIRCLE	PITCH	NUMBER IN EACH ROW	DIAMETER	NUMBER OF ROWS
1/2	3 3/4	1/2	1/2	1 3/8		3/16	1 15/16	1/8	3	1/2	2 1/4	2.880	NONE	NONE	NONE
3/4	4	1/2	1/2	1 5/8		"	2 3/16	"	4	1/2	3	2.356	"	"	"
1	4 3/8	9/16	9/16	2		"	2 9/16	"	4	1/2	3 3/8	2.650	'	"	"
1 1/4	5 1/8	9/16	9/16	2 1/4		"	2 15/16	"	5	5/8	3 7/8	2.434	"	"	"
1 1/2	5 3/8	9/16	9/16	2 1/2		"	3 3/16	"	5	5/8	4 1/8	2.592	"	"	"
2	6 1/8	5/8	5/8	3 1/8		1/4	3 15/16	"	6	5/8	4 7/8	2.552	"	"	"
2 1/2	6 5/8	5/8	5/8	3 5/8		"	4 7/16	"	7	5/8	5 3/8	2.412	"	"	"
3	7 5/8	11/16	11/16	4 1/4		"	5 1/8	3/16	8	3/4	6 3/16	2.430	"	"	"
3 1/2	8 1/8	11/16	11/16	4 3/4		"	5 5/8	"	8	3/4	6 11/16	2.626	"	"	"
4	8 3/4	3/4	3/4	5 3/8		"	6 1/4	"	9	3/4	7 5/16	2.553	"	"	"
4 1/2	9 1/4	3/4	3/4	5 7/8		"	6 3/4	"	9	3/4	7 13/16	2.728	"	"	"
5	9 7/8	13/16	13/16	6 1/2		"	7 3/8	"	10	3/4	8 7/16	2.651	"	"	"
5 1/2	10 3/8	13/16	13/16	7		"	7 7/8	"	11	3/4	8 15/16	2.552	"	"	"
6	11 3/4	7/8	3 3/16	7 7/8	7 3/8	1/4	8 7/8	3/16	11	7/8	10 1/16	2.874	11	5/8	ZIG 2 ZAG
6 1/2	12 5/16	7/8	"	8 7/16	7 15/16	"	9 7/16	"	12	"	10 5/8	2.781	12	"	"
7	12 13/16	15/16	"	8 15/16	8 7/16	"	9 15/16	"	"	"	11 1/8	2.912	13	"	"
7 1/2	13 3/8	15/16	"	9 1/2	9	"	10 1/2	"	"	"	11 11/16	3.060	14	"	"
8	14 1/4	1	"	10	9 1/2	"	11 1/16	"	"	1	12 3/8	3.239	15	"	"
8 1/2	14 3/4	1	"	10 1/2	10	"	11 9/16	"	13	"	12 7/8	3.111	16	"	"
9	15 5/16	1 1/16	"	11 1/16	10 9/16	"	12 1/8	"	"	"	13 7/16	3.247	17	"	"
9 1/2	15 13/16	1 1/16	"	11 9/16	11 1/16	"	12 5/8	"	14	"	13 15/16	3.128	18	"	"
10	16 3/8	1 1/8	"	12 1/8	11 5/8	"	13 3/16	"	15	"	14 1/2	3 037	19	"	"
10 1/2	17 1/8	"	3 1/2	12 7/8	12 3/8	"	13 15/16	"	"	"	15 1/4	3.194	18	3/4	"
11	17 11/16	"	"	13 7/16	12 15/16	"	14 1/2	"	"	"	15 13/16	3.311	19	"	"
11 1/2	18 11/16	1 3/16	"	13 15/16	13 7/16	"	15 1/8	"	'	1 1/8	16 9/16	3.469	20	"	"
12	19 1/4	"	"	14 1/2	14	"	15 11/16	"	16	1 1/8	17 1/8	3.362	20	"	"
13	20 5/16	1 1/4	"	15 9/16	15 1/16	"	16 3/4	"	17	1 1/8	18 3/16	3.361	22	"	"
14	21 3/8	1 5/16	"	16 5/8	16 1/8	"	17 13/16	"	18	1 1/8	19 1/4	3.359	24	"	"
15	22 13/16	1 3/8	"	17 11/16	17 3/16	"	18 15/16	"	"	1 1/4	20 1/2	3.578	26	"	"

PRESSURES FROM 301 TO 500 LBS. PER SQ. INCH.

INTERNAL DIAM. OF PIPE.	DIMENSIONS OF FLANGE								BOLTS				RIVETS		
	DIAMETER	MEAN THICKNESS A	DEPTH OF BOSS B	DIAM. OF BOSS C	DIAM. OF BOSS D	RAD. OF FILLET E	DIAM. OF FEMALE PART F	DEPTH OF FEMALE PART G	NUMBER	DIAMETER	DIAM. OF PITCH CIRCLE	PITCH	NUMBER IN EACH ROW	DIAMETER	NUMBER OF ROWS
1/2	3 13/16	1/2	1/2	1 7/16		3/16	2	1/8	4	1/2	2 13/16	2.209	NONE	NONE	NONE
3/4	4 11/16	9/16	9/16	1 13/16		"	2 1/2	"	4	5/8	3 7/16	2.700	"	"	"
1	4 15/16	9/16	9/16	2 1/16		"	2 3/4	"	5	5/8	3 11/16	2.317	"	"	"
1 1/4	5 5/16	5/8	5/8	2 7/16		"	3 1/8	"	5	5/8	4 1/16	2.552	"	"	"
1 1/2	5 9/16	5/8	5/8	2 11/16		"	3 3/8	"	6	5/8	4 5/16	2.258	"	"	"
2	6 11/16	11/16	11/16	3 5/16		1/4	4 3/16	"	6	3/4	5 1/4	2.749	"	"	"
2 1/2	7 1/4	11/16	11/16	3 7/8		"	4 3/4	"	7	3/4	5 13/16	2.608	"	"	"
3	7 7/8	3/4	3/4	4 1/2		"	5 3/8	3/16	8	3/4	6 7/16	2.527	"	"	"
3 1/2	8 1/2	13/16	13/16	5 1/8		"	6	"	9	3/4	7 1/16	2.465	"	"	"
4	9 9/16	13/16	13/16	5 11/16		"	6 11/16	"	9	7/8	7 7/8	2.749	"	"	"
4 1/2	10 3/16	7/8	7/8	6 5/16		"	7 5/16	"	9	7/8	8 1/2	2.967	"	"	"
5	11 3/16	15/16	15/16	6 15/16		"	8	"	9	1	9 5/16	3.251	"	"	"
5 1/2	11 3/4	15/16	15/16	7 1/2		"	8 9/16	"	10	1	9 7/8	3.102	"	"	"
6	12 7/8	1	3 3/16	8 1/8	7 5/8	1/4	9 5/16	3/16	10	1 1/8	10 3/4	3.377	11	5/8	ZIG 2 ZAG
6 1/2	13 7/16	1 1/16	"	8 11/16	8 3/16	"	9 7/8	"	"	1 1/8	11 5/16	3.554	12	"	"
7	14	1 1/16	"	9 1/4	8 3/4	"	10 7/16	"	11	1 1/8	11 7/8	3.391	13	"	"
7 1/2	14 1/8	1 1/8	"	9 3/4	9 1/4	"	10 15/16	"	"	1 1/8	12 3/8	3.534	14	"	"
8	15 7/16	1 3/16	"	10 5/16	9 13/16	"	11 9/16	"	"	1 1/4	13 1/8	3.748	15	"	"
8 1/2	16	1 3/16	"	10 7/8	10 3/8	"	12 1/8	"	12	"	13 11/16	3.584	16	"	"
9	16 9/16	1 1/4	"	11 7/16	10 15/16	"	12 11/16	"	"	"	14 1/4	3.730	17	"	"
9 1/2	17 1/8	1 5/16	"	12	11 1/2	"	13 1/4	"	13	"	14 13/16	3.580	18	"	"
10	17 3/4	1 3/8	"	12 1/2	12	"	13 7/8	"	"	"	15 7/16	3.730	19	"	"
10 1/2	18 15/16	1 3/8	3 1/2	13 5/16	12 13/16	"	14 11/16	"	"	1 3/8	16 3/8	3.958	18	3/4	"
11	19 1/2	1 7/16	"	13 7/8	13 3/8	"	15 1/4	"	"	"	16 15/16	4.094	20	"	"
11 1/2	20 1/16	1 1/2	"	14 7/16	13 15/16	"	15 13/16	"	14	"	17 1/2	3.927	20	"	"
12	20 5/8	1 1/2	"	15	14 1/2	"	16 1/8	"	"	"	18 1/16	4.053	22	"	"
13	22 3/16	1 5/8	"	16 1/16	15 9/16	"	17 5/8	"	"	1 1/2	19 7/16	4.362	22	"	"
14	23 1/16	1 11/16	"	17 3/16	16 11/16	"	18 7/8	"	15	"	20 11/16	4.333	24	"	"
15	25 3/8	1 3/4	"	18 1/4	17 3/4	"	20 1/16	'	16	"	21 7/8	4.295	26	"	"

（千九百五年十一月二日 アメリカン、マシニスト）