A Note on the Backing of an Arch.

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If an arch, with parallel intrados aud extrados, could be constructed whose line of resistance is parallel to them, it might have been considered as a most reliable, and those tedious and laborious graphic solutions of a line of resistance often employed in this country may be wholly dispensed with, at least for a particular given loading and in fact in many other cases when the live load is small compared with the dead—a condition fulfilled in many an arch. The solution of the equations of such curves, however, seems to involve a difficulty, but the amount of backing which makes a line of resistance of a given arch satisfy this condition may be found very simply by the following method.

§ I. *General Considerations.*

Let the given arch be normal and of uniform thickness, that the extrados and intrados may be parallel and let it be symmetric about the vertical line through the crown, the free surface being horizontal. Take a line in the free surface parallel to the head of the arch as the axis of x, and the vertical line through the crown as that of y, the positive direction of the former being taken leftward and that of the latter downward. Let $y=f(x)$ be the equation of the extrados*, $\eta=\phi(x$, that of the upper limit of the backing, and $\eta'=\psi(x)$, $\eta''=\chi(x)$ those of the upper and lower boundaries of hollow space to be provided in the backing. Then we must have evidently

$$\eta \leqq \eta' \leqq \eta'' \leqq y.$$

Lastly let

w=unit weight of earth,

w'=ditto of backing,

W=ditto of arch ring $=\sigma w$ say,

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* It is a more common practice to make the intrados assume a required form, as circular or catenarian, but there will be no inconvenience to have the extrados given, as it is almost a universal practice to take an approximate baskethandled form for the actual arch, whatever the ideal theoretical form may be. The equation of the intrados may be easily written down; thus

$$y_0=y+t\cos\theta,$$
$$x_0=x-t\sin\theta,$$

in which x_0, y_0 are coordinates of a point of the required intrados, $y=f(x)$, $tg\theta=f'(x)$ and t is the thickness of the arch ring.

t=thickness of arch ring,
a=surcharge at crown (Ueberschüttungshöhe),
a_0=depth of backing at crown below the free surface,
b=rise plus ring thickness plus surcharge at crown,
λ=half span plus ring thickness,
$c=(1-sin\phi)/(1+sin\phi)$, ϕ being the angle of repose of the earth.

Then, since the line of resistance is parallel to the extrados, we have

$$1). \begin{cases} d(T\cos\theta) = -cw\eta d\eta \\ d(T\sin\theta) = w\eta dx + w'(y-\eta''+\eta'-\eta)dx + Wtds - \frac{Wt^2}{2}d\theta, \end{cases}$$

ds being an elementary arc length and $d\theta$ the angle of contingence of the extradosal curve, and T the thrust at the section under consideration.

If we take

$$|T|_{\theta=0} \equiv \frac{cw}{2}(b_0^2 - a_0^2),$$

and

$$w\eta + w'(y-\eta''+\eta'-\eta) = wy$$

or

$$2). \qquad \eta''-\eta' = \frac{w'-w}{w'}(y-\eta), \text{*}$$

then equations 1) give

$$d\left\{\frac{cw}{2}(b_0^2-\eta^2)\frac{dy}{dx}\right\} = wydx + Wds - \frac{Wt^2}{2}d\theta,$$

whence by integration and by putting $dy/dx=p$,

$$3.) \qquad b_0^2-\eta^2 = \frac{2}{c}\frac{1}{p}\int ydx + \frac{2\sigma t}{c}\frac{s}{p} - \frac{\sigma t^2}{c}\frac{\theta}{p}.$$

This enables us to find the function ϕ, and the backing can easily be determined by the relation 2).

§ 2. *Application to a Circular Arch.*

Let the equation of the extrados be

$$4). \qquad y = a + r - \sqrt{r^2-x^2},$$

r being the radius of the circle. In this case

$$p = \frac{x}{\sqrt{r^2-x^2}},$$

$$\int ydx = \int (a+r-\sqrt{r^2-x^2})dx$$

* This is evidently always possible.

$$= (a+r)x - \tfrac{1}{2}x\sqrt{r^2-x^2} - \frac{r^2}{2}\sin^{-1}\frac{x}{r},$$

$$s = r\sin^{-1}\frac{x}{r},$$

$$\theta = \sin^{-1}\frac{x}{r}.$$

Putting these values in 3), we obtain

$$5).\quad c(\eta^2 - b_0^2) = r^2 - x^2 - 2(a+r)\sqrt{r^2-x^2} + (r^2 - 2\sigma tr + \sigma t^2)\frac{\sqrt{r^2-x^2}}{x}\sin^{-1}\frac{x}{r}$$

$$= r^2\cos^2\theta - 2(a+r)r\cos\theta + (r^2 - 2\sigma tr + \sigma t^2)\theta\cot\theta$$

in which

$$0 \leqq \sin^{-1}\frac{x}{r} = \theta \leqq \frac{\pi}{2}.$$

§ 3. *Application to a Transformed Catenarian Arch.*

The equation of a transformed catenarian arch can readily be found to be

$$6).\qquad y = \frac{a}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right),$$

where

$$m = \lambda / \log\frac{b \pm \sqrt{b^2-a^2}}{a}. \ *$$

In this case

$$p = \frac{\sqrt{y^2-a^2}}{m},$$

$$\int y\,dx = \int \frac{a}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right)dx = m\sqrt{y^2-a^2},$$

$$s = \int\sqrt{1 + \left|\frac{dx}{dy}\right|^2}\,dy = \int\sqrt{\frac{y^2+m^2-a^2}{y^2-a^2}}\,dy.$$

To find this integral put

* It is immaterial which of double signs is taken, since

$$\log\frac{b+\sqrt{b^2-a^2}}{a} = -\log\frac{b-\sqrt{b^2-a^2}}{a}$$

$$\frac{1}{a^2}-\frac{1}{y^2}=\frac{z^2}{a^2} \quad *$$

and

$$k=\left|\frac{\sqrt{m^2-a^2}}{m}\right|.$$

Then

$$s=\int\frac{y^2+m^2-a^2}{\sqrt{y^2-a^2}\,\sqrt{y^2+m^2-a^2}}dy$$

$$=m\int\frac{dz}{\sqrt{1-z^2.\,1-k^2z^2}}+\frac{a^2}{m}\int\frac{z^2dz}{(1-z^2)\,\sqrt{1-z^2.1-k^2z^2}}.$$

The latter integral can readily be found to depend on an elliptic integral of second kind, by virtue of the relation

$$sn^{2n-3}u\,cn\,u\,dn\,u=(2n-3)\int sn^{2n-4}u\,du$$

$$-(2n-2)(1+k^2)\int sn^{2n-2}u\,du+(2n-1)\,k^2\int sn^{2n}u\,du, \quad **$$

so that we shall obtain

$$s=m\left\{\int\frac{dz}{\sqrt{1-z^2.\,1-k^2z^2}}-\int\frac{\sqrt{1-k^2z^2}}{\sqrt{1-z^2}}dz+\frac{z\sqrt{1-k^2z^2}}{\sqrt{1-z^2}}\right\}.$$

These integrals can be calculated by the wellknown transformation of Landen; thus if

$$k_r^1=|\sqrt{1-k_r^2}|,$$

$$k_{r+1}=\frac{1-k_r^1}{1+k_r^1},$$

$$tg(\phi_{r+1}-\phi_r)=k_r^1\,tg\,\phi_r,$$

$$u_r=\frac{k_r}{\prod_{p=1}^{r}(1+k_p)},$$

and if we put $z=sin\,\phi_1$, then

$$\int_0^z\frac{dz}{\sqrt{1-z^2.1-k_1^2z^2}}=\int_0^{\phi_1}\frac{d\phi_1}{\sqrt{1-k_1^2\,sin^2\,\phi_1}}=\prod_{r=1}^{\infty}(1+k_r)\,\underset{n=\infty}{Lim}\frac{\phi_n}{2^n}=\frac{2K}{\pi}\underset{n=\infty}{Lim}\frac{\phi_n}{2^n}, \quad ***$$

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* See *Appell et Lacour—Principes de la theorie des fonctions elliptiques*, p. 244.

** See also *Lévy—Fonctions elliptiques, Ch. VI, Ex.* 3.

*** *Durège—Theorie der elliptischen Funktionen.*

$$\int_0^{z}\frac{\sqrt{1-k_1^2z^2}}{\sqrt{1-z^2}}dz=\int_0^{\phi_1}\sqrt{1-k_1^2\sin^2\phi_1}\,d\phi_1=1-\sum_{r=1}^{\infty}2^r\frac{n_r}{k_r}\int_0^{\phi_1}\frac{d\phi_1}{\sqrt{1-k_1^2\sin^2\phi_1}}+\sum_{r=1}^{\infty}n_r\sin\phi_r, \text{ *}$$

where K denotes the complete elliptic integral of the first kind.

They can also be most simply calculated by the aid of tables ** if such were at hand.

Putting the equation of s in the form after Legendre, we have

$$s=m\left\{F(k,\phi)-E(k,\phi)+tg\ \phi\sqrt{1-k^2\ \sin^2\phi}\right\},$$

taken between suitable limits, ϕ being defined by

$$\sin\phi=\frac{\sqrt{y^2-a^2}}{y},\qquad 0\leqq\phi\leqq\frac{\pi}{2}.$$

Substituting the values thus found in 3). we arrive at

$$7).\quad c(\eta^2-b_0^2)=\frac{2atm}{\sqrt{y^2-a^2}}\left\{mE(k,\phi)-mF(k,\phi)-\frac{\sqrt{y^2-a^2}\sqrt{y^2+m^2-a^2}}{y}+\frac{t}{2}tg^{-1}\frac{\sqrt{y^2-a^2}}{m}\right\}-2m^2$$

in which

$$0\leqq tg^{-1}\frac{\sqrt{y^2-a^2}}{m}\leqq\frac{\pi}{2}.$$

§ 4. *Application to a Geostatic Arch.*

The tracing of a geostatic curve by the transformation of a hydrostatic, as pointed out by Rankine, involves much tedious, laborious and yet inaccurate graphic methods, and it may well be substituted by an analytical solution, which gives directly the equation of a geostatic arch. The required equation can clearly be obtained by solving the equations

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* *Cayley—Elliptic Functions.*

** *Legendre —Fonctions elliptiques.*

Lévy—Fonctions elliptiques

I have annexed the tables, with their explanations, at the end of this paper, as such are not found in the current pocket books for engineers.

$$d(T\cos\theta) = -c_0 wy\,dy,$$
$$d(T\sin\theta) = wy\,dx,$$

$\sqrt{c_0}$ being what is termed "the ratio of transformation" by Rankine, so that

$$8).\qquad \frac{dy}{dx} = \frac{\sqrt{(b^2-a^2)^2-(b^2-y^2)^2}}{\sqrt{c_0}\,(b^2-y^2)},$$

and

$$9).\qquad x = b\sqrt{c_0}\left|mE(k,\phi) - \frac{1}{m}F(k,\phi)\right|_{\frac{\pi}{2}}^{\phi},$$

in which

$$m = \sqrt{2b^2-a^2}/b,$$
$$E(k,\phi) = \int\sqrt{1-k^2\sin^2\phi}\,d\phi,$$
$$F(k,\phi) = \int\frac{d\phi}{\sqrt{1-k^2\sin^2\phi}},$$
$$k = \left|\sqrt{\frac{2(b^2-a^2)}{2b^2-a^2}}\right|,$$
$$\sin^2\phi = (2b^2-a^2-y^2)/2(b^2-a^2), \qquad 0 \leqq \phi \leqq \frac{\pi}{2},$$

c_0 being determined by the equation

$$\lambda = b\sqrt{c_0}\left\{m\int_{\frac{\pi}{2}}^{\frac{\pi}{4}}\sqrt{1-k^2\sin^2\phi}\,d\phi - \frac{1}{m}\int_{\frac{\pi}{2}}^{\frac{\pi}{4}}\frac{d\phi}{\sqrt{1-k^2\sin^2\phi}}\right\}.$$

These elliptic integrals can be calculated by the methods explained in § 3, or directly by the annexed tables.

Thus we obtain

$$\int y\,dx = \int\frac{y}{p}\,dy = \frac{\sqrt{c_0}}{2}\sqrt{(b^2-a^2)^2-(b^2-y^2)^2},$$

and

$$s = \int\sqrt{1+\left|\frac{dx}{dy}\right|^2}\,dy = \int\frac{\sqrt{(b^2-a^2)^2-(1-c_0)(b^2-y^2)^2}}{\sqrt{(b^2-a^2)^2-(b^2-y^2)^2}}\,dy.$$

As this integral, which is a hyperelliptic integral * of first order, cannot be easily calculated, the value of s may be best found by a graphic solution, either by subdividing the arc into a number of sufficiently small segments or by the use of a universal map measurer.

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* For an elementary treatise on the subject, see *Königsberger—Vorlesungen über die Theorie der hyperelliptischen Integrale.*

These values, by virtue of 3), give rise to the formula

10). $c(\eta^2-b_0^2)=c_0(y^2-b^2)-\frac{\sigma t}{p}(2s-t\,tg^{-1}p)$

in which

$$o\leqq tg^{-1}p\leqq\tfrac{\pi}{2}.$$

§. 5. *Application to an Elliptic Arch.*

Let the equation of the extrados be

11). $$\frac{(b-y)^2}{(b-a)^2}+\frac{x^2}{\lambda^2}=1.$$

In this case

$$p=\frac{b-a}{\lambda}\;\frac{x}{\sqrt{\lambda^2-x^2}},$$

$$\int ydx=\int\left(b-\frac{b-a}{\lambda}\sqrt{\lambda^2-x^2}\right)dx=bx-\frac{b-a}{2\lambda}\left(x\sqrt{\lambda^2-x^2}+\lambda^2 sin^{-1}\frac{x}{\lambda}\right),$$

$$s=\lambda E(k,\phi),$$

where

$$E=\int_0^{\phi}\sqrt{1-k^2\,sin^2\phi}\;d\phi,$$

$$sin\phi=x/\lambda \qquad o\leqq\phi\leqq\tfrac{\pi}{2},$$

$$k=|\sqrt{\lambda^2-(b-a)^2}\,|/\lambda.$$

Thus we have

12). $$c(\eta^2-a_0^2)=\frac{2\lambda\sqrt{\lambda^2-x^2}}{b-a}\left\{\frac{b-a}{2\lambda}\left(\sqrt{\lambda^2-x^2}+\frac{\lambda^2}{x}sin^{-1}\frac{x}{\lambda}\right)-b\right\}$$

$$-\frac{\sigma t\lambda}{b-a}\;\frac{\sqrt{\lambda^2-x^2}}{x}\left\{2\lambda E(k,\phi)-t\,tg^{-1}\frac{(b-a)x}{\lambda\sqrt{\lambda^2-x^2}}\right\}$$

$$=\lambda^2\left(cos^2\phi+\phi cot\phi-\frac{2b}{b-a}\,cos\phi\right)$$

$$-\frac{\sigma t\lambda}{b-a}cot\phi\left\{2\lambda E(k,\phi)-t\,tg^{-1}\left(\frac{b-a}{\lambda}tg\phi\right)\right\},$$

in which

$$o\leqq tg^{-1}\frac{(b-a)x}{\lambda\sqrt{\lambda^2+x^2}}\leqq\frac{\pi}{2}\quad\text{and}\quad o\leqq sin^{-1}\frac{x}{\lambda}=\phi\leqq\frac{\pi}{2}.$$

For $\lambda = b - a = r$, 12) reduces to 5), as it should be.

§. 6. *Note on the Tables.*

Table I contains the values of the complete elliptic integrals of the first and the second kind (K and E), for various values of the angle of modulus $\theta = sin^{-1}k$.

Table II contains the values of the elliptic integrals of the first kind for various values of the angle of modulus and of the argument $\phi = sin^{-}\ sn\ u$.

Table III contains the corresponding values of the elliptic integrals of the second kind. All these tables, as will appear at the first glance, serve also to calculate the values of the elliptic integrals not included therein, simply by means of proportional parts.

Table IV is not required for the subject in question so far, but I have added it by the way. It may serve to calculate the values of the elliptic integrals of the third kind. Thus putting an integral of this kind in the form

$$\int_0^u \frac{du}{sn^2u - sn^2a},$$

where a is a constant not of the form $2mK + 2m'iK'$, we have, adopting the notations of Jacobi,

$$2sn\ a\ cn\ a\ dn\ a\int_0^u \frac{du}{sn^2u - sn^2a} = log\frac{H(u-a)}{H(u+a)} + 2u\frac{(\theta)'(a)}{(\theta)(a)}.$$

The functions H and θ may be determined by any method,*but, in numerical calculations it is most convenient to express them in trigonometric series, which are very rapidly convergent. In many cases it will be found that one or at most two terms in q will give sufficiently accurate results.

These series are

$$H(u) = 2\sum_{n=0}^{\infty}(-1)^n q^{\left(\frac{2n+1}{2}\right)^2} sin(2n+1)\frac{\pi u}{2K},$$

$$\theta(u) = 1 + \sum_{n=1}^{\infty}(-1)^n q^{n^2} cos\frac{n\pi u}{K},$$

in which, if ω_1 and ω_2 are respectively real and imaginary periods of the corresponding $p\acute{e}$ function,

$$q = e^{\frac{\pi\omega_2 i}{\omega_1}} = e^{-\frac{\pi K'}{K}},$$

* Refer to any elementary treatise on elliptic functions.

K' being the complete elliptic integral of the first kind with the modulus complementary to that of K.

TABLE 1.

θ	K	E	θ	K	E	θ	K	E	θ	K	E
0°	1,57080	1,57080	40°	1,78677	1,39314	70°.0′	2,50455	1,11838	84°.0′	3,65186	1,01724
1	092	068	41	9922	38489	30	2729	11399	12	8525	628
2	127	032	42	1,81216	37650	71. 0	5073	10964	24	3,71984	534
3	187	1,56972	43	2560	36800	30	7490	10533	36	5572	443
4	271	888	44	3957	35938	72. 0	9982	106	48	9298	354
5	379	781	45	5407	35064	30	2,62555	09683	85. 0	3,83174	266
6	511	650	46	6915	34181	73. 0	5214	265	12	7211	181
7	668	495	47	8481	33287	30	7962	8851	24	3,91423	099
8	849	296	48	1,90108	32384	74. 0	2,70807	443	36	5827	018
9	1,58054	114	49	1800	31473	30	3752	039	48	4,00437	0940
10	284	1,55889	50	3558	30554	75. 0	6806	7641	86, 0	5276	865
11	539	640	51	5386	29628	30	9975	248	12	4,10366	792
12	820	368	52	7288	28695	76. 0	2,83267	6861	24	5736	721
13	1,59125	073	53	9267	27757	30	6691	480	36	4,21416	653
14	457	1,54755	54	2,01327	26815	77. 0	2,90256	106	48	7444	588
15	814	415	55	3472	25868	30	3974	5738	87. 0	4,33865	526
16	1,60198	052	56	5706	24918	78. 0	7857	378	12	40733	466
17	608	1,53667	57	8036	23966	30	3,01918	024	24	8115	410
18	1,61045	260	58	2,10466	23013	79. 0	6173	4679	36	56190	356
19	510	1,52831	59	3002	22059	30	3,10640	4341	48	64765	306
20	1,62003	380	60	2,15652	21106	80. 0	5339	011	88, 0	74272	258
21	523	1,51908	61	8421	0154	12	7288	3882	12	84785	215
22	1,63073	415	62	2,21319	19205	24	9280	754	24	96542	174
23	632	1,50901	63	4355	18259	36	3,21317	628	36	5,09876	137
24	1,64260	366	64	7538	17318	48	3400	503	48	25274	104
25	900	1,49811	65	2,30879	16383	81. 0	5530	379	89. 0	43491	075
26	1,65570	237	66	4390	15455	12	7711	257	6	54020	062
27	6272	1,48643	67	8087	14535	24	9945	126	12	65792	050
28	7006	029	68	2,41984	13624	36	3,32234	017	18	79140	039
29	7773	1,47397	69	6100	12725	48	4580	2900	24	94550	030
30	1,68575	1,46746	70	2,50455	11838	82. 0	6987	784	30	6,12778	021
31	9411	077				12	9457	670	36	35038	014
32	1,70284	1,45391				24	3,41994	558	42	63854	008
33	1192	44687				36	4601	447	48	7,04398	004
34	2139	43966				48	7282	338	54	73711	001
35	3125	229				83. 0	3,50042	231	90	∞	000
36	4150	42476				12	2884	126			
37	5217	41707				24	5814	023			
38	6326	40924				36	8837	1921			
39	7479	126				48	3,61959	821			

TABLE II,

φ	θ									
	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
1°	0,01745	0,01745	0,01745	0,01745	0.01745	0,01745	0,01745	0,01745	0,01745	0,01745
2	03491	03491	03491	03491	03491	03491	03491	03491	03491	03491
3	05236	05236	05236	05236	05236	05236	05237	05237	05237	05237
4	06981	06981	06981	06982	06982	06982	06983	06983	06984	06984
5	08727	08727	08727	08727	08728	08729	08729	08730	08731	08732
6	10472	10472	10473	10473	10474	10475	10477	10478	10480	10482
7	12217	12218	12218	12219	12221	12223	12225	12227	12230	12233
8	13963	13963	13964	13966	13968	13977	13974	13978	13981	13985
9	15708	15708	15710	15712	15715	15719	15724	15729	15735	15740
10	17453	17454	17456	17459	17464	17469	17475	17482	17490	17498
11	19199	19200	19202	19206	19212	19220	19228	19237	19247	19258
12	20944	20945	20949	20954	20962	20971	20982	20994	21007	21021
13	22689	22691	22695	22702	22712	22724	22738	22753	22770	22787
14	24435	24436	24442	24451	24463	24478	24495	24514	24535	24556
15	26180	26182	26189	26200	26215	26233	26254	26278	26303	26330
16	27925	27928	27936	27949	27967	27989	28015	28044	28075	28107
17	29671	29674	29684	29699	29721	29748	29779	29813	29850	29889
18	31416	31420	31431	31450	31475	31507	31544	31585	31629	31675
19	33161	33166	33179	33201	33231	33268	33312	33360	33412	33466
20	34907	34912	34927	34953	34988	35031	35082	35138	35199	35262
21	36652	36658	36676	36706	36746	36796	36855	36920	36990	37063
22	28397	38404	38425	38459	38505	38563	38630	38705	38786	38871
23	40143	40151	40174	40213	40266	40331	40408	40494	40587	40683
24	41888	41897	41924	41968	42027	42102	42189	42287	42392	42503
25	43633	43643	43674	43723	43791	43875	43973	44084	44203	44328
26	45379	45390	45424	45479	45555	45650	45761	45885	46020	46161
27	47124	47137	47174	47236	47321	47427	47551	47690	47841	48000
28	48869	48883	48925	48994	49089	49207	49345	49500	49669	49846
29	50615	50630	50677	50753	50858	50988	51142	51315	51503	51700
30	52360	52377	52428	52513	52628	52773	52943	53134	53343	53562
31	54105	54124	54181	54273	54401	54560	54747	54959	55189	55432
32	55851	55871	55933	56035	56175	56349	56555	56788	57042	57310
33	57596	57619	57686	57797	57950	58141	58367	58623	58902	59197
34	59341	59366	59439	59561	59727	59936	60483	60463	60769	61093
35	61087	61113	61193	61325	61506	61734	62003	62308	62643	62998
36	62832	62861	62948	63090	63287	63534	63827	64159	64524	64912
37	64577	64609	64702	64857	65070	65337	65655	66016	66413	66836
38	66323	66356	66457	66624	66854	67144	67487	67879	68309	68769
39	68068	68104	68213	68393	68641	68953	69324	69747	70214	70713
40	69813	69852	69969	70162	70429	70765	71165	71622	72126	72667
41	71558	71600	71726	71933	72219	72580	73010	73502	74047	74632
42	73304	73349	73483	73704	74011	74398	74860	75389	75976	76608
43	75049	75097	75240	75477	75805	76219	76714	77282	77914	78594
44	76794	76846	76998	77251	77600	78043	78573	79182	79860	80592
45	0,78540	0,78594	0,78756	0,79025	0,79398	0,79871	0,80437	0,81088	0,81815	0,82602

TABLE II.

φ	θ								
	50°	55°	60°	65°	70°	75°	80°	85°	90°
1°	01745	0,01745	0,01745	0,01745	0,01745	0,01745	0,01745	0,01745	0.01745
2	03491	03491	03491	03491	03491	03491	03491	03491	03491
3	05237	05238	05238	05238	05238	05238	05238	05238	05238
4	06985	06985	06986	06986	06986	06987	06987	06987	06987
5	08733	08734	08735	08736	08736	08737	08737	08738	08738
6	10483	10485	10486	10488	10489	10490	10491	10491	10491
7	12235	12238	12240	12242	12244	12246	12247	12248	12248
8	13989	13993	13997	14000	14003	14005	14007	14008	14008
9	15746	15751	15757	15761	15765	15769	15771	15772	15773
10	17505	17513	17520	17526	17532	17536	17540	17542	17543
11	19268	19278	19288	19296	19304	19310	19314	19317	19318
12	21034	21047	21059	21071	21080	21088	21094	21098	21099
13	22804	22821	22836	22851	22863	22873	22880	22865	22886
14	24578	24599	24618	24636	24652	24664	24674	24680	24681
15	26356	26382	26406	26428	26448	26463	26475	26482	26484
16	28139	28171	28200	28227	28251	28270	28284	28293	28295
17	29927	29965	30001	30034	30062	30085	30102	30112	30116
18	31721	31766	31809	31848	31881	31909	31929	31942	31946
19	33520	33574	33624	33670	33710	33742	33766	33781	33786
20	35326	35388	35447	35501	35548	35586	35615	35632	35638
21	37137	37210	37279	37342	37396	37441	37474	37494	37501
22	38956	39040	39119	39192	39255	39307	39346	39369	39377
23	40782	40878	40969	41053	41126	41186	41230	41257	41266
24	42614	42724	42829	42925	43008	43077	43128	43159	43169
25	44455	44580	44699	44808	44904	44982	45040	75075	45088
26	46304	46445	46580	46704	46812	46901	46967	470 8	47021
27	48161	48320	48472	48612	48735	48835	48910	48956	48972
28	50027	50206	50377	50534	56672	50785	50870	50922	50939
29	51902	52102	52293	52470	52624	52752	52847	52905	52925
30	53787	54009	54223	54420	54593	54736	54843	54908	54931
31	55681	55928	56166	56386	56579	56739	56858	56931	55956
32	57586	57860	58123	58367	58582	58760	58893	58975	59003
33	59501	59803	60095	60365	60604	60802	60950	61042	61073
34	61427	61760	62082	62381	62646	62865	63029	63131	63166
35	63364	63730	64085	64415	64707	64950	65132	65245	65284
36	65313	65715	66104	66468	66790	67058	67260	67385	67428
37	67273	67713	68141	68540	68895	69131	69414	69552	69599
38	69246	69727	70195	70633	71023	71349	71594	71747	71799
49	71232	71756	72267	72746	73175	73533	73804	73972	74029
40	73231	73801	74358	74882	75352	75745	76043	76228	76291
41	75243	75862	76469	77041	77555	77987	78313	78517	78586
42	77269	77940	78600	79224	79786	80258	80617	80841	80917
43	79308	80035	80752	81432	82045	82562	82954	83200	83284
44	81362	82149	82926	83665	84333	84898	85329	85598	85690
45	0,83431	0,84281	0,85122	0,85925	0,86653	0,87270	0,87741	0,88037	0,88137

TABLE II.

φ	θ									
	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
46°	0,80285	0,80343	0,80515	0,80801	0,81198	0,81701	0,82305	0,83001	0,83779	0,84523
47	82030	82092	82275	82578	82999	83535	84178	84920	85752	86656
48	83776	83841	84025	84356	84803	85371	86055	86846	87734	88701
49	85521	85590	85795	86135	86609	87211	87937	88779	89725	90759
50	87266	87339	87556	87915	88416	89054	89825	90719	91725	92829
51	89012	89088	89317	89697	90226	90901	91716	92665	93735	94912
52	90757	90838	91078	91479	92037	92750	93613	94618	95755	97007
53	92502	92587	92841	93262	93850	94603	95514	96578	97784	0,99115
54	94248	94337	94603	95047	95666	96458	97420	0,98545	0,99822	1,01237
55	95993	96086	96366	96832	97483	0,98317	0,99331	1,00519	1,01871	03371
56	97738	97836	98130	0,98618	0,99302	1,00179	1,01247	02499	03928	05519
57	0,99484	0,99586	0,99894	1,00406	1,01123	02044	03167	04487	05996	07680
58	1,01229	1,01336	1,01658	02194	02946	03912	05092	06481	08073	09854
59	02974	03086	03423	03984	04770	05783	07021	08482	10159	12042
60	04720	04837	05188	05774	06597	07657	08955	10490	12256	14243
61	06465	06587	06954	07566	08425	09534	10894	12504	14361	16457
62	08210	08338	08720	09358	10255	11414	12837	14525	16476	18685
63	09956	10088	10486	11151	12087	13296	14784	16552	18601	20926
64	11701	11839	12253	12945	13920	15182	16735	18586	20735	23180
65	13446	13590	14020	14740	15755	17070	18691	20626	22877	25447
66	15192	15340	15787	16536	17592	18961	20651	22672	25029	27727
67	16937	17091	17555	18333	19430	20854	22615	24724	27190	30020
68	18682	18842	19324	20130	21269	22750	24583	26782	29359	32325
69	20428	20593	21092	21928	23110	24648	26555	58846	31537	34642
70	22173	22345	22861	23727	24953	26548	28530	30915	33723	36972
71	23918	24096	24630	25527	26796	28451	30509	32990	35917	39313
72	25664	25847	26400	27328	28641	30356	32491	35070	38118	41666
73	27409	27599	28169	29129	30488	32263	34477	37155	40328	44030
74	29154	29350	29939	30930	32335	34172	36466	39244	42544	46404
75	30900	31102	31710	32733	34184	36083	38457	41339	44767	48788
76	32645	32853	33480	34535	36034	47996	40452	43437	46997	51183
77	34390	34605	35251	36339	37884	39911	42449	45540	49232	53586
78	36136	36356	37022	38143	39736	41827	44449	47647	51474	55999
79	37881	38108	38793	39947	41588	43744	46451	49757	53721	58419
80	39626	39860	40565	41752	43442	45663	48455	51870	55973	60848
81	41372	41612	42336	43557	45296	47583	50462	53987	58230	63283
82	43117	43364	44108	45362	47150	49504	52470	56106	60491	65725
83	44862	45115	45879	47168	49005	51426	54479	58228	62756	68172
84	46608	46867	47651	48974	50861	53350	56490	60352	65024	70625
85	48353	48619	49423	50781	52717	55273	58503	62478	67295	73082
86	50098	50371	51195	52587	54574	57198	60516	64605	69569	75542
87	51844	52123	52968	54394	56431	59123	62530	66734	71844	78006
88	53589	53875	54740	56200	58288	61048	64545	68864	74121	80472
89	55334	55627	56512	58007	60145	62974	66560	70994	76399	82939
90	1.57080	1,57379	1,58284	1,59814	1,62003	1,64900	1,68575	1,73125	1,78677	1,85407

TABLE II.

φ	θ 50°	55°	60°	65°	70°	75°	80°	85°	90°
46°	0,85515	0,86431	0,87342	0,88231	0,89005	0,89678	0,90193	0,90517	0,90628
47	89614	88601	89585	90529	91390	92124	92687	93042	93163
48	80729	90791	91853	92875	93811	94610	95226	95614	95747
49	91860	93001	94146	95252	96267	97139	0,97810	0,98235	0,98381
50	94008	95232	96465	0,97660	0,98762	0,99711	1,00444	1,00909	1,01068
51	96171	97484	0.98811	1,00102	1,01397	1,02329	03129	03638	03812
52	0,98352	0,99759	1,01185	02578	03876	04995	05868	06425	06616
53	1,00550	1,02055	03587	05089	06491	07711	08665	09274	09483
54	02765	04374	06018	07637	09155	10481	11521	12188	12418
55	04998	06716	08479	10223	11855	13307	14442	15171	15423
56	07248	09082	10971	12848	14624	16190	17430	18229	18505
57	09517	11472	13494	15513	17433	19136	20488	21364	21667
58	11803	13886	16050	18220	20295	22145	23623	24582	24916
59	14108	16325	18638	20970	23212	25223	26837	27890	28257
60	16432	18788	21254	23764	26186	28371	30135	31292	31696
61	18773	21277	23916	26604	29219	31594	33524	34795	35240
62	21134	23792	26606	29490	32314	34897	37008	38407	38899
63	23513	26332	29332	32425	35473	38281	40594	42135	42679
64	25910	28898	32094	35409	38699	41753	44288	45989	46591
65	28326	81491	34893	38443	41994	45316	48098	49977	50645
66	30760	34109	37728	41529	45360	38976	52081	54112	54855
67	33212	36753	40600	44668	48800	52738	56096	58404	59232
98	35683	39423	43510	47860	52317	56606	60308	62868	63794
69	38171	42119	46457	51107	55913	60586	64661	67518	68557
70	40677	44840	49441	54410	59591	64684	69181	72872	73542
71	43200	47587	52463	57768	63352	68905	73877	77450	78771
72	45739	50359	55522	61182	67198	73256	78759	82774	84273
73	48296	53155	58618	64653	71132	77743	83844	88370	90079
74	50867	55974	61750	68180	75155	82371	89146	1,94267	1,96226
75	53455	58817	64918	71763	79269	87145	1,94682	2,00499	2,02759
76	56056	61682	68120	75401	83473	92073	2,00470	07106	09732
77	58672	64569	71356	79094	87764	1,97157	06529	14136	17212
78	61302	67476	74625	82840	92154	2,02403	12878	21614	25280
79	63943	70403	77924	86637	1,96630	07813	19538	29694	34040
80	66597	73347	81253	90484	2,01193	13390	26527	38365	43625
81	69261	76309	84609	94377	05840	19131	33866	47748	54209
82	71935	79286	87991	1,98313	10568	25035	41569	57954	66031
83	74618	82278	91395	2,02290	15371	31097	49648	69109	79422
84	77309	85281	94821	06303	20244	37309	58105	81362	2,94870
85	80006	88296	1,98264	10348	25178	43658	66935	2,94869	3,13130
86	82710	91320	2,01723	14421	30166	50129	76116	3,09782	35467
87	85418	94351	05194	18515	35198	56703	85612	26198	3,64253
88	88129	1,97388	08674	22627	40265	63357	2,95366	44116	4,04813
89	90843	2,00429	12161	26750	45354	70068	3,05304	63279	4,74135
90	1,93558	2,03472	2,15652	2,30879	2,50455	2,76806	3,15339	3,83174	∞

TABLE III.

φ	θ									
	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
1°	0,01745	0,01745	0,01745	0,01745	0,01745	0,01745	0,01745	0,01745	0,01745	0,01745
2	03491	03491	03492	03491	03491	03491	03490	03490	03490	03490
4	05236	05236	05236	05236	05236	05236	05235	05235	05235	05235
3	06981	06981	06981	06981	06981	06980	06980	06979	06979	06978
5	08727	08727	08726	08726	08725	08725	08724	08723	08722	08721
6	10472	10472	10471	10471	10470	10469	10467	10466	10464	10462
7	12217	12217	12216	12215	12214	12212	12210	12207	12205	12202
8	13963	13962	13961	13960	13957	13955	13951	13948	13944	13940
9	15708	15707	15706	15704	15700	15696	15692	15687	15681	15676
10	17453	17453	17451	17447	17443	17438	17431	17427	17417	17409
11	19199	19198	19195	19191	19185	19178	19169	19160	19150	19140
12	20944	20943	20939	20934	20926	20917	20906	20894	20881	20868
13	22639	22688	22683	22676	22667	22655	22641	22626	22609	22593
14	24434	24433	24427	24119	24406	24392	24374	24355	24335	24314
15	26180	26178	26171	26160	26145	26127	26106	26083	26058	26032
16	27925	27923	27914	27901	27883	27861	27836	27807	27777	27746
17	29671	29667	29658	29642	29629	29594	29563	29529	29493	29455
18	31416	31412	31401	31382	31357	31325	31289	31248	31205	31161
19	33161	33157	33143	33121	33092	33055	33012	32965	32914	32862
20	34907	34901	31886	34860	34825	34783	34733	34678	34619	34553
21	36652	36646	36628	36598	36558	36509	36451	36387	36319	36249
22	38397	38390	38370	38336	38290	38233	38167	38094	38015	37934
23	40143	40135	40111	40073	40020	39955	39880	39796	39707	39614
24	41888	41879	41852	41809	41749	41676	41590	41496	41394	41289
25	43633	43623	43593	43544	43477	43394	43298	43191	43076	42958
26	45379	45367	45333	45278	45203	45110	45002	44882	44753	44620
27	47124	47111	47074	47012	46928	46824	46703	46569	46425	46279
28	48869	48855	48813	48745	48651	48536	48402	48252	48092	47926
29	50615	50599	50553	50477	50373	50245	50097	49931	49753	49569
30	52360	52343	52292	52208	52094	51953	51788	51605	51409	51205
31	54105	54086	54030	53938	53813	53657	53476	53275	53059	52834
32	55851	55830	55768	55667	55530	55360	55161	54940	54708	54456
33	57596	57573	57506	57396	57245	57059	56842	56600	56341	56070
34	59341	59317	59243	59123	58959	58756	58520	58256	57972	57677
35	61087	61060	60980	60850	60672	60451	60194	59907	59598	59276
36	62832	62803	62716	62575	62382	62143	61864	61552	61217	60868
37	64577	64546	64452	64300	64091	63832	63530	63193	62830	62451
38	66323	66289	66188	66023	65798	65519	65193	64828	64436	64027
39	68068	68031	67923	67746	67503	67203	66851	66459	66035	65594
40	69813	69774	69658	69467	69207	68884	68506	68084	67628	67153
41	71558	71517	71392	71188	70909	70562	70157	69703	69214	68703
42	73304	73259	73126	72907	72609	72238	71804	71318	70798	70245
43	75049	75001	74859	74626	74307	73910	73446	72927	72365	71778
44	76794	70744	76592	76343	76003	75580	75085	74530	73931	73303
45	0,78540	78486	0,78324	0,78059	0,77697	0,77247	0,76720	0,76128	0,75489	0,74819

TABLE III.

φ	θ								
	50°	55°	60°	65°	70°	75°	80°	85°	90°
1°	0,01745	0,01745	0,01745	0,01745	0,01745	0,01745	0,01745	0,01745	0,01745
2	03490	03490	03490	03490	03490	03490	03490	03490	03490
3	05235	05234	05234	05234	05234	05234	05234	05234	05234
4	06978	06978	06977	06977	06976	06976	06976	06976	06976
5	08720	08719	08718	08718	08717	08716	08716	08716	08716
6	10461	10459	10458	10456	10455	10454	10453	10453	10453
7	12199	12197	12195	12192	12190	12189	12188	12187	12187
8	13936	13932	13929	13925	13923	13920	13919	13918	13917
9	15670	15665	15660	15655	15651	15648	15645	15644	15643
10	17401	17394	17387	17381	17375	17371	17367	17365	17365
11	19130	19120	19110	19102	19095	19089	19084	19082	19081
12	20855	20842	20830	20819	20809	20801	20796	20792	20791
13	22576	22559	22544	22530	22518	22508	22501	22497	22495
14	24293	24272	24253	24236	24221	24209	24200	24194	24192
15	26006	25981	25957	25936	25917	25902	25891	25884	25882
16	27714	27684	27655	27629	27606	27588	27575	27567	27564
17	29418	29381	29347	29315	29288	29267	29250	29241	29237
18	31116	31073	31032	30995	30963	30937	30917	30906	30902
19	32809	32758	32710	32666	32629	32598	32575	32561	32557
20	34496	34437	34381	34330	34286	34250	34224	34207	34202
21	36178	36109	36044	35985	35934	35892	35862	35843	35837
22	37853	37773	37699	37631	37572	37525	37490	37468	37461
23	39521	39431	39345	39268	39201	39146	39106	39081	39073
24	41183	32080	40983	40895	40819	40757	40711	40683	40674
25	42838	42722	42612	42513	42426	42356	42304	42273	42262
26	44486	44355	44232	44120	44023	43944	43885	43849	43837
27	46126	45980	45842	45716	45607	45518	45453	45413	45399
28	47759	47595	47441	47301	47180	47081	47007	46962	46947
29	49383	49202	49031	48875	48740	48629	48548	48498	48481
30	51000	50799	50609	50437	50287	50165	50074	50019	50000
31	52608	52386	52177	51986	51821	51686	51586	51525	51504
32	54207	53964	53733	53524	53341	53193	53082	53015	52992
33	55798	55531	55278	55048	54848	54684	54563	54489	54464
34	57379	57087	56811	56559	56340	56161	56028	55947	55919
35	58952	58634	58332	58057	57818	57622	57477	57388	57358
36	60515	60169	59841	59541	59280	59067	58909	58811	58779
37	62068	61693	61337	61011	60727	60495	60323	60217	60182
38	63612	63206	62828	62467	62159	61907	61720	61605	61566
39	65146	64707	64290	63908	63574	63302	63099	62974	62932
40	66671	66197	65746	65334	64974	64679	64459	64324	64279
41	68185	67675	67189	66745	66356	66038	65801	65655	65606
42	69688	69140	68619	68140	67722	67379	67124	66966	66913
43	71182	70594	70034	69520	69070	68701	68426	68257	68200
44	72665	72036	71435	70884	70401	70005	69710	69527	69466
45	0,74137	0,73465	0,72822	0,72232	0,71715	0,71289	0,70972	0,70777	0,70711

TABLE III.

φ	θ									
	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
46°	0,80285	0,80228	0,80056	0,79775	0,79390	0,78911	0,78350	0,77721	0,77040	0,76324
47	82030	81969	81787	81489	81081	80573	79977	79308	78584	77824
48	83776	83711	83518	83202	82770	82231	81599	80890	80121	79313
49	85521	85453	85249	84914	84457	83887	83217	82466	81651	80794
50	87266	87194	86979	86626	86142	85539	84832	84036	83173	82265
51	89012	88936	88709	88336	87826	87189	86442	85601	84689	83728
52	90757	90677	90438	90045	89507	88836	88048	87161	86197	85182
53	92502	92418	92166	91753	91187	90481	89650	88715	87698	86627
54	94248	94159	93895	93450	92865	92122	91248	90264	89193	88063
55	95993	95900	95622	95166	94541	93761	92843	91807	90680	89490
56	97738	97641	97350	96872	96216	95397	94433	93345	92160	90908
57	0,99484	0,99381	0,99077	0,98576	97889	97030	96019	94878	93634	92318
58	1,01229	1,01122	1,00803	1,00279	0,99560	0,98661	97602	96405	95100	93719
59	02974	02863	02529	01981	1,01129	1,00289	99180	97928	96560	95111
60	04720	04603	04255	03683	02897	01915	100756	0,99445	98013	96495
61	06465	06343	05980	05383	04563	03538	02327	1,00957	0,99460	97871
62	08210	08084	07705	07083	06228	05158	03895	02465	1,00900	0,99238
63	09956	09824	09430	08781	07891	06776	05459	03967	02334	1,00598
64	11701	11564	11154	10479	09553	08392	07020	05465	03762	01949
65	13446	13304	12878	12176	11213	10005	08577	06958	05183	03293
66	15192	15043	14601	13873	12871	11616	10132	08447	06599	04629
67	16937	16783	16324	15568	14529	13225	11683	09932	08009	05957
68	18682	18523	18047	17263	16185	14832	13231	11412	09413	07279
69	20428	20262	19769	18957	17839	16437	14776	12888	10812	08593
70	22173	22002	21491	20650	19493	18040	16318	14360	12205	09901
71	23918	23741	23213	22343	21145	19640	17857	15828	13594	11202
72	25664	25481	24935	24034	22796	21239	19394	17293	14977	12497
73	27409	27220	26656	25726	24446	22837	20928	18754	16356	13786
74	29154	28959	28377	27417	26094	24432	22459	20211	17731	15068
75	30900	30698	30097	29107	27742	26026	23989	21666	19101	16346
76	32645	32437	31818	30796	29389	27619	25516	23117	20467	17618
77	34390	34176	33538	32486	31035	29210	27041	24566	21830	18885
78	36136	35915	35258	34174	32680	30800	28565	26012	23189	20148
79	37881	37654	36978	35862	34325	32389	30086	27456	24544	21407
80	39626	39393	38698	37550	35968	33976	31606	28897	25897	22661
81	41372	41132	40417	39238	37611	35563	33124	30336	27246	23912
82	43117	42871	42137	40925	39254	37148	34641	31773	28594	25159
83	44862	44610	43856	42612	40896	38733	36157	33209	29939	26404
84	46608	46349	45575	44299	42537	40317	37672	34643	31282	27646
85	48352	48087	47294	45985	44174	41900	39186	26076	32623	28886
86	50098	49826	49013	47671	45819	43483	40699	37508	33963	30124
87	51844	51565	50732	49357	47459	45066	42211	38939	35302	31360
88	53589	53304	52451	51043	49100	46648	43723	40369	36640	32596
89	55334	55042	54170	52729	50740	48230	45235	41799	37977	33830
90	1,57080	0,56781	0,55889	0,54415	1,52380	1,49811	1,46746	1,43229	1,39314	1,35064

TABLE III.

φ	50°	55°	60°	65°	70°	75°	80°	85°	90°
46	0,75599	0,74881	0,74195	0,73564	0,73010	0,72554	0,72215	0,72005	0,71934
47	77050	76285	75553	74879	74287	73800	73436	73211	73135
48	78490	77676	76896	76177	75546	75025	74636	74396	74314
49	79920	79054	78225	77459	76786	76230	75815	75558	75471
50	81338	80419	79538	78724	78008	77414	76971	76697	76604
51	82746	81772	80836	79971	79208	78578	78106	77814	77715
52	84143	83111	82120	81202	80391	79720	79218	78907	78801
53	85529	84438	83388	82415	81554	80842	80307	79976	79864
54	86904	85752	84641	83610	82698	81941	81374	81021	80902
55	88269	87052	85879	84788	83822	83020	82417	82042	81915
56	89622	88340	87101	85949	84926	84076	83436	83039	82904
57	90965	89614	88308	87092	86011	85110	84432	84010	83867
58	92297	90876	89500	88217	87075	86122	85404	84957	84805
59	93619	92125	90677	89325	88119	87112	86352	85878	85717
60	94930	93362	91839	90415	89144	88080	87276	86773	86603
61	96231	94586	92986	91488	90148	89025	88175	87843	87462
62	97521	95797	94118	92543	91132	89948	89049	88486	88295
63	0,98802	96996	95236	93581	92096	90848	89898	89303	89101
64	1,00072	98183	95339	94602	93041	91725	90273	90094	89879
65	01333	0,99358	97427	95606	93965	92580	91523	90858	90631
66	02585	1,00522	98502	96593	94870	93412	92297	91595	91355
67	03827	01674	0,99562	97564	95756	94222	93047	92305	92050
68	05060	02815	1,00609	98518	96622	95010	93771	92987	92718
69	06284	03945	01643	0,99456	97469	95775	94470	93642	93358
70	07500	05064	02664	1,00379	98298	96519	95144	94270	93969
71	08707	06173	03672	01286	99108	97240	95793	94870	94552
72	09907	07272	04668	02178	0,99900	97940	96417	95442	95106
73	11098	08362	05651	03056	1,00674	98619	97016	95987	95630
74	12283	09442	06624	03919	01431	99278	97590	96503	96126
75	13460	10513	07586	04769	02172	0,99916	98141	96992	96593
76	14631	11577	08537	05607	02896	1,00534	98667	97453	97030
77	15795	12632	09478	06432	03605	01133	99170	97887	97437
78	16954	13680	10410	07245	04300	01714	0,99650	98293	97815
79	18107	14721	11333	08047	04981	02277	1'00107	68671	98162
80	19255	15755	12249	08839	05648	02823	00543	99023	98481
81	20399	16784	13156	09621	06304	03354	00958	99348	98769
82	21538	17807	14057	10395	06948	03870	01354	99646	99027
83	22673	18825	14952	11161	07582	04372	01731	0,99920	99255
84	23805	19839	15841	11920	08207	04863	02091	1,00168	99452
85	24934	20850	16726	12673	08825	05343	02436	00394	99619
86	26061	21857	17606	13421	09435	05813	02768	00598	99256
87	27186	22862	18484	14165	10041	06277	03089	00784	99863
88	28310	23865	19359	14906	10642	06735	03401	00954	99939
89	29432	24867	20233	15645	11241	07188	03708	01113	0,99985
90	1,30554	1,25868	1,21106	1,16383	1,11838	1,07641	1,04011	1,01266	1,00000

TABLE IV. —— Values of log q

θ	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′
0°	−∞	$\bar{7}$,12127	$\bar{7}$.72333	$\bar{6}$,07552	$\bar{6}$,32539	$\bar{6}$,51922	$\bar{6}$,67758	$\bar{6}$,81148	$\bar{6}$,92746	$\bar{5}$,02977	$\bar{5}$,12129	$\bar{5}$,20408
1	$\bar{5}$,27936	$\bar{5}$.34918	$\bar{5}$,41356	$\bar{5}$,47349	$\bar{5}$,52955	$\bar{5}$,58221	$\bar{5}$,68187	$\bar{5}$,67883	$\bar{5}$,72339	$\bar{5}$.76578	$\bar{5}$,80619	$\bar{5}$,84481
2	$\bar{5}$,88178	$\bar{5}$,91725	$\bar{5}$ 95132	$\bar{5}$,93411	$\bar{4}$,01571	$\bar{4}$,04620	$\bar{4}$,07565	$\bar{4}$,10414	$\bar{4}$,13173	$\bar{4}$,15847	$\bar{4}$,18441	$\bar{5}$,20960
3	$\bar{4}$,23403	$\bar{4}$,25789	$\bar{4}$,28106	$\bar{4}$,30363	32564	34710	36805	38819	40847	42801	44711	46581
4	48411	50203	51930	53681	55369	57025	58651	60246	61813	63352	64864	66351
5	67813	69250	70664	72056	73426	74775	76103	77411	78700	79970	81222	82456
6	83673	84874	86078	$\bar{4}$,87223	$\bar{4}$,88379	$\bar{4}$,89516	$\bar{4}$.90639	$\bar{4}$,91748	$\bar{4}$,92843	$\bar{4}$,93925	$\bar{4}$,94993	$\bar{4}$.96049
7	$\bar{4}$,97091	$\bar{4}$,98122	$\bar{4}$,99140	$\bar{3}$,00147	$\bar{3}$,01143	$\bar{3}$,02127	$\bar{3}$,03100	$\bar{3}$,04063	$\bar{3}$,05015	$\bar{3}$,05957	$\bar{3}$,06888	$\bar{3}$,07811
8	$\bar{3}$,08723	$\bar{3}$,09326	$\bar{3}$,10520	11405	12281	13148	14007	14858	15700	16534	17361	18180
9	18991	19795	20592	21381	22164	22939	23708	24470	25226	25975	26718	27454
10	28185	28910	29328	30341	31048	31750	32445	33136	33321	34501	35176	35846
11	36510	37170	37825	38475	39120	39760	40396	41028	41655	42277	42896	43510
12	44119	44725	45326	45924	46517	47107	47693	48274	48852	49427	49997	50564
13	51128	51688	52244	52797	53346	53893	54436	54975	55511	56045	56575	57101
14	57625	58146	58664	59178	59690	60199	60705	61208	61709	62206	62701	63194
15	63683	64170	64654	65136	65615	66091	66566	67037	67506	67973	68437	68899
16	69359	69316	70271	70724	71174	71622	72068	47512	72954	73393	73831	74266
17	74699	75130	75560	75987	76402	76835	77256	77675	78093	78508	78922	79333
18	79743	80151	80558	80932	81365	81763	86824	82562	82958	83352	83744	84135
19	84524	84911	85297	85681	83064	86445	91259	87202	87578	87953	88326	88698
20	89038	89437	89804	90170	90535	90898	91259	91620	91979	92336	92692	93047
21	93400	93752	94103	94452	94801	95147	95493	95387	$\bar{3}$,96180	$\bar{3}$,96522	$\bar{3}$,96863	$\bar{3}$,97202
22	$\bar{3}$,97540	$\bar{3}$,97877	$\bar{3}$,98213	$\bar{3}$,98547	$\bar{3}$,98880	$\bar{3}$,99212	$\bar{3}$,99543	$\bar{3}$,99873	$\bar{2}$,00202	$\bar{2}$,00529	$\bar{2}$,00855	$\bar{2}$,01181
23	$\bar{2}$,01505	$\bar{2}$,01828	$\bar{2}$,02150	$\bar{2}$,02471	$\bar{2}$,02791	$\bar{2}$,03109	$\bar{2}$,03427	$\bar{2}$,03744	04059	04374	04687	05000
24	05311	05621	05931	06239	06547	06853	07159	07433	07767	08069	08371	08672
25	08971	09270	09568	09865	10161	10456	10751	11044	11336	11628	11919	12209
26	12498	12786	13073	13360	13645	13930	14214	14497	14780	15061	15312	15622
27	15901	16179	16457	16734	17010	17285	17559	17833	18106	18378	18650	18920
28	19190	19459	19728	19996	20263	20529	20795	21060	21324	21588	21851	22113
29	22374	22635	22895	23155	23414	23672	23929	24186	24442	24698	24953	25207
30	25461	25714	25966	26218	26469	26719	26969	27219	27467	27715	27963	28210
31	28456	28702	28947	29191	29435	29679	29922	30164	30406	30647	30887	31127
32	31367	31606	31844	32082	32319	32556	32792	33028	33263	33498	33732	33966
33	34199	34431	34664	34895	35126	35357	35587	35816	36046	46274	36502	36730
34	36957	37184	37410	37636	37861	38086	38310	38534	38757	38980	39203	39425
35	39646	39867	40038	40308	40523	40747	40966	41185	41403	41620	41838	42054
36	42271	42487	42702	42917	43132	43346	43560	43774	43987	44199	44411	44623
37	44835	45046	45256	45467	45677	45886	46095	46304	46512	46720	46928	47135
38	47342	47548	47754	47960	48166	48371	48575	38779	48983	49187	49390	49595
39	49798	49998	50200	50410	50602	50803	51003	51204	51403	51603	51802	52001
40	52199	52397	52595	52792	52990	53186	53383	53579	53775	53971	54166	54361
41	54555	54750	54944	55137	55331	55524	55717	55909	56101	56293	56485	56676
42	56867	57058	57249	57439	57629	57818	58007	58197	58385	58574	58762	58950
43	59138	59325	59512	59699	59885	60072	60258	60443	60629	60814	60999	61184
44	$\bar{2}$,61368	$\bar{2}$,61553	$\bar{2}$,61737	$\bar{2}$,61920	$\bar{2}$,62104	$\bar{2}$,62287	$\bar{2}$,62470	$\bar{2}$,62653	$\bar{2}$,62835	$\bar{2}$,63017	$\bar{2}$,63199	$\bar{2}$,63381

TABLE VI. —— Values of log$g$

θ	0′	5′	10′	15′	20′	25′	30′	35′	40′	45′	50′	55′
45°	$\bar{2}$,63562	$\bar{2}$,63744	$\bar{2}$,63925	$\bar{2}$,64105	$\bar{2}$,64286	$\bar{2}$,64466	$\bar{2}$,64646	$\bar{2}$,64826	$\bar{2}$,65006	$\bar{2}$,65185	$\bar{2}$,65364	$\bar{2}$,65543
46	65722	65900	66078	66256	66434	66611	66789	66966	67143	67320	67496	67672
47	67848	68024	68200	68375	68550	68725	68900	69074	69249	69423	69597	69771
48	69944	70118	70291	70464	70636	70809	70981	71154	71326	71497	71669	71840
49	72012	72183	72353	72524	72695	72865	73035	73205	73375	73544	73714	73883
50	74052	74221	74390	74558	74726	74895	75063	75231	75398	75566	75733	75900
51	76067	76234	76401	76567	76733	76900	77066	77232	77398	77563	77729	77894
52	78059	78224	78389	78554	78718	78883	79047	79211	79375	79539	79703	79866
53	80030	80193	80356	80519	80682	80845	81007	81170	81332	81494	81656	81818
54	81980	82142	82303	82464	82626	82787	82948	83109	83270	83430	83591	83751
55	83911	84072	84232	84392	84552	84711	84871	85030	85190	85349	85508	85667
56	85826	85985	86144	86302	86461	86619	86778	86936	87094	87252	87410	87568
57	87726	87883	88041	88198	88356	88513	88670	88827	88984	89141	89298	89454
58	89611	89768	89924	90080	90237	90393	90549	90705	90861	91017	91173	91329
59	91484	91640	91795	91951	92106	92261	92417	92572	92727	92882	93037	93192
60	93347	93501	93656	93811	93965	94120	94274	94429	94583	94737	94891	95046
61	95200	95354	95508	95662	95816	95970	96123	96277	96431	96584	96738	96892
62	97045	97199	97352	97506	97659	97812	97966	98119	$\bar{2}$,98272	$\bar{2}$,98425	$\bar{2}$,98579	$\bar{2}$,98732
63	$\bar{2}$,98885	$\bar{2}$,99038	$\bar{2}$,99191	$\bar{2}$,99344	$\bar{2}$,99497	$\bar{2}$,99650	$\bar{2}$,99803	$\bar{2}$,99956	$\bar{1}$,00109	$\bar{1}$,00261	$\bar{1}$,00414	$\bar{1}$,00567
64	$\bar{1}$.00720	1,40873	1,01026	$\bar{1}$,01178	$\bar{1}$,01331	$\bar{1}$,01484	$\bar{1}$,01637	$\bar{1}$,01789	01942	02095	02247	02400
65	02553	02704	02858	03011	03163	03316	03469	03621	03774	03927	04079	04232
66	04385	04537	04690	04843	04996	05148	05301	05454	05607	05760	05912	06065
67	06218	06371	06524	06677	06830	06983	07136	07289	07442	07595	07748	07902
68	08055	08208	08361	08515	08668	08822	08975	09129	09282	09436	09590	09743
96	09897	10051	10205	10359	10513	10667	10821	10975	11130	11284	11439	11953
70	11748	11902	12057	12212	12367	12522	12677	12832	12987	13142	13298	13453
71	13609	13765	13920	14076	14232	14388	14544	14701	14857	15014	15170	15327
72	15484	15641	15798	15955	16113	16270	16428	16585	16743	16901	17059	17218
73	17376	17535	17693	17852	18011	18170	18330	18489	18649	18809	18969	19129
74	19289	19450	19610	19771	19932	20094	20255	20417	20578	20740	20903	21065
75	21228	21391	21554	21717	21880	22044	22208	22372	22537	22701	22866	23031
76	23197	23362	23528	23694	23861	24027	24194	24361	24529	24697	24865	25033
77	25202	25371	25540	25709	25879	26050	26220	26391	26562	26734	26906	27078
78	27250	27423	27597	27770	27944	28119	28294	28469	28645	28821	28997	29174
79	29351	29529	29707	29886	30065	30244	30424	30605	30786	30968	31150	31332
80	31515	31699	31883	32067	32253	32438	32625	32812	32999	33187	33376	33566
81	33756	33946	34138	34330	34523	34716	34910	35105	35301	35497	35694	35892
82	36091	36291	36491	36692	36894	37097	37301	37506	37712	37919	38127	38335
83	38545	38756	38968	39181	39395	39610	39827	40044	40263	40483	40705	40928
84	41152	41377	41604	41833	42063	42294	42527	42762	42998	43237	43476	43718
85	43962	44207	44455	44704	44956	45210	45466	45724	45985	46248	46514	46783
86	47054	47328	47605	47886	48169	48456	48746	49040	49338	49639	49945	50255
87	50569	50889	51213	51542	51877	52218	52565	52918	53278	53646	54021	54405
88	54798	55200	55613	56036	56472	56921	57384	57863	58359	58875	59412	59974
89	$\bar{1}$,60564	$\bar{1}$,61185	$\bar{1}$,61844	$\bar{1}$,62547	$\bar{1}$,63302	$\bar{1}$,64122	$\bar{1}$,65025	$\bar{1}$,66035	$\bar{1}$,67196	$\bar{1}$,68579	$\bar{1}$,70342	$\bar{1}$,72938