

# 縦斷曲線の計算に就いて

新潟縣岩船  
築港事務所

山田與三郎

從來道路築設に對し平面曲線を挿入したる時でも、縦斷曲線は餘り重きを置かなかつたが、近來高速度の交通増加に伴ひ其の必要を認めらるゝやうになつた。而して兩接線對稱の場合に其の計算簡であつて手数を要せなくとも非對稱の場合では高等數理に基いた計算を要する爲め其の理解に苦む所である。是等を簡易にする意味に於て公式を案出し別表を作つたのである幸に批評を乞ふ。

## 兩接線對稱の場合

第一圖に於てA、Bを四等分しA、Cの線より曲線迄の長を夫々 $P_1, P_2, P_3, P_4$ とすれば $P_1 = 2P_2, P_2 = 3P_3, P_3 = 4P_4$ 。(勾配に關係なく)なるは周知のことなり。而して $P_1$ は兩勾配に依て既定せらるゝ數なれば $P_1 = \frac{P}{4^2} \times 1^2 = 0.0625P$ 。

$$P_2 = \frac{P}{4^2} \times 2^2 = 0.2500P, \quad P_3 = \frac{P}{4^2} \times 3^2 = 0.5625P, \quad P_4 = \frac{P}{4^2} \times 4^2 = 0.6250P$$

兩接線非對稱の場合にも亦如斯き形に導き得以下例に就

き解説を試みるとす。

## 兩接線非對稱の場合

第二圖に於て $(1+d_1)^2 : P_1^2 :: (2+d_2)^2 : P_2^2 :: (3+d_3)^2 : P_3^2 :: (4+d_4)^2 : P_4^2$ なる事は兩接線對稱の場合と同様なり而して以下各式は皆相似三角形の相應邊なるを以て夫々比例す。

$$P_1^2 : P_2^2 :: P_3^2 : P_4^2, \text{ 故に } (1+d_1)^2 : P_1^2 = (2+d_2)^2 : P_2^2 \\ P_2^2 : (3+d_3)^2 :: P_3^2 : (4+d_4)^2, \text{ 之れと同様に } (1+d_1)^2 : P_1^2 = (2+d_2)^2 : P_2^2 \\ (3+d_3)^2 : P_3^2 :: (4+d_4)^2 : P_4^2, \text{ 而して } d_1 = d_2 = d_3 = d_4$$

に掲記すれば

て既知なる故に之等の關係よくして  $d_2$  を求めることを得  
 $(1+d_1)^2 \cdot d_1 = (4+d_2)^2 \cdot d_2$   $d_4 = l_1 - l_2 = 8 - 1 = 7$  を代入すれば

$$(1+d_1)^2 \cdot d_1 = (4+2)^2 \cdot 2$$

$$= (1+2d_1+d_1^2) \cdot d_1 = 36$$

$$\therefore \therefore 36d_1 = 2d_1^2 + 4d_1 + 2$$

$$2d_1^2 - 32d_1 + 2 = 0$$

$$d_1^2 - 16d_1 + 1 = 0$$

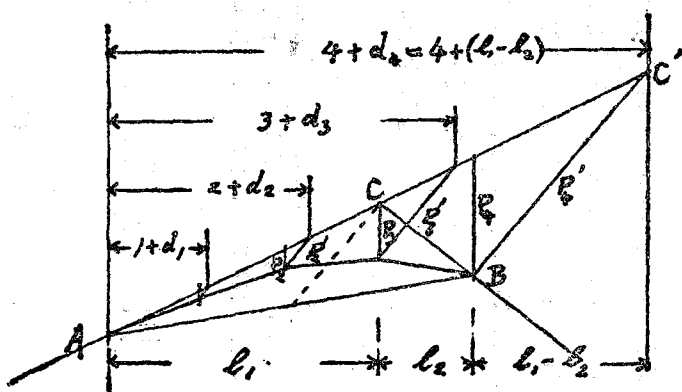
$$= 2 \left[ \left( d_1 - \frac{16}{2} \right)^2 - \left( \frac{16}{2} \right)^2 + 1 \right]$$

$$- 1 = 2 \left( d_1 - \frac{16}{2} \right)^2 - 1$$

$$\left( \frac{\sqrt{18 \times 14}}{2} \right)^2 = 2 \left\{ \left( d_1 - \frac{16}{2} \right) - \frac{\sqrt{18 \times 14}}{2} \right\}$$

$$\left\{ d_1 - \frac{16}{2} \right\}$$

圖 二



同様の方法に  
 $\left. \begin{aligned} + \frac{\sqrt{18 \times 14}}{2} \\ - \frac{\sqrt{18 \times 14}}{2} \end{aligned} \right\} = 0 \therefore d_1 = \frac{16 - \sqrt{18 \times 14}}{2}$   
 因の  $d_2 = \frac{14 - \sqrt{18 \times 10}}{2}$   $d_3 = \frac{12 - \sqrt{18 \times 6}}{2}$  を得これを表

$x_n$	$d_n$	$x_n + d_n$	$(x_n + d_n)^2$	$P_n = \frac{(x_n + d_n)^2}{(2l_1)^2} \times 100$ { $P_n = 100,000$ } に對する比
1	$\frac{16 - \sqrt{18 \times 14}}{2}$	$\frac{18 - \sqrt{18 \times 14}}{2} = 1.063$	1.130	$\frac{1.130}{(2 \times 3)^2} \times 100 = 3.139$
2	$\frac{16 - \sqrt{18 \times 10}}{2}$	$\frac{18 - \sqrt{18 \times 10}}{2} = 2.292$	5.253	$\frac{5.253}{(2 \times 3)^2} \times 100 = 14.592$
3	$\frac{16 - \sqrt{18 \times 6}}{2}$	$\frac{18 - \sqrt{18 \times 6}}{2} = 3.804$	14.470	$\frac{14.470}{(2 \times 3)^2} \times 100 = 40.194$
4	$\frac{16 - \sqrt{18 \times 2}}{2}$	$\frac{18 - \sqrt{18 \times 2}}{2} = 6.000$	36.000	$\frac{36.000}{(2 \times 3)^2} \times 100 = 100.000$

上表中の  $x_n + d_n$  を檢するに次の如き一般公式にて得らるゝことを知る

$$x_n + d_n = \frac{2l_1^2 - \sqrt{2l_1^2 \times (2l_1^2 - (1-l_2) \times 2x)}}{l_1 - l_2}$$

上式を用へ  $l_2 = 10$ . に對し  $l_3 = 8$  の六種を作れり 表中  $P_n$  は最終の  $P$  を 100 としたる比  $\triangle$ 印は CB 線よりの同上百分比なり

$l_1=10 \quad l_2=3$ の場合		$l_1=10 \quad l_2=4$ の場合	
$X_n$	$X_n + dn$	$X_n$	$(X_n + dn)$
1	$\frac{200 - \sqrt{200 \times 188}}{7} = 1.018$	1	$\frac{200 - \sqrt{200 \times 183}}{6} = 1.015$
2	$\frac{200 - \sqrt{200 \times 172}}{7} = 2.075$	2	$\frac{200 - \sqrt{200 \times 175}}{6} = 2.064$
3	$\frac{200 - \sqrt{200 \times 158}}{7} = 3.177$	3	$\frac{200 - \sqrt{200 \times 164}}{6} = 3.140$
4	$\frac{200 - \sqrt{200 \times 144}}{7} = 4.328$	4	$\frac{200 - \sqrt{200 \times 152}}{6} = 4.274$
5	$\frac{200 - \sqrt{200 \times 130}}{7} = 5.536$	5	$\frac{200 - \sqrt{200 \times 140}}{6} = 5.445$
6	$\frac{200 - \sqrt{200 \times 116}}{7} = 6.812$	6	$\frac{200 - \sqrt{200 \times 128}}{6} = 6.667$
	$P_n = \frac{(X_n + dn)^2}{(2l)^2} \times 100$		$P_n = \frac{(X_n + dn)^2}{(2l)^2} \times 100$
	1.036 400 $\times 100 = 0.259$		1.030 400 $\times 100 = 0.258$
	4.306 400 $\times 100 = 1.076$		4.260 400 $\times 100 = 1.065$
	10.093 400 $\times 100 = 2.523$		9.916 400 $\times 100 = 2.479$
	18.732 400 $\times 100 = 4.683$		18.269 400 $\times 100 = 4.567$
	30.647 400 $\times 100 = 7.662$		29.648 400 $\times 100 = 7.412$
	46.413 400 $\times 100 = 11.601$		44.444 400 $\times 100 = 11.111$

7	$\frac{200 - \sqrt{200 \times 102}}{7} = 8.167$	$\frac{66,700}{400} \times 100 = 16,675$	
8	$\frac{200 - \sqrt{200 \times 88}}{7} = 9.619$	$\frac{92,525}{400} \times 100 = 23,131$	
9	$\frac{200 - \sqrt{200 \times 74}}{7} = 11.192$	$\frac{125,261}{400} \times 100 = 31,315$	
10	$\frac{200 - \sqrt{200 \times 60}}{7} = 12.922$	$\frac{166,926}{400} \times 100 = 41,732$	
11	$\frac{200 - \sqrt{200 \times 46}}{7} = 14.862$	$\frac{231,057}{400} \times 100 = 57,764$ $\Delta 21,939$	
12	$\frac{200 - \sqrt{200 \times 32}}{7} = 17.143$	$\frac{293,875}{400} \times 100 = 73,469$ $\Delta 6,803$	
13	$\frac{200 - \sqrt{200 \times 18}}{7} = 20.000$	$\frac{400,000}{400} \times 100 = 100,000$ $\Delta 0$	
7	$\frac{200 - \sqrt{200 \times 116}}{6} = 7.947$		$\frac{63,155}{400} \times 100 = 15,789$
8	$\frac{200 - \sqrt{200 \times 104}}{6} = 9.291$		$\frac{86,416}{400} \times 100 = 21,604$
9	$\frac{200 - \sqrt{200 \times 92}}{6} = 10.726$		$\frac{115,047}{400} \times 100 = 28,762$
10	$\frac{200 - \sqrt{200 \times 80}}{6} = 12.251$		$\frac{150,086}{400} \times 100 = 37,522$
11	$\frac{200 - \sqrt{200 \times 68}}{6} = 13.827$		$\frac{193,133}{400} \times 100 = 48,283$ $\Delta 23,283$
12	$\frac{200 - \sqrt{200 \times 56}}{6} = 15.695$		$\frac{246,333}{400} \times 100 = 61,583$ $\Delta 11,583$
13	$\frac{200 - \sqrt{200 \times 44}}{6} = 17.639$		$\frac{313,252}{400} \times 100 = 78,313$ $\Delta 3,313$
14	$\frac{200 - \sqrt{200 \times 32}}{6} = 20.000$		$\frac{400,000}{400} \times 100 = 100,000$ $\Delta 0$

$l_1=10$ $l_2=5$ の場合		$l_1=10$ $l_2=6$ の場合	
$X_n$	$X_n+dn$	$X_n$	$X_n+dn$
1	$\frac{200-\sqrt{200 \times 190}}{5} = 1.013$	1	$\frac{200-\sqrt{200 \times 192}}{4} = 1.010$
2	$\frac{200-\sqrt{200 \times 180}}{5} = 2.053$	2	$\frac{200-\sqrt{200 \times 184}}{4} = 2.042$
3	$\frac{200-\sqrt{200 \times 170}}{5} = 3.122$	3	$\frac{200-\sqrt{200 \times 176}}{4} = 3.096$
4	$\frac{200-\sqrt{200 \times 160}}{5} = 4.223$	4	$\frac{200-\sqrt{200 \times 168}}{4} = 4.174$
5	$\frac{200-\sqrt{200 \times 10}}{5} = 5.359$	5	$\frac{200-\sqrt{200 \times 160}}{4} = 5.279$
6	$\frac{200-\sqrt{200 \times 140}}{5} = 6.534$	6	$\frac{200-\sqrt{200 \times 152}}{4} = 6.411$
7	$\frac{200-\sqrt{200 \times 130}}{5} = 7.751$	7	$\frac{200-\sqrt{200 \times 144}}{4} = 7.574$
	$P_n = \frac{(X_n+dn)^2}{(2l_1)^2} \times 100$		$P_n = \frac{(X_n+dn)^2}{(2l_1)^2} \times 100$
	1.036 $\times 100 = 0.257$ 40)		1.020 $\times 100 = 0.255$ 400
	$\frac{4.215}{400} \times 100 = 1.054$		$\frac{4.170}{400} \times 100 = 1.042$
	$\frac{9.747}{400} \times 100 = 2.437$		$\frac{9.585}{400} \times 100 = 2.396$
	$\frac{17.831}{400} \times 100 = 4.458$		$\frac{17.422}{400} \times 100 = 4.355$
	$\frac{28.719}{400} \times 100 = 7.180$		$\frac{27.868}{400} \times 100 = 6.969$
	$\frac{42.693}{400} \times 100 = 10.673$		$\frac{41.101}{400} \times 100 = 10.275$
	$\frac{60.078}{400} \times 100 = 15.019$		$\frac{57.365}{400} \times 100 = 14.341$

8	$\frac{200 - \sqrt{200 \times 120}}{5} = 9.016$	$\frac{81,288}{400} \times 100 = 20,322$
9	$\frac{200 - \sqrt{200 \times 110}}{5} = 10,385$	$\frac{106,812}{400} \times 100 = 26,703$
10	$\frac{200 - \sqrt{200 \times 100}}{5} = 11,716$	$\frac{137,265}{400} \times 100 = 34,316$
11	$\frac{200 - \sqrt{200 \times 90}}{5} = 13,167$	$\frac{173,369}{400} \times 100 = 43,342$ $\Delta 23,342$
12	$\frac{200 - \sqrt{200 \times 80}}{5} = 14,702$	$\frac{216,148}{400} \times 100 = 54,037$ $\Delta 14,037$
13	$\frac{200 - \sqrt{200 \times 70}}{5} = 16,386$	$\frac{266,868}{400} \times 100 = 66,716$ $\Delta 6,716$
14	$\frac{200 - \sqrt{200 \times 60}}{5} = 18,021$	$\frac{327,284}{400} \times 100 = 81,821$ $\Delta 1,821$
15	$\frac{200 - \sqrt{200 \times 50}}{5} = 20,000$	$\frac{400,000}{400} \times 100 = 100,000$ $\Delta 0$
8	$\frac{200 - \sqrt{200 \times 136}}{4} = 8,769$	$\frac{76,835}{400} \times 100 = 19,224$
9	$\frac{200 - \sqrt{200 \times 128}}{4} = 10,000$	$\frac{100,000}{400} \times 100 = 25,000$
10	$\frac{200 - \sqrt{200 \times 120}}{4} = 11,270$	$\frac{127,013}{400} \times 100 = 31,753$
11	$\frac{200 - \sqrt{200 \times 112}}{4} = 12,583$	$\frac{158,332}{400} \times 100 = 39,583$ $\Delta 22,916$
12	$\frac{200 - \sqrt{200 \times 104}}{4} = 13,945$	$\frac{194,458}{400} \times 100 = 48,615$ $\Delta 15,282$
13	$\frac{200 - \sqrt{200 \times 96}}{4} = 15,353$	$\frac{235,892}{400} \times 100 = 58,975$ $\Delta 8,975$
14	$\frac{200 - \sqrt{200 \times 88}}{4} = 16,834$	$\frac{283,376}{400} \times 100 = 70,844$ $\Delta 4,177$
15	$\frac{200 - \sqrt{200 \times 80}}{4} = 18,377$	$\frac{337,772}{400} \times 100 = 84,443$ $\Delta 1,110$
16	$\frac{200 - \sqrt{200 \times 72}}{4} = 20,000$	$\frac{400,000}{400} \times 100 = 100,000$ $\Delta 0$

$l_1=10 \quad l_2=7$ の場合			$l_1=10 \quad l_2=8$ の場合		
$X_n$	$X_n+dn$	$P_n = \frac{(X_n+dn)^2}{(2n)^2} \times 100$	$X_n$	$X_n+dn$	$P_n = \frac{(X_n+dn)^2}{(2n)^2} \times 100$
1	$\frac{200 - \sqrt{200 \times 194}}{3} = 1,008$	$\frac{1,016}{400} \times 100 = 0,254$	1	$\frac{200 - \sqrt{200 \times 196}}{2} = 1,005$	$\frac{1,010}{400} \times 100 = 0,253$
2	$\frac{200 - \sqrt{200 \times 188}}{3} = 2,031$	$\frac{4,125}{400} \times 100 = 1,031$	2	$\frac{200 - \sqrt{200 \times 192}}{2} = 2,010$	$\frac{4,080}{400} \times 100 = 1,020$
3	$\frac{200 - \sqrt{200 \times 182}}{3} = 3,071$	$\frac{9,431}{400} \times 100 = 2,358$	3	$\frac{200 - \sqrt{200 \times 188}}{2} = 3,046$	$\frac{9,278}{400} \times 100 = 2,320$
4	$\frac{200 - \sqrt{200 \times 176}}{3} = 4,128$	$\frac{17,040}{400} \times 100 = 4,260$	4	$\frac{200 - \sqrt{200 \times 184}}{2} = 4,083$	$\frac{16,671}{400} \times 100 = 4,168$
5	$\frac{200 - \sqrt{200 \times 170}}{3} = 5,203$	$\frac{27,071}{400} \times 100 = 6,768$	5	$\frac{200 - \sqrt{200 \times 180}}{2} = 5,132$	$\frac{21,337}{400} \times 100 = 5,334$
6	$\frac{200 - \sqrt{200 \times 164}}{3} = 6,297$	$\frac{39,652}{400} \times 100 = 9,913$	6	$\frac{200 - \sqrt{200 \times 176}}{2} = 6,192$	$\frac{38,341}{400} \times 100 = 9,585$
7	$\frac{200 - \sqrt{200 \times 158}}{3} = 7,412$	$\frac{54,988}{400} \times 100 = 13,747$	7	$\frac{200 - \sqrt{200 \times 172}}{2} = 7,264$	$\frac{52,766}{400} \times 100 = 13,191$
8	$\frac{200 - \sqrt{200 \times 152}}{3} = 8,543$	$\frac{73,068}{400} \times 100 = 18,267$	8	$\frac{200 - \sqrt{200 \times 168}}{2} = 8,349$	$\frac{69,703}{400} \times 100 = 17,426$



9	$\frac{200 - \sqrt{200 \times 146}}{3} = 9.707$	$\frac{91,236}{400} \times 100 = 22,809$	$\frac{200 - \sqrt{200 \times 164}}{2} = 9.446$	$\frac{89,227}{400} \times 100 = 22,307$
10	$\frac{200 - \sqrt{200 \times 140}}{3} = 10,883$	$\frac{118,570}{400} \times 100 = 29,643$	$\frac{200 - \sqrt{200 \times 160}}{2} = 10,567$	$\frac{111,430}{400} \times 100 = 27,867$
11	$\frac{200 - \sqrt{200 \times 134}}{3} = 12,083$	$\frac{146,361}{400} \times 100 = 36,590$ $\Delta 22,304$	$\frac{200 - \sqrt{200 \times 156}}{2} = 11,683$	$\frac{156,438}{400} \times 100 = 39,117$ $\Delta 21,617$
12	$\frac{200 - \sqrt{200 \times 128}}{3} = 13,333$	$\frac{177,769}{400} \times 100 = 44,442$ $\Delta 15,871$	$\frac{200 - \sqrt{200 \times 152}}{2} = 12,823$	$\frac{164,403}{400} \times 100 = 41,101$ $\Delta 16,101$
13	$\frac{200 - \sqrt{200 \times 122}}{3} = 14,538$	$\frac{213,101}{400} \times 100 = 53,275$ $\Delta 10,418$	$\frac{200 - \sqrt{200 \times 148}}{2} = 13,977$	$\frac{195,355}{400} \times 100 = 48,839$ $\Delta 11,339$
14	$\frac{200 - \sqrt{200 \times 116}}{3} = 15,895$	$\frac{252,649}{400} \times 100 = 63,162$ $\Delta 6,019$	$\frac{200 - \sqrt{200 \times 144}}{2} = 15,147$	$\frac{229,432}{400} \times 100 = 57,358$ $\Delta 7,358$
15	$\frac{200 - \sqrt{200 \times 110}}{3} = 17,225$	$\frac{236,700}{400} \times 100 = 74,175$ $\Delta 2,746$	$\frac{200 - \sqrt{200 \times 140}}{2} = 16,334$	$\frac{266,798}{400} \times 100 = 66,699$ $\Delta 4,198$
16	$\frac{200 - \sqrt{200 \times 104}}{3} = 18,538$	$\frac{345,615}{400} \times 100 = 86,424$ $\Delta 0,710$	$\frac{200 - \sqrt{200 \times 136}}{2} = 17,538$	$\frac{307,581}{400} \times 100 = 76,895$ $\Delta 1,895$
17	$\frac{200 - \sqrt{200 \times 98}}{3} = 20,000$	$\frac{400,000}{400} \times 100 = 100,000$ $\Delta 0,0$	$\frac{200 - \sqrt{200 \times 132}}{2} = 18,760$	$\frac{351,937}{400} \times 100 = 87,934$ $\Delta 0,484$
18			$\frac{200 - \sqrt{200 \times 128}}{2} = 20,000$	$\frac{400,000}{400} \times 100 = 100,000$ $\Delta 0,0$