

論說及報告

On the Thickness of a Retaining Wall under some Particular Conditions.

By S. C. E.

Eleven months ago I wrote a short problem under the same subject which was published in Kōgakukwaishi №. 178. I intend here to make some extensions of the problem in the following two cases.

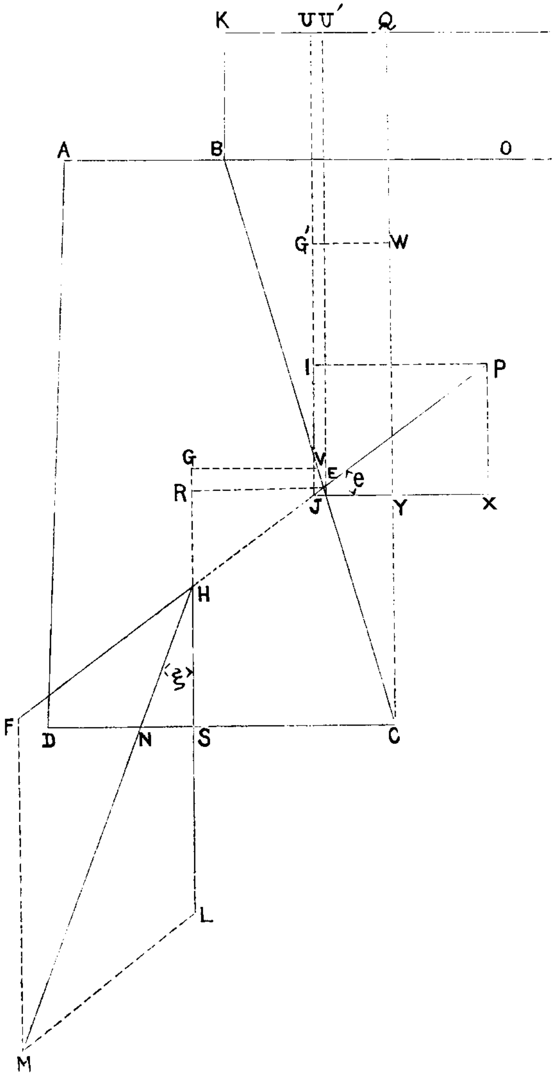
Case I.

To find the thickness of a wall retaining the earth under a uniform loading with the free surface in the same level plane as the top of the wall having uniform batters in its front and back surface.

I have stated in the above mentioned periodical that the true theory of the pressure of the earth is at present unknown even in its free state, and the problem becomes much more complicated when it is under an external loading I shall, as is done by some engineers, regard the external load to cause the earth the same result as if the latter were superposed with the earth whose weight is equal to the external load of any point.

Let ABCD be a wall retaining the earth under the uniform external load which is equivalent to the weight of the earth O B K Q superposed upon the free level surface O B of the earth retained by the wall, K Q being a level plane. Draw C Q perpendicular to K Q and let G′ be the centre of gravity of the trapezoid B K Q C, and G that of the wall A B C D. Let

$$AB = a$$
$$CD = b$$
$$\text{Height of wall} = h$$
$$\text{Batter of } AD = s : 1$$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" " $BC = r : 1$,

so that

$$a = b + \lambda h,$$

where

$$\lambda = r + s.$$

Let also

Weight of unit volume of the earth $= w$

" " " " " " " wall $=$ W.

Taking the unit thickness of the wall, we observe that the external forces exerted upon the wall are the vertical earth pressure equal to the weight of the earth C B K Q and the horizontal earth pressure. The line of the former passes through G′, and that of the latter through Y such that 3 C Y = C Q. Let IJ be the vertical and X J the horizontal pressure of the earth. The diagonal P J of the parallelogram XJIP represents the resultant pressure of the earth both in magnitude and direction. Produce PJ to meet a vertical line through G at H and take H F = P J. In the prolongation of GH set off HL equal to the weight of the wall and complete the parallelogram H L M F. The diagonal HM represents the resultant force of the pressure of the earth and the weight of the wall both in magnitude and direction. Let N be the point at which H M meets the base C D of the wall.

Let

$$BK = x;$$

then we have

$$G'U = \frac{1}{3}\,\frac{3x^2 + 3xh + h^2}{2x + h}$$

$$G'W = \frac{rh}{3}\,\frac{3x+h}{2x+h}$$

$$IJ = \frac{wrh}{2}(2x+h)$$

$$XJ = \frac{wch}{2}(2x+h)$$

$$\mathrm{tg}\,\theta = \frac{IJ}{XJ} = \frac{r}{c},$$

where $c=(1-\sin\phi)/(1+\sin\phi)$, ϕ being the angle of repose of the earth.

Now it can easily be shewn that

$$EU' = \frac{2}{3}\left\{x+h+\frac{x^2r^2}{(c+r^2)(2x+h)}\right\}$$

$$\therefore\ RS = x+h-EU'$$

$$= \frac{1}{3}\left\{x+h-\frac{2x^2r^2}{(c+r^2)(2x+h)}\right\}.$$

But

$$Gs = \frac{h}{3}\,\frac{3a+\lambda h}{2a+\lambda h}$$

and

$$GV = \frac{3a^2+3a\lambda h+\lambda^2h^2}{3(2a+\lambda h)}$$

$$\therefore\ GR = GS-RS$$

$$= \frac{ah}{3(2a+\lambda h)} - \frac{cx(2x+h)+xhr^2}{3(c+r^2)(2x+h)},$$

and consequently

$$RE = GV + r.GR$$

$$= \frac{3a^2+3a\lambda h+\lambda^2h^2+rah}{3(2a+\lambda h)} - \frac{r}{3}\,\frac{cx(2x+h)+xhr^2}{(c+r^2)(2x+h)}$$

$$\mathrm{RH} = \mathrm{RE}\,\mathrm{tg}\,\theta = \frac{r}{c}\,\mathrm{RE}$$

$$\mathrm{HS} = \mathrm{RS} - \mathrm{RH}$$

$$= \frac{h}{3} - \frac{r}{c}\,\frac{3a^2+3a\lambda h+\lambda^2h^2+rah}{3(2a+\lambda h)} + \frac{2cx^2+(c+r^2)hx}{3c(2x+h)}$$

$$\mathrm{NS} = \mathrm{HS}\,\mathrm{tg}\,\xi,$$

where

$$\mathrm{tg}\,\xi = \frac{wc(2x+h)}{wr(2x+h)+(2a+\lambda h)\mathrm{W}.}$$

Again
$$\mathrm{DS} = \frac{3a^2+3\lambda ah+\lambda^2h^2+3ash+\lambda sh^2}{3(2a+\lambda h)}$$

and
$$\mathrm{DN} = \mathrm{DS} - \mathrm{NS}.$$

Various authorities take different values for the ratio C D : D N. Let n denote the ratio for generalization. Then we get

$$\frac{\mathrm{CD}}{n} = \mathrm{DN},$$

i e.,

$$\frac{a+\lambda h}{w} = \frac{3a^2+3\lambda ah+\lambda^2h^2+3ash+\lambda sh^2}{3(2a+\lambda h)}$$

$$-\frac{w(2x+h)}{wr(2x+h)+\mathrm{W}(2a+\lambda h)}\left[\frac{2ach+c\lambda h^2-3ra^2-3\lambda rah-r^2ah-\lambda^2rh^2}{3(2a+\lambda h)}+\frac{2cx^2+(c+r^2)hx}{3(2x+h)}\right].$$

Transforming this equation we arrive at, after some simple reductions the following result : —

$$\left[(3n-6)a^2+(3n-9)\lambda ah+(n-3)\lambda^2h^2+3nash+n\lambda sh^2\right](2a+\lambda h)\,\mathrm{W}$$

$$=\left[nch(2a+\lambda h)-r\{(3n-3)a+(2n-3)\lambda h\}(2a+\lambda h)-nshr(2a+\lambda h)\right]wh$$

$$+\left[nch(2a+\lambda h)-r\{(6n-6)a+(4n-6)\lambda h\}(2a+\lambda h)-2nshr(2a+\lambda h)\right.$$

$$- nhr^2(2a+\lambda h) - 2ncx(2a+\lambda h)\rbrack wx.$$

But

$$2a + \lambda h \neq 0.$$

Removing, therefore, this common factor from both sides of the equation we obtain :

$$\lbrack(3n-6)a^2 + (3n-9)\lambda ah + (n-3)\lambda^2h^2 + 3nash + n\lambda sh^2\rbrack W$$
$$= \lbrack nch - r\{(3n-3)a + (2n-3)\lambda h\} - nshr\rbrack wh$$
$$+ \lbrack nch - r\{(6n-6)a + (4n-6)\lambda h\} - 2nshr - nhr^2 - 2wcx\rbrack wx.$$

This is a quadratic equation of a. Hence transposing we have

$$(3n-6)Wa^2 + \lbrack(3n-9)\lambda hW + 3nshW + (3n-3)rhw + (6n-6)rxw\rbrack a$$
$$= ncwh^2 - (2n-3)\lambda rwh^2 - nrswh^2 + ncwhx - (4n-6)\lambda rwhx$$
$$- 2nrswhx - nr^2whx - 2ncwx^2 - (n-3)w\lambda^2h^2 - n\lambda sWh^2.$$

Put

$$x \equiv ph.$$

Then the last equation becomes

$$(3n-6)Wa^2 + 3\lbrack\{(n-3)\lambda + ns\}W + \{(n-1)r + (2n-2)rp\}w\rbrack ha$$
$$= \lbrack\{nc - (2n-3)\lambda r - nrs + ncp - (4n-6)\lambda rp - 2nrsp - nr^2p - 2ncp^2\}w$$
$$- \{(n-3)\lambda^2 + n\lambda s\}W\rbrack h^2.$$

Put

$$\alpha \equiv 3n - 6$$
$$\beta \equiv (n-3)\lambda + ns$$
$$\gamma \equiv (n-1)r + 2(n-1)rp$$

$$\delta \equiv nc - (2n-3)\lambda r - nrs + ncp - (4n-6)\lambda rp - 2nrsp - nr^2p - 2ncp^2$$

$$\epsilon \equiv (n-3)\lambda^2 + n\lambda s;$$

then

$$\alpha W a^2 + 3(\beta W + \gamma w) h a = (\delta w - \epsilon W) h^2,$$

whence

$$a = \frac{-3(\beta W + \gamma w) \pm \sqrt{9(\beta W + \gamma w)^2 + 4\alpha W(\delta w - \epsilon W)}}{2\alpha W} h.$$

The upper or lower one of the double signs is to be taken according as $\delta w > \epsilon W$ or $\delta w < \epsilon W$.

Restoring the values of $\alpha, \beta, \gamma, \delta$ and ϵ, we obtain, after some reductions,

$$\frac{a}{h} = \frac{1}{2(n-2)W}\Big[-\{(n-3)r + (2n-3)s\}W - (n-1)(1+2p)rw$$

$$\pm\sqrt{\Big|\frac{1}{3}\{-(n-3)(n+1)r^2 - 6(n-1)rs + (2n-3)(2n-1)s^2\}W^2}$$

$$+ \frac{2}{3}\{(1+2p)[-(n^2-2n+3)r^2 + 3(n-1)rs] + 2(n-2)n(c+cp-pr^2-cp^2)\}Ww$$

$$+ \{(n-1)r(1+2p)\}^2 w^2\Big|\Big].$$

Put, for the sake of brevity,

$$m \equiv \frac{1}{3}\{-(n-3)(n+1)r^2 - 6(n-1)rs + (2n-3)(2n-1)s^2\}$$

$$p \equiv \frac{2}{3}\{(1+2p)[-(n^2-2n+3)r^2 + 3(n-1)rs] + 2n(n-2)(c+cp-pr^2-cp^2)\}$$

$$q \equiv \{(n-1)r(1+2p)\}^2;$$

then we finally get the following result : —

$$\frac{a}{h} = \frac{1}{2(n-2)W}\Big[-\{(n-3)r + (2n-3)s\}W - (w-1)(1+2p)rw \pm \sqrt{mW^2 + pWw + qw^2}\Big].$$

If, for instance, we take $n=3$,

then

$$\frac{a}{h}=\frac{1}{2W}\left[-6sW-2(1+2p)rw\pm\sqrt{m^1W^2+p^1Ww+qw^2}\right]$$

where

$$m^1=-4rs+\delta s^2$$

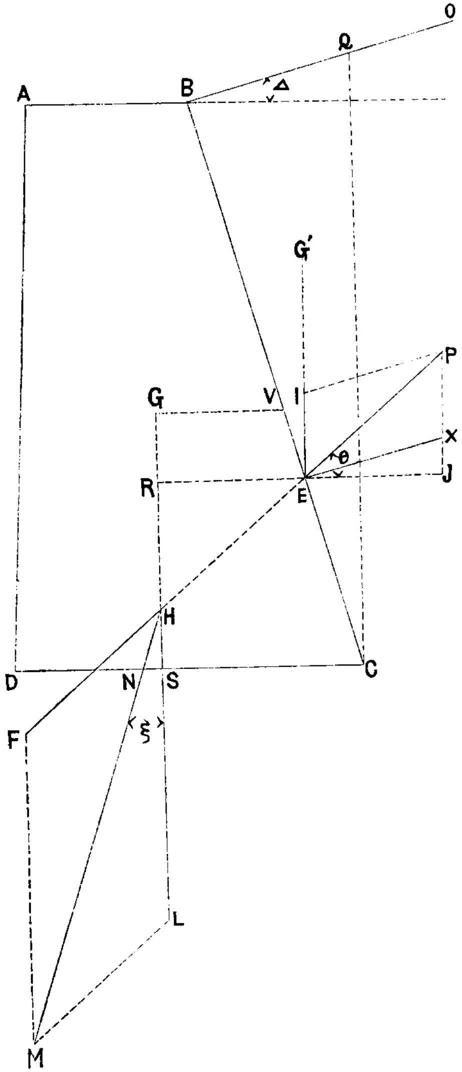
$$p^1=4(1+2p)(-r^2+rs)+4(c+cp-pr^2-cp^2)$$

$$q^1=4r^2(1+2p)^2.$$

Case II.

To find the thickness of a wall retaining the earth, under no external loading, with the free surface making an acute angle with and above the prolongation of the top level surface of the wall having uniform batters in its front and back surface.

Let ABCD be a wall retaining the earth whose free surface is the plane BO making an acute angle Δ with AB produced. Draw CQ perpendicular to CD meeting BO at Q. Taking the unit thickness of the wall the external forces acting upon the wall are the vertical pressure of the earth equal to the weight of the earth BCQ and the conjugate earth pressure whose direction is parallel to BO. Let IE denote the former and XE the latter pressure, both of which acting at a point E such that 3CE=CB. Complete the parallelogram IEXP; then PE denote the resultant earth pressure both in magnitude and direction. Produce PE to meet at H a vertical line through G, the centre of gravity of the wall, and take H F=P E. Seting off H L equal to the weight of the wall in the prolongation of GH, and completing the parallelogram HLMF, we have HM representing the resultant of the earth pressure and the weight of the wall both in magnitude and direction. Let N be the point at



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which HM cut the base CD.

Let

$$AB = a$$

$$CD = b$$

Height of wall $= h$

Batter of AD $= s : 1$

„ „ BC $= r : 1$.

so that $b = a + \lambda h,$

where

$$\lambda = r + s.$$

Let also

Weight of unit volume of earth = w

„ „ „ „ „ wall = W.

Then we have

$$IE = \frac{w}{2}(1 + r\,\mathrm{tg}\triangle)rh^2$$

$$EX = \frac{w}{2}(1 + r\,\mathrm{tg}\triangle)^2 ch^2,$$

where

$$c = \cos\triangle\;\frac{\cos\triangle - \sqrt{\cos^2\triangle - \cos^2\phi}}{\cos\triangle + \sqrt{\cos^2\triangle - \cos^2\phi}},$$

ϕ being the angle of repose of the earth.

Put

$$1 + \mathrm{tg}\,\triangle \equiv \tau;$$

then

$$\mathrm{I\,E} = \frac{w}{2}\tau r h^2$$

$$\mathrm{EX} = \frac{w}{2}\tau^2 c h^2.$$

Let θ be the angle which PE makes with the line EJ perpendicular to IE. Then

$$\mathrm{E\,J} = \frac{w}{2}\tau^2 c h^2 \cos\triangle$$

$$\mathrm{X\,J} = \frac{w}{2}\tau^2 c h^2 \sin\triangle,$$

so that

$$\mathrm{tg}\,\theta = \frac{\mathrm{PX+XJ}}{\mathrm{EJ}} = \frac{\mathrm{IE+XJ}}{\mathrm{EJ}}$$

$$= \frac{r + \tau c \sin\triangle}{\tau c \cos\triangle}.$$

Now

$$\mathrm{R\,E} = \frac{3a^2 + 3\lambda a h + \lambda^2 h^2 + r a h}{3(2a + \lambda h)}$$

$$\mathrm{RH} = \mathrm{RE}\,\mathrm{tg}\,\theta$$

$$\mathrm{R\,S} = \frac{h}{3}$$

$$\mathrm{S\,N} = \mathrm{HS}\,\mathrm{tg}\,\xi,$$

where

$$\operatorname{tg}\xi = \frac{wc\tau^2\cos\triangle h}{w(\tau r + c\tau^2\sin\triangle)h + W(2a+\lambda h)}$$

Denoting the ratio CD : DN by n we obtain the following result: —

$$\frac{1}{n}(a+\lambda h) = \frac{1}{3(2a+\lambda h)}(3a^2+3\lambda ah+\lambda^2h^2+3ash+\lambda sh^2$$

$$-\frac{wc\tau^2\cos\triangle h}{w(\tau r+c\tau^2\sin\triangle)h+W(2a+\lambda h)}\left\{\frac{h}{3}-\frac{3a^2+3\lambda ah+\lambda^2h^2+rah}{3(2a+\lambda h)}\frac{r+\tau c\sin\triangle}{\tau c\cos\triangle}\right\}$$

Transforming the above equation we have after some simple reductions

$$[(3n-6)a^2+(3n-9)\lambda ah+(n-3)\lambda^2h^2+3nash+n\lambda sh^2](2a+\lambda h)W$$

$$=[nch\tau^2\cos\triangle(2a+\lambda h)-(3n-3)a\tau r(2a+\lambda h)-(2n-3)\lambda r\tau h(2a+\lambda h)$$

$$-nr\tau sh(2a+\lambda h)+\tau^2\sin\triangle\{3ac(2a+\lambda h)+3ch\lambda(2a+\lambda h)$$

$$-3acn(2a+\lambda h)-2ch\lambda n(2a+\lambda h)-cnhs(2a+\lambda h)\}]wh.$$

But

$$2a+\lambda h \neq o.$$

Hence removing this common factor from both members of the equation we obtain

$$[(3n-6)a^2+(3n-9)\lambda ah+(n-3)^2h^2+3nash+n\lambda sh^2]W$$

$$=[nch\tau^2\cos\triangle-(3n-3)a\tau r-(2n-3)\lambda\tau rh-nr\tau sh$$

$$+\tau^2\sin\triangle(3ac+3ch\lambda-3acn-2ch\lambda-n-cnhs)]wh.$$

This is a quadratic equation of a, and transforming we have

$$(3n-6)Wa^2+3[\{(n-3)\lambda+ns)\}W+\{(n-1)\tau r-c\tau^2\sin\triangle+cn\tau^2\sin\triangle\}w]ha$$

$$=[\{uc\tau\cos\triangle-(2n-3)\lambda r-nrs+3c\lambda\tau\sin\triangle-2c\lambda n\tau\sin\triangle-cns\tau\sin\triangle\}\tau w$$

$$-\{(n-3)\lambda+ns\}\lambda W]h^2.$$

Put

$$\alpha\equiv 3n-6$$

$$\beta\equiv (n-3)\lambda+ns$$

$$\gamma\equiv (n-1)\tau r-c\tau^2\sin\triangle+en\tau^2\sin\triangle$$

$$\delta\equiv nc\tau^2\cos\triangle-(2n-3)\lambda r\tau-nrs\tau+3c\lambda\tau^2\sin\triangle-2c\lambda n\tau^2\sin\triangle$$
$$-cns\tau^2 sin\triangle$$

$$\varepsilon\equiv (n-3)\lambda^2+ns\lambda.$$

Then

$$\alpha Wa^2+3(\beta W+\gamma w)ha=(\delta w-\varepsilon W)h^2,$$

whence we get

$$a=\frac{h}{2\alpha W}\left[-3(\beta W+\gamma w)\pm\sqrt{9(\beta W+\gamma w)^2+4\alpha W(\delta w-\varepsilon W)}\right].$$

The upper or lower one of the double signs is to be taken according as $\delta w>\varepsilon W$ or $\delta w<\varepsilon W$.

Restoring the values of $\alpha,\beta,\gamma,\delta$ and ε in the last equation we finally have

$$\frac{a}{h}=\frac{1}{2(n-2)W}\Big[-\{(n-3)r+(2n-3)s\}W-(n-1)\tau(r+c\tau\sin\triangle)w$$
$$\pm\sqrt{\Big|\frac{1}{3}\{-(n-3)(n+1)r^2-6(n-1)rs+(2n-3)(2n+1)s^2\}W^2}$$
$$+\frac{2}{3}\{-\overline{(n^2-2n+3)r^2+3(n-1)rs}\,|\,\tau+\overline{(-n^2-2n+5)r-(n-5)s}\,|\,c\tau^2\sin\triangle$$
$$+2n(n-2)c\tau^2\cos\triangle\}Ww+\{(n-1)\tau(r+c\tau\sin\triangle)\}^2w^2|\Big].$$

Put, for the sake of brevity,

$$m \equiv \frac{1}{3}\{-(n-3)(n+1)r^2-6(n-1)rs+(2n-3)(2n-1)s^2$$

$$p \equiv \frac{2}{3}\{-(n^2-2n+3)r^2+3(n-1)rs\}\tau+\{(-n^2-2n+5)r-(n-5)s\}c\tau^2\sin\triangle$$
$$+2n(n-2)c\tau^2\cos\triangle\}$$

$$q \equiv \{(n-1)\tau(r+c\tau\sin\triangle)\}^2.$$

Then we have

$$\frac{a}{h}=\frac{1}{2(n-2)W}\left[-\{(n-3)r+(2n-3)s\}W-(n-1)\tau(r+c\tau\sin\triangle)w\right.$$
$$\left.\pm\sqrt{mW^2+pWw+qw^2}\right].$$

If, for instance, we take $n=3$, then the above equation becomes

$$\frac{a}{h}=\frac{1}{2W}\left[-6sW-2\tau(r+c\tau\sin\triangle)w\pm\sqrt{m^1W^2+p^1Ww+wq^1w^2}\right],$$

where

$$m^1=-4rs+5s^2$$

$$p^1=4(-r^2+rs)+\frac{4}{3}(-5r+s)c\tau^2\sin\triangle+4c\tau^2\cos\triangle$$

$$q^1=4\tau^2(r+c\tau\sin\triangle)^2$$

抜萃

○世界第一ノ長橋

世界第一ノ長橋ハルーマニア國セルナヴォダニ於テダニユウブ江ニ架セル鐵道橋ニシテ頃日竣工シタルモノナリ此長橋ハダニユウブ江其物ト所謂浸水地(毎年一定期ニ水ノ氾濫スル地ヲ云フ)トヲ横斷シ其長サ九哩以上ニ達セリ河水ノ主部ヲ横斷セ