On the Thickness of a Retaining Wall under some Particular Conditions,

By S. C. E.

The following are the formulæ for the thickness of a wall retaining the earth under no external loading. I know not whether the formulæ of this description have ever been devised : my poor knowledge is unable to detect any such. The subject is, in fact, limited to some particular cases as described below, but, though it is not worthy a notice of our readers, it may not be utterly useless in forming a rough idea about the thickness of a retaining wall.

It is a well-known fact that the true phenomena about the pressure of the earth are not and can not be (at least at the present state of our knowledge) fully recognized and many an assumption and hypothesis have been taken up by various' authorities. I shall, in the present problem, follow to the Rankine's theory of the pressure of the earth, as, although his theory is not sufficiently rigorous and some of his opinions are not universally true, still it may be regarded as the best of the theories concerning the subject.

The present problem is :— To find the thickness of a wall retaining the earth (under no external loading) with the free surface in the same level plane as the top of the wall having uniform batters in its front and back surface.

Let AB CD be a wall retaining the earth whose free surface is a plane in the same level as AB. Taking the unit length of the wall, we see that the forces acting thereupon are the weight of the earth BCQ and the horizontal pressure of the earth. Both of these forces act at a point E such that $3CE = BC$.

Let

$AB = a$

Batter of $AD = 1 : \frac{1}{s}$

„ „ $CB = 1 : \frac{1}{r}$

$CD = b$

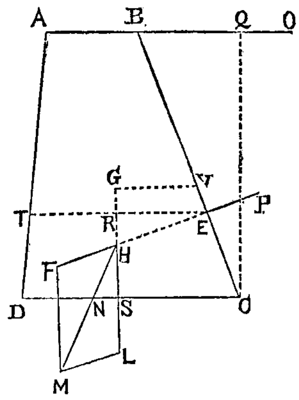
Height of wall $= h$

Weight of unit volume of earth $= w$

„ „ „ „ „ wall $= W.$

then $b = a + \lambda h,$

where $\lambda = r + s.$



The vertical external force is $\frac{wrh^2}{2}$, and the horizontal pressure $\frac{cwh^2}{2}$, where $c = (1-\sin\phi)$ I $(1+\sin\phi)$, ϕ being the angle of repose of the earth.

Let G be the centre of gravity of the wall and PE the resultant of the pressures of the earth. Produce PE to meet the vertical line GL at H and take HF equal to PE. Take HL egual to the weight of the wall and complete the parallelogram HLMF. Then HM gives the resultant of the pressure of the earth and the weight of the mall both in magnitude and direction. Let N be the point at which HM meets the base CD. Rankine takes DN — $\frac{1}{3}$ CD for the safety of the stability of the wall.

Now we have

$$RE = \frac{3a^2 + 3\lambda ah + \lambda^2h^2 + rah}{3(2a+\lambda h)}$$

$$RH = RE \text{ ten } BREH = \frac{r}{c} RE.$$

$$HS = RS - RH = \frac{h}{3} - \frac{r}{c} RE$$

$$SN = HS \text{ ten } SHN$$

$$= HS \frac{cwh}{rwh + (2a + \lambda h)W}$$

$$DS = \frac{3a^2 + 3\lambda ah + \lambda^2 h^2 + 3ash + \lambda sh^2}{3(2a + \lambda h)}.$$

Hence if we take

$$DN = \frac{1}{n} cD,$$

then we have

$$\frac{1}{a}(a + \lambda h) = \frac{1}{3(2a + \lambda h)}\Big[3a^2 + 3\lambda ah + \lambda^2 h^2 + 3ash + \lambda sh^2 - \frac{wh}{rwh + (2a + \lambda h)W}(2ach + c\lambda h^2 - 3ra^2 - 3\lambda rah - r^2 ah - \lambda^2 rh^2)\Big] \quad \cdots\cdots\cdots (1)$$

This is the most general equation for the stability of the retaining wall.

Clearing the fractions we have after some simple reductions :

$$[(3n - 6)a^2 + (3n - 9)\lambda ah + (n - 3)\lambda^2 h^2 + 3nash + n\lambda sh^2](2a + \lambda h)W$$

$$= wh[2nach + nc\lambda h^2 - (6n - 6)ra^2 - (6n - 9)\lambda rah - nr^2 ah - (2n - 3)\lambda^2 rh^2 - 3narsh - nr\lambda sh^2].$$

$$= wh[nch(2a + \lambda h) - r\{(6n - 6)a^2 + (7n - 9)\lambda ah + (2n - 3)\lambda^2 h^2\} - nshr(2a + \lambda h)].$$

$$= wh[nch(2a + \lambda h) - r\{(3n - 3)a + (2n - 3)\lambda h\}\{2a + \lambda h\} - nshr(2a + \lambda h)].$$

But $\quad 2a + \lambda h \neq 0.$

Hence we obtain

$$[(3n-6)a^2+(3n-9)\lambda ah+(n-3)\lambda^2h^2+3nash+n\lambda sh^2]W$$

$$= wh[nch-r\{(3n-3)a+(2n-3)\lambda h\}-nshr].$$

Which, transforming, becomes

$$(3n-6)Wa^2+3h[(n-3)\lambda W+nsW+(n-1)rw]a$$

$$= h^2[ncw-(2n-3)\lambda rw-nrsw-(n-3)\lambda^2W-n\lambda sW].$$

This is a quadratic equation of a ; hence solving we have

$$a = \frac{h}{2(n-2)W}\Big[-\{(n-3)\lambda W+nsW+(n-1)rw\} + \sqrt{\frac{1}{3}\Big\{\{3n^2s^2-(n^2-2n-3)\lambda^2-(10n-2n^2)\lambda s\}W^2 + \{(4n^2-8n)c-(2n^2-4n+6)\lambda r+(2n^2+2n)rs\}Ww+3(n-1)^2r^2w^2\Big\}}\Big]$$

Put

$$3n^2s^2-(n^2-2n-3)\lambda^2-(10n-2n^2)\lambda s \equiv m$$

$$(4n^2-8n)c-(2n^2-4n+6)\lambda r+(2n^2+2n)rs \equiv p$$

$$3(n-1)^2r^2 \equiv q,$$

Then the above equation becomes

$$a = \frac{h}{2(n-2)W}\Big[-\{(n-3)\lambda W+nsW+(n-1)rw\}+\sqrt{\frac{1}{3}(mW^2+pWw+qw^2)}\Big] \quad \cdots\cdots\cdots (2$$

This is the most general equation for the top width of the retaining wall.

If, according to Rankine, we take $n=3$, then the equation (2) becomes

$$a = \frac{h}{2W}\left[-(3sW + 2rw) + \sqrt{m^1W^2 + p^1Ww + q^1w^2}\right] \qquad \text{(3)}$$

where $\qquad m^1 = 9s^2 - 4\lambda s$

$\qquad p^1 = 8rs + 4c - 4r\lambda$

$\qquad q^1 = 4r^2.$

If, on the other hand, we take $n = 4$, then

$$a = \frac{h}{4W}\left[-(\lambda W + 4sW + 3rw) + \sqrt{\frac{1}{3}(m^{11}W^2 + p^{11}Ww + q^{11}w^2)}\right] \qquad \text{(4)}$$

where $\qquad m^{11} = 48s^2 - 5\lambda^2 - 8\lambda s$

$\qquad p^{11} = 32c - 22\lambda r + 40rs$

$\qquad q^{11} = 27r^2.$

If we take

$\qquad w = 100$ lbs

$\qquad W = 120$ lbs

then the equations (3) and (4) become respectively

$$a = \frac{h}{12}\left[-(18s + 10r) + \sqrt{36m^1 + 30p^1 + 25q^1}\right] \qquad \text{(3. A)}$$

$$a = \frac{h}{24}\left[-(6\overline{\lambda + 4s} + 15r) + \sqrt{\tfrac{1}{3}(36m^{11} + 30p^{11} + 25q^{11})}\right] \qquad \text{(4. A)}$$

If we assume the batter of the front face 1 : 24 and that of the back face 1 : 6, so that

$\qquad s = \frac{1}{24}$

$$r = \tfrac{1}{8}$$

$$\lambda = s + r = \tfrac{5}{24}$$

and $$\phi = 30^{0} \text{ or } c = \tfrac{1}{3},$$

then the equations (3. A) and (4. A) become respectively

$$a = .3233h \quad \text{or} \quad b = .532h \quad \cdots\cdots (3.\ a)$$

$$a = .2303h \quad \text{or} \quad b = .439h \quad \cdots\cdots (4.\ a)$$

If the front face be vertical so that $s = 0$, then the equations (3) and (4) become respectively

$$a = \frac{h}{W}\left[-rw + \sqrt{(c-r^2)Ww + r^2w^2}\right] \cdots\cdots (3.\ B)$$

$$a = \frac{h}{4W}\left[-r(W+3w) + \sqrt{\tfrac{1}{3}\{27r^2w^2 + (32c - 22r^2)wW - 5r^2W^2\}}\right] \cdots\cdots (4.\ B)$$

If we take as before

$$w = 100 \text{ lbs}$$

$$W = 120 \text{ lbs}$$

$$r = \tfrac{1}{8}$$

$$\phi = 30^{0},$$

then the equations (3. B) and (4. B) give respectively

$$a = .3854h \quad \text{or} \quad b = .551h \quad \cdots\cdots (3.\ b)$$

$$a = .2814h \quad \text{or} \quad b = .458h \quad \cdots\cdots (4.\ b)$$

These equations (3. a), (3. b), (4. a), (4. b) give at once the dimensions of the wall whenever the height h is known.

The above four formulæ have been derived on the assumption that w = 100 lbs, W = 120 lbs, $r = \frac{1}{6}$, $s = \frac{1}{24}$ and $c = \frac{1}{3}$. For other values of these quantities any one can easily deduce the requisite formulæ from the equations (3) and (4). Whether 3 or 4 or other values are to be used for n must be determined by each person with his own authority: I have only shewn here some instances of the use of the formula (2).

It is a fact that the batter of the back surface of a retaining wall is not in general uniform, and in such cases the above formulæ cannot be used with perfect rigorousness. They can, however, be made use of by substituting a uniform batter for the varying one which is most probable to give the equal stability as those in question, the choice of such a batter depending however merely on our skill. They will much facilitate the determination of the dimensions of the wall when its different parts are of different heights.

抜萃

○蒸氣ローラート瓦斯管

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