Appendix

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Calculation respecting the Discharge of Water in Open Channels.

By A. Inokuchi, M. E.

The following calculations which I have made respecting the discharge of water in open channels are only particular problems in hydraulics. The later one is a practical instance of which the solution gave a simple and interesting result.

For the erection of an hydraulic motor, a long straight open channel is to be constructed leading from a large river, reservoir, or lake to the site of the prime mover. The transverse profile of the channel is a rectangle of constant area and the difference of levels of water in the reservoir and the tail race is h ft. It is required to give such an inclination to the bed of the channel that the greatest possible amount of power may be obtained from the water. (see Fig. 1)

Suppose that the ratio of the total head h to the length l of the channel is very small, so that the velocity of flow may be assumed to be uniform throughout the whole length of the channel. let the total fall of water in the channel be denoted by z, then we have the following formula for uniform motion :—

$$z = f\frac{l}{m}\frac{v^2}{2g},$$

where f the coefficient of friction varies to some extent according to the magnitude of the velocity v, but in an investigation like the present it may safely be considered to be uniform.

Now the water power is

$$E = w\,(h-z)\,Q$$

where w is the weight of water in lbs. per cubic foot and Q the quantity in cubic feet per second. But if A be the area of the cross section of the channel, $Q=v\,A$;

$$E = wA\,(h-z)\,v$$

$$= wA\,(h-z)\,\sqrt{\frac{2gz}{fl}}$$

$$= constant \times (h-z)\,\sqrt{z}.$$

From the usual rule of the defferential calculus the power is maximum when

$$z = \frac{1}{3}\,h.$$

Thus it is seen that the power obtainable from the water would be the greatest possible if one third of the total available head be employed for the fall of the channel.

Taking another problem.

A straight open channel of great length receives its supply of water from a large reservoir and discharges it freely at the end. The transverse section of the channel is rectangular and its bed is quite horizontal being at a depth of h feet below the level of the water in the reservoir. It is required to compute the discharge. (See Fig. 2)

Since the water is freely discharged at the end B of the channel, it is evident that the surface of the stream assumes a certain natural slope as C D in the figure.

Conceive an imaginary surface B E passing through the bottom end B of the channel and parallel to the water surface D C, we may then assume as an approximate truth that the flow is equivalent to one of uniform motion taking place in the inclined channel E

Fig. 1.

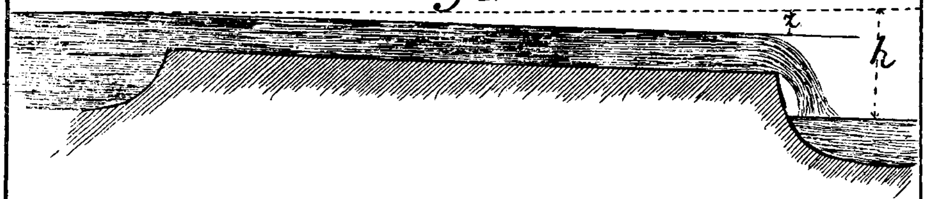


Fig. 2

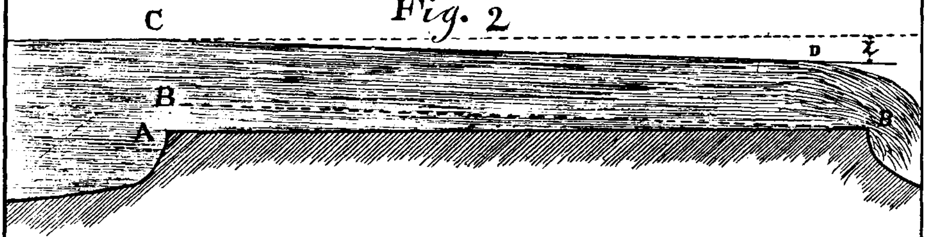
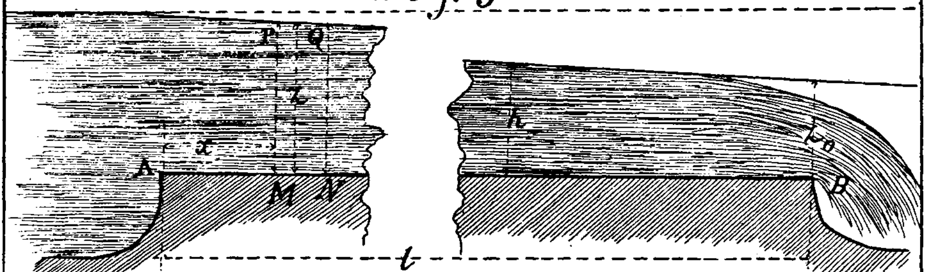


Fig. 3



C D B of constant transverse sectional area, as if the water occupying the wedge-shaped portion A B E had turned solid. Under this supposition we are at liberty to employ the following formula :—

$$z = f\frac{l}{m}\frac{v^2}{2g},$$

where we shall in the mean time assume that the hydraulic mean depth m is constant; then since $Q = v A$ & $A = b(h-z)$,

$$z = f\frac{l}{m}\frac{Q_2}{2gb^2(h-z)^2}$$

or $$Q = constant \times (h-z)\sqrt{z.}$$

If to z in this equation we assign various values, there will be obtained corresponding values for Q. Now because the fall of the surface of water in the channel has taken place by itself and also because it is the natural tendency for water to flow down, we see at once that the value of z must be such as will give the greatest value to Q.

The discharge Q will be a maximum when

$$z = \frac{1}{3}h.$$

Therefore the fall of the water surface is about one-third of the depth of the channel, and the discharge Q can now be easily found from the foregoing formula; thus

$$Q = A\sqrt{\frac{2glm}{f}}$$

$$= \frac{2}{3}hb\sqrt{\frac{2g\,\frac{1}{3}h\,m}{fl}}$$

A nearer approximation to z or Q will be obtained by employing the following value for the hydraulic mean depth,

$$m = \frac{b(h-z)}{2(h-z)+b}.$$

Then $$Q=\sqrt{\frac{2gb^3z(h-z)^3}{2(h-z)+b}},$$

which will be found to be a maximum when

$$z=\frac{1}{3}(2h+b)-\frac{1}{3}\sqrt{(h+b)^2+\tfrac{1}{2}hb.}$$

This may be expanded into the following series :—

$$z=\frac{h}{3}\left\{1-\frac{1}{4}\frac{b}{h+b}+\frac{1}{32}\frac{hb^2}{(h+b)^3}\cdots\right\}$$

of which the first two terms would be sufficient to take as our result, seeing that the formaula was obtained by assuming the flow to be uniform. Then we have

$$z=\frac{h}{3}\left(1-\frac{1}{4}\frac{b}{h+b}\right).$$

Considering the problem more closely and taking into account the change of velocity of water in the channel, let P M and Q N (Fig. 3) be two transverse sections situated very near to each other; denote A M by x, P M by z, and the mean velocity in the section P M by v, then the corresponding quantities for the section Q N may be denoted by $x+dx$,z-dz, and $v+dv$.

The actual head of water in the section PM is z and the head due to velocity is $\frac{v^2}{2g}$, and the sum of these two must be equal to the corresponding sum of the heads in the section Q N together with the head lost in overcoming the frictional resistance of the passage from P M to Q N, and

$$z+\frac{v^2}{2g}=(z-dz)+\frac{(v+dv)^2}{2g}+f\frac{dx}{m}\frac{v^2}{2g},$$

from which we obtain

$$\frac{dz}{dx}=\frac{v}{g}\frac{dv}{dx}+f\frac{S}{A}\frac{v^2}{2g}$$

where $S=$ wetted perimeter.

But $A=bz$, $\quad v=\frac{Q}{A}=\frac{Q}{bz}$, & $\frac{dv}{dx}=-\frac{Q}{bz^2}\frac{dz}{dx}$,

$$\therefore \quad \frac{dz}{dx} = \frac{Q}{gbz}\left(-\frac{Q}{bz^2}\frac{dz}{dx}\right) + f\,\frac{S}{bz}\,\frac{Q^2}{2gb^2z^2},$$

or $$2gb^3z^3\,\frac{dz}{dx} = Q^2\left(fS - 2b\frac{dz}{dx}\right).$$

The tangent of the surface inclination, $\frac{dz}{dx}$, may be considered as constant, being practically so, and equal to $(h-z)\div x$; then put z_0 for z, $2z_0+b$ for s, and $(h-z_0)\div l$ for $\frac{dz}{dx}$, and we obtain

$$2gb^3z_0^3\,\frac{h-z_0}{l} = Q^2\left\{\mathrm{f}\,(2z_0+b) - 2b\frac{h-z_0}{l}\right\}$$

or $$Q = K\sqrt{\frac{z_0^3(h-z_0)}{2z_0+c}}$$

where $$K = \sqrt{\frac{2gb^3}{fl+b}},\ \&\ c = b\,\frac{fl-2h}{fl+b}.$$

If we put z' for $h-z_0$ and $h-z'$ for z_0, we obtain

$$Q = K\sqrt{\frac{z'(h-z')^3}{2(h-z')+c}},$$

This value of Q differs only in the constants K and c from that which we got before; so that proceeding in the same manner, we expect to obtain the following:—

$$z' = \frac{h}{3}\left\{1 - \frac{1}{4}\frac{c}{h+c} + \frac{1}{32}\,\frac{c^2h}{(h+c)^3}\,- - - - -\right\}$$

where $c = b\dfrac{fl-2h}{fl+b}$, which is always less than b.

When $c = o = fl - 2h$, z' is exactly $\frac{1}{3}h$, that is, when $\frac{h}{l} = \frac{f}{2}$ or when the length of the channel is about 264 times the depth, the total fall of the water surface is $\frac{1}{3}h$ and the inclination of the surface would be about 1 in 790.

We may now lay down the following two sets of formulae :—

1. For rough and approximate calculations,

$$z = \frac{h}{3}\left(1 - \frac{1}{4}\,\frac{b}{h+b}\right)$$

& $$Q = \sqrt{\frac{2gb^3}{fl+b}}\sqrt{\frac{z(h-z)^3}{2(h-z)+c}}.$$

2. For more exact calculations,

$$z = \frac{h}{3}\left(1 - \frac{1}{4}\,\frac{c}{h+c}\right)$$

& $$Q = \sqrt{\frac{2gb^3}{fl+b}}\sqrt{\frac{z(h-z)^3}{2(h-z)+c}},$$

where $$c = b\,\frac{fl-2h}{fl+b}.$$

In the preceding calculations, the free surface of the water in the channel has been assumed to be uniformly sloping down the stream, in reality however it would be some sort of curved surface and not a simple plane.

This point we shall now consider.

From the formula

$$2gb^3 z^3\,\frac{dz}{dx} = Q^2\left(fS - 2b\,\frac{dz}{dx}\right),$$

since $S = 2z + b$, we obtain

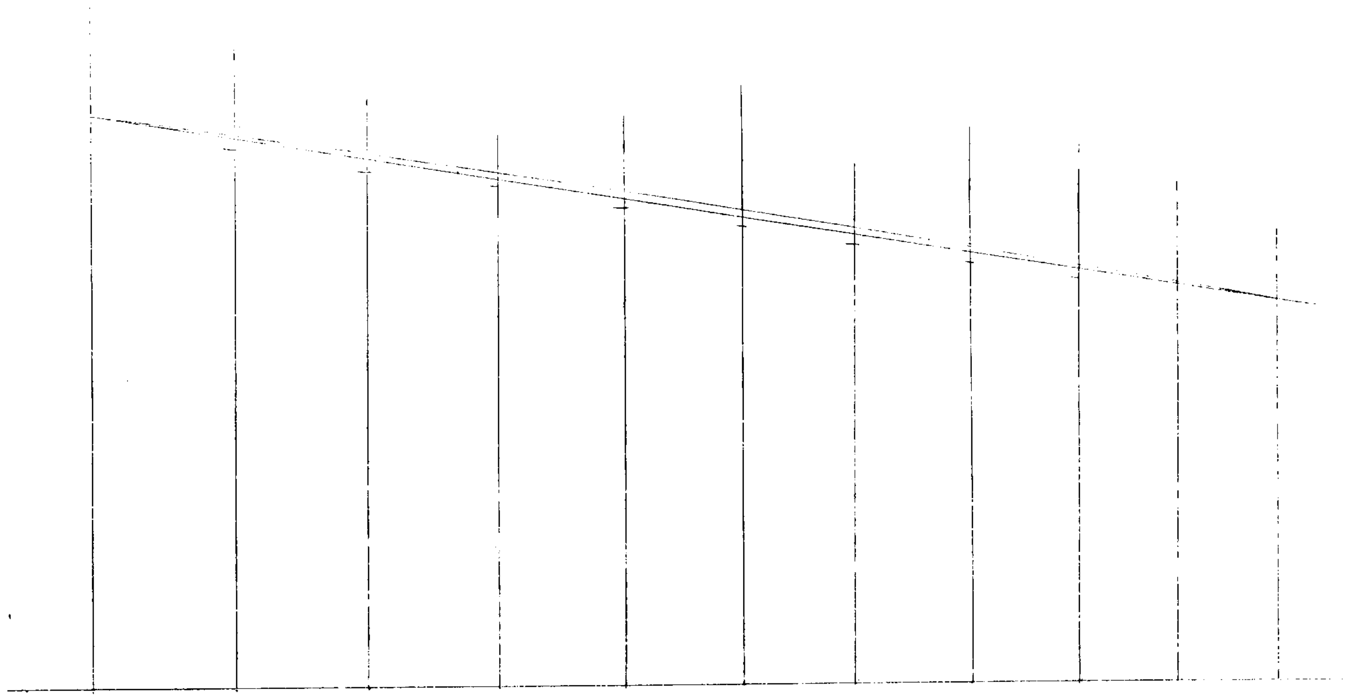
$$\frac{z^3 + bc^2}{z + \frac{1}{2}b}\cdot\frac{dz}{dx} = fc^2,$$

where $$c^2 = \frac{Q^2}{gb^3}.$$

But $$\int \frac{z^3 + bc^2}{z + \frac{1}{2}b}\,\frac{dz}{dx}\,dx = \int\left(z^2 - \frac{1}{2}bz + \frac{1}{4}b^2 + \frac{bc^2 - \frac{1}{8}b^3}{z + \frac{1}{2}b}\right)dz$$

$$= \frac{1}{3}z^3 - \frac{1}{4}bz^2 + \frac{1}{4}b^2 z + \left(bc^2 - \frac{1}{8}b^3\right)\log\left(z + \frac{1}{2}b\right),$$

$$\therefore\quad fc^2x = A + \frac{B(h-z)}{z+(h+b)} - z\left(\frac{1}{3}z^2 - \frac{1}{4}bz + \frac{1}{4}b^2\right),$$



bers of Kōgaku-Kai is the characteristic features of where $A=h\left(\frac{1}{3}h^2-\frac{1}{4}bh+\frac{1}{4}b^2\right)$

& $B=2\left(bc^2-\frac{1}{8}b^3\right)$

The above result assumes that

$$\log\frac{h+\frac{1}{2}b}{z+\frac{1}{2}b}=\frac{2(h-z)}{z+(h+b)}\text{ nearly.}$$

As might be expected the curve of free water surface is very slightly convex; in the following diagram showing the general appearance of the curve, z is assumed to be equal to 3 ft., b 10 ft., and Q 180 cubic ft. per second, so that we have

$$fc^2x-61.5-\frac{210(3-z)}{15+z}-z\left(\frac{1}{3}z^2+25-\frac{10}{4}z\right),$$

where $fc^2 = 1 \div 131$,

when $z = 3.0$,	$x = 0$,
„ $= 2.9$,	„ $= 75{\cdot}5$,
„ $= 2{\cdot}8$,	„ $= 148$,
„ $= 2{\cdot}7$,	„ $= 216$,
„ $= 2{\cdot}6$,	„ $= 283$,
„ $= 2{\cdot}5$,	„ $= 347$,
„ $= 2{\cdot}4$,	„ $= 407$,
„ $= 2{\cdot}3$,	„ $= 466$,
„ $= 2{\cdot}2$,	„ $= 525$,
„ $= 2{\cdot}1$,	„ $= 579$,
„ $= 2{\cdot}0$,	„ $= 633$,

The exaggeration of the scale for the depth of the water is 100 and yet the curve approximates very fairly to a right line.

The characteristic features of the American Locomotive Engine.

by M. Crizuka.

The subject which I wish to present to the members of Kōgaku-Kai is the characteristic features of the American Locomotive.

It is not my intention to cover in these notes the whole subject of American Locomotive building, but simply to convey to you a brief outline of the American Locomotive, pointing but a few of the most characteristic differences between its constraction and that of the standard English engines. Many papers have appeared under the heading of American and English Engines in the leading mechanical fournals published both in England and America. Shoud I try to take up the subject, most of my arguments would almost neceesarily consist of repetitions of the other writers words, so therefore I have directed my attention to the above subject alone which is quite new, at least to the readers of Kōguku Kai journals. The facts here stated have been obtained mainly from my personal enperience and observations during the last four years in the Kobe Railway works, Japan and at the Boldioni Loco. works Philadelphia America. Commencing with this paper it is my desire to lay before the public the systems and plans of American Locomotive building without referring to the system adopted in England.

If would be of great benefit and interest both to the writer and to the public if any reader of the paper would criticise the subject in whole or in part.

It may be well to add that in this paper I have no design to criticise the present system of the railway business as carried on in Japan.

I. The Characteristic features of American engines are that the frames are made of forged bars, the driving wheels have equalizing levers connecting the springs and the truck wheels are also equalized together Some of the engines in Japan of English make, have the same plans with the exception of the frames, but as is known to every-body American railroads particularly in the new sections of the country and in the hilly regeons are built with steep grades and short curves in many instances and the mechanical engineer had to follow with engines adapted to these conditions; the engines therefore are characterised by extreme flexibility and last of mention, doing the least possible damage to the track and themselves while running.

II. This is accomplished by distributing the loads uniformly on the wheels by means of equalizers and supporting the front part of the engine on a separate truck in effect, on a single point.

III. It is easy to see after a little consideration that the effect of the equalizing levers is that each side of the locomotive is supported in such a way that the action is the same as it would be, if it was supported on point one.

Thus all the American typical engines are made on the principle of a tripod or a three legged stool, which is well known to the students of surveying, will adjust itself to any surface, however uneven and stand firmly in any position. One wheel may rise and another fall but the load on each is constant. (*See figure*)

IV. Another effect of taking the carrying wheels out of the rigid frame necessarily occupied by the driving wheels and placing them in a truck (the whole carried on a point centrally placed under the front end of the boiler), is to reduce the rigid wheel base to the distance between the centre of the front and back drivers and thus allow the leading wheels to follow, the sharp curves without finding.

V. In addition to the ordinary swivelling movement, a swinging or lateral motion of the truck is avoided.

This is done in American practice by connecting the bearing block upon which the truck swivels to the truck frame by swinging links. The links are placed at a slight angle from the perpendicular. By this arrangement the gravity of the engine constantry tends to restore the truck to the exact tangent line of the engine after it has moved laterally in passing a curve. This has the advantage over the mode adopted in the Japanese engines in two ways-firstly less friction on the bearing surface of the block,and secondly gravity does restore the equilibrium in American engines, whereas in the latter the lateral springs are depended upon to restore the truck to the exact tangent line of the locomotive.

VI. Thus it is easy to undestand that the American engines can move from side to side, rise and fall and in fast conform to the minor irregulalities of the truck while the engine is not affected at all.

Under such conditions there is little wonder that the American engine is a favorite on both good and poor roads, its adhesion is utilized at all times.

VII. The method of framing universally employed in American locomotives is the square forged bar frame. The pedestals are usually welded on, in some cases however they are bolted on.

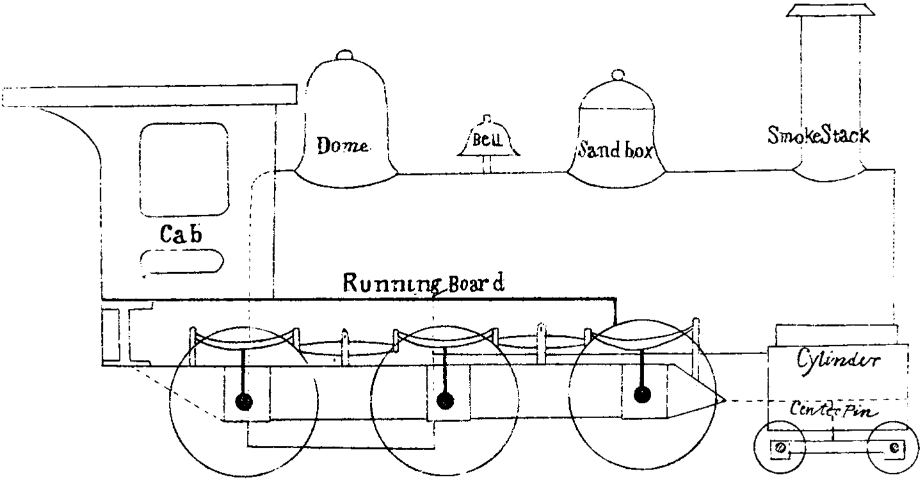
Frames of this description in addition to the advantages of stiffness and strength as compared with that of the plate frame are less likely to collect and retain the dust under the engine and about the machinery and gives free access to any part of the machniery—a point of no inconsiderable importance.

Another important condition in this method of constructing engine frames is that as the frames are planed on are sides, they furnish true surfaces on which to fit accurately all pieces of machinery &c which are planed up without any special chipping and fitting. In some class of engines the main frames to which the pedestals are welded are connected with the first rails of the frames by bolts, it will be seen therefore that by removing the bolts at these connections the wheels, frames &c about the fire box, can be removed when it is desirable to do any work on the fire box without disturbing the machinery, cylinders &c.

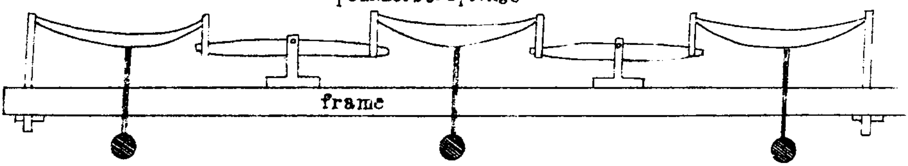
VIII. Another important feature, in the construction of American locomotives, is the casting of the cylinders in one piece with half of the saddle. (See Figure)

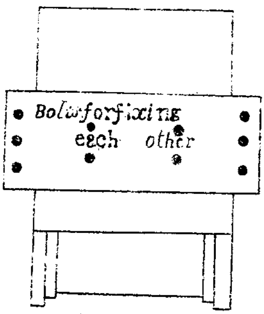
As the rule in American practice is to place the cylinders horizontally, the pattren for the cylinder with half saddle is made so that it is reversible; there are no rights and lefts.

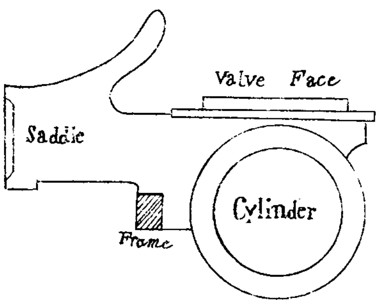
If two castings of the above frame are made, they can be bolted together in the middle and the result will be the required pair of cylinders perfectly horizontal and with the centre lines perfectly true. This method of



Arrangement of Equalizing Levers
pualizers & Springs







construction gives in effect a bedplate of ample strength at the front end of the locomotive for supporting the boiler. The smoke box is fastened firmly to the cylinders and saddle, and an latter to the engine frames. The expantion is provided for at the back end of the boiler.

From this method of making the cylinders, it follws that one pattern suffices for a cylinder with half of the saddle attached, for either side of the locomotive. Economy in construction and repairs results.

IX. Another point which is worthy to be mentioned in connection with the cylinder is that the steam chests on American locomotives are always outside and on top of the cylinders.

One might think badly of the Rocker Arms but when the advantage of ready access for facing off the valve face &c is considered, the merits thereby obtained surpasses by far the demerits incurred, (See figure.)

X. In America, it is a common practice to make the whole centres of cast iron and the practice of making it with wrought iron is still the rule in England.

When this is considered from the natural law regarding the strength of materials it might seem unreasonable to make the fast moving part of the machinery and which is not unfrequently subjected to sudden shocks of the matter ial inferior in tenacily.

But one will find in time that cast iron can be had in America that will show a tensile strength of 30,000lbs per sq.inch wheels of this material have a better record for the past seven years than the English wrought iron wheels, to say the least. In Connection with this, I may add that the engines which five weeks ago, begin to run the express trains between New York and Chicago, a distance of nine hundred and eight miles in twenty hours and thirty minutes are engines with cast iron wheels and no better by any means than common American express engines.

As I lack at present better statistics of the statements of the relative coal consumption of the American and English engines, I shall not touch the subject until I am ready to do so. I believe the American engines will make *long runs* at high rates of speed; their performances challenge admiration, and are in all respects a machine on which it is quite proper to rely.
