# EVALUATION OF MODAL DAMPING RATIO BASED ON STRAIN ENERGY PROPORTIONAL DAMPING METHOD

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Presented in this paper are a model free-oscillation test and an analysis for clarifying the strain energy proportional damping method which is widely adopted for evaluating modal damping ratios of structures. Because damping ratios of structural components are sometimes different even in the same structure, it is required in the modal dynamic response analysis to evaluate the damping ratio for each mode. In this paper it is presented from experiments and analyses that the strain energy proportional damping method is quite accurate in evaluating the modal damping ratios of structures for seismic analysis.

*Key Words: Seismic Design, Damping Ratio, Energy Dissipation, Bridges*

## 1 INTRODUCTION

The damping ratio is an important parameter for evaluating seismic response of structures. Although the mass and stiffness can be accurately computed from mass and stiffness distribution in structures, the damping ratio can not be evaluated in the similar manner because the prediction of energy dissipation in structures is very complex. Therefore instead of directly evaluating energy dissipation of structures it is generally adopted to evaluate the energy dissipation of structures in terms of damping ratio. The damping ratio is an appropriate parameter for assigning the damping characteristics of structures. It should, however, be kept in mind that the damping ratio can be defined only for a single-degree-of-freedom oscillator. From the analogy of the orthogonal condition of modal matrix with respect to stiffness and

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mass matrices, the damping ratio has been assumed for seismic analysis of structures.

The damping ratios are different depending upon structural components even in the same structure. Therefore it is required to evaluate the modal damping ratio based on the different damping ratio assigned for each structural component. The strain energy proportional damping method has been widely used for such purposes [1) 2)]. Because most of energy dissipation is developed due to the deformation of structures, it has been considered reasonable to employ this method. But experimental verification has not yet fully been made.

This paper attempts to verify the strain energy proportional damping method for the application to the seismic design of structures. Based on model oscillation tests which are carefully designed for evaluating energy dissipation of structural components as well as of the entire structure, accuracy and application of the strain energy proportional damping method is clarified. A bridge is considered for the analysis.

## 2 ENERGY PROPORTIONAL DAMPING

For a linear discrete structure, the equations of motion may be written as

$$M\ddot{u} + C\dot{u} + Ku = -M\{1,0,0,\cdots 1,0,0\}^T \ddot{Z}_{gx} \qquad (1)$$

where

$M$ : mass matrix

$C$ : damping matrix

$K$ : stiffness matrix

$u$, $\dot{u}$, $\ddot{u}$ : displacement, velocity and acceleration vectors

$\ddot{Z}_{gx}$ : ground acceleration in x axis

Denoting $w_i$ and $\phi_i$ as the undamped angular natural frequency and the mode shape for the $i$-th mode, $u$, $\dot{u}$, $\ddot{u}$ may be expressed as

$$u = \sum_{i=1}^{k} \phi_i q_i(t)$$
$$\dot{u} = \sum_{i=1}^{k} \phi_i \dot{q}_i(t) \qquad (2)$$
$$\ddot{u} = \sum_{i=1}^{k} \phi_i \ddot{q}_i(t)$$

in which $q_i$ represents the generalized coordinate for the $i$-th mode and $k$ represents the degree of freedom.

From the orthogonal condition, the stiffness matrix $K$ and mass matrix $M$ become as

$$\begin{matrix} \phi m^T K \phi_n = 0 \\ \phi m^T M \phi_n = 0 \end{matrix} \quad (m \neq n) \qquad (3)$$

Assuming that the damping matrix $C$ can also be orthogonalized as

$$\phi m^T C \phi_n = 0 \qquad (4)$$

one can have $k$ sets of uncoupled equations of motion as

$$\ddot{q}_i + 2h_i w_i \dot{q}_i = -\Gamma_{ix} \ddot{Z}_{gx} \qquad (5)$$

in which $h_i$ and $\Gamma_{ix}$ represent the modal damping ratio and the mode participation factor for the $i$-th mode, respectively.

It is important to note here that the modal damping ratio $h_i$ which is defined only for a single–degree–of–freedom oscillator has been adopted in a dynamic response analysis of a multi–degree–of–freedom system in the form as in Eq.(5) because there is no other practical definition for energy dissipation for a multi–degree–of–freedom system.

It is then required to evaluate the modal damping ratio in Eq.(5) based on the energy dissipation mechanism of the structure. For such a purpose it is assumed to divide a structure into several structural segments in which energy dissipation has the same mechanism as shown in **Fig. 1** [3) 4)]. The segment is designated hereinafter as a substructure. A substructure may be a deck, a pier or a foundation as shown in **Fig. 2** in the analysis of a bridge.

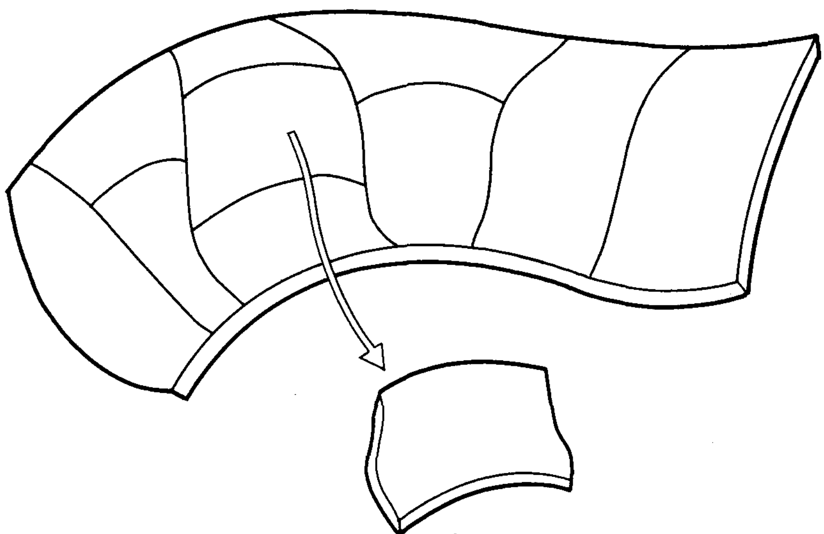


**Fig. 1** Substructures

Strain energy in the $j$–th substructure for the $i$–th mode may be evaluated as

$$E_{ij}=\phi_{ij}^{T}\cdot k_j\cdot\phi_{ij} \tag{6}$$

where

$E_{ij}$ : strain energy of $j$–th substructure for $i$–th mode

$\phi_{ij}$ : mode shape of $j$–th substructure for $i$–th mode

$k_j$ : stiffness matrix of $j$–th substructure

Denoting the energy dissipation in the $j$–th substructure for the $i$–th mode as $\triangle E_{ij}$, the damping ratio of the $j$–th substructure may be evaluated as

$$h_{ij}=\frac{1}{4\pi}\cdot\frac{\triangle E_{ij}}{E_{ij}} \tag{7}$$

Because the damping ratio of a substructure depends on its mode shapes and boundary conditions, it needs to be the one evaluated when the substructure of interest deforms as a part of the entire structure in the $i$–th mode shape.

Then the strain energy and energy dissipation of the entire structure for the $i$-th mode can be evaluated as

$$E_i = \sum_{j=1}^{n} E_{ij}$$
$$\Delta E_i = \sum_{j=1}^{n} \Delta E_{ij} \tag{8}$$

in which $n$ represents the number of substructures. The damping ratio of the entire structure (modal damping ratio) for the $i$-th mode may be written from Eq.(8) as

$$h_i = \frac{1}{4\pi} \cdot \frac{\Delta E_i}{E_i} \tag{9}$$

Substituting Eqs.(7) and (8) into Eq.(9), one obtains an equation for evaluating the modal damping ratio as

$$h_i = \frac{\sum_{j=1}^{n} E_{ij} h_{ij}}{\sum_{j=1}^{n} E_{ij}} \tag{10}$$

Eq.(10) is called as the strain energy proportional damping because the averaged damping ratio over each substructure is taken with the strain energy of the substructure being the weighting function. Substituting Eq.(6) into Eq.(10), Eq.(10) can be modified into a form which is more appropriate for computation

$$h_i = \frac{\sum_{j=1}^{n} \phi_{ij}^T \cdot h_{ij} \cdot k_j \cdot \phi_{ij}}{\sum_{j=1}^{n} \phi_{ij}^T \cdot k_j \cdot \phi_{ij}} \tag{11}$$

It should be noted that the natural periods of substructures are not necessarily the same as that of the entire structure. Therefore, if the frequency dependence of the damping ratios of substructures is significant, Eq.(11) can not be adopted.

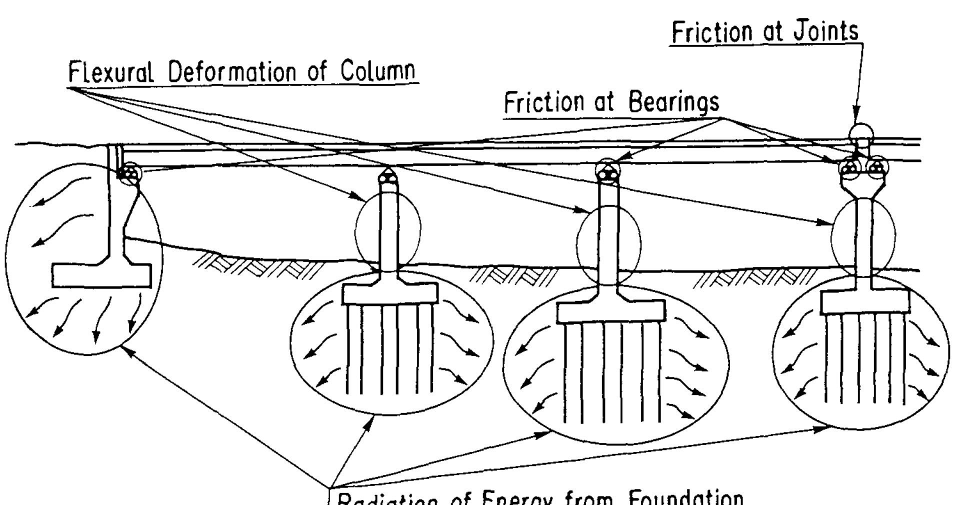


**Fig. 2** Substructures of Bridges in View of Energy Dissipation

## 3 MODEL FREE OSCILLATION TEST

**Fig. 3** shows the model bridge studied for verifying the effectiveness of the strain energy proportional damping method. **Photos 1** and **2** show the bridge model and the bearing. The girder is of a rigid steel plate so that energy dissipation during excitation is negligibly small. The girder has a span length of 2.1 m and a weight of 611.7 kgf. Each column with a height of 95 cm consists of four steel plates with the top of each two plates being rigidly connected. The bottom of the columns are rigidly connected to the base frame for preventing energy dissipation at the bottom.

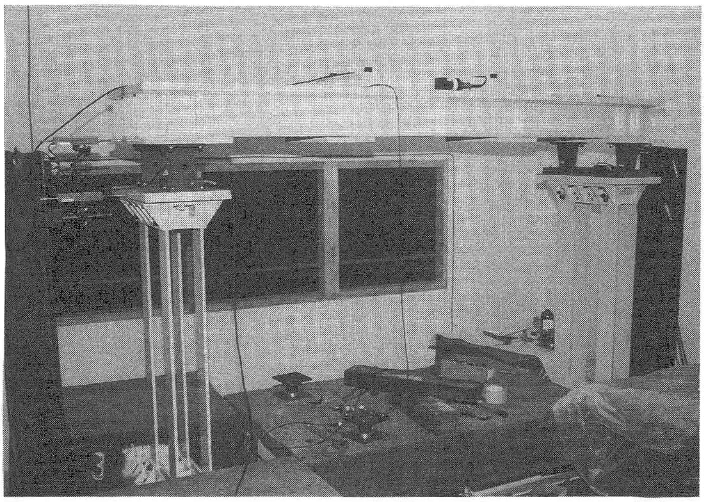

**Photo 1** Bridge Model

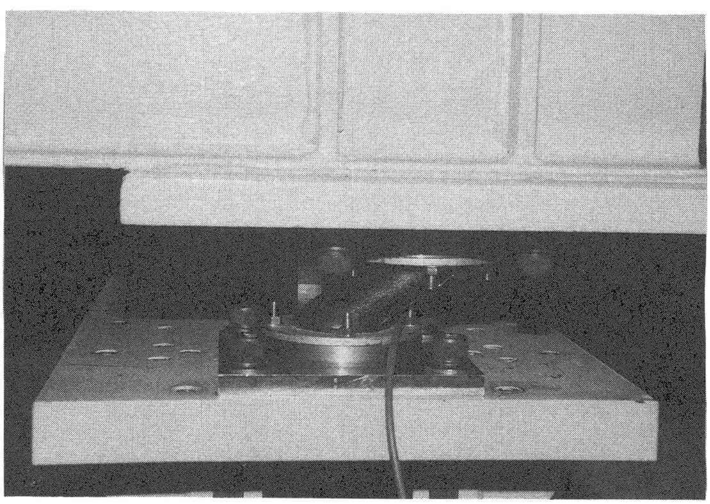

**Photo 2** Bearings

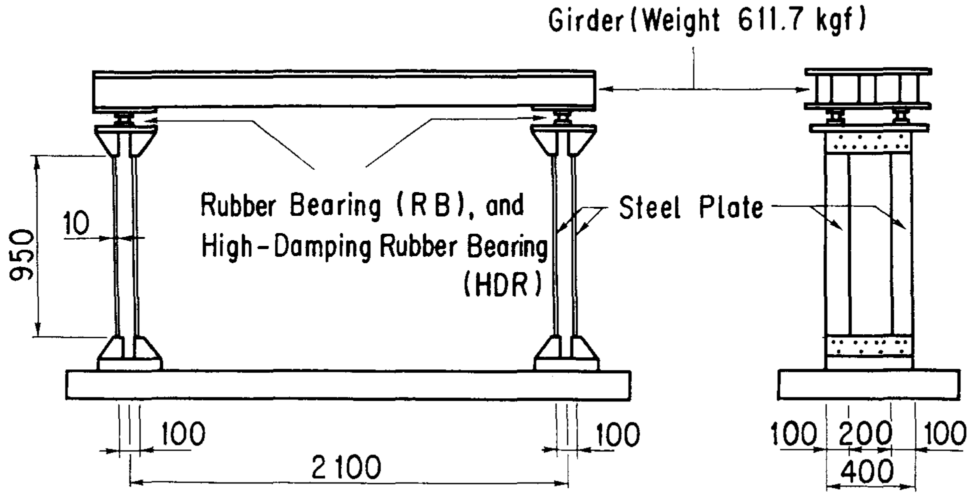


**Fig. 3** Model Bridge used for Free Oscillation Test

Four bearings were used for supporting the deck. Two types of bearings as shown in **Fig. 4** were used for the test. One is the laminated natural rubber bearing, which is called hereinafter as RB. This was made by laminating 25 natural rubber plates with a thickness of 0.9 mm each and 24 steel plates with a thickness of 0.3 mm each. The total thickness of rubber is 22.5 mm. It has elastic restoring force characteristics. The other is the laminated high damping rubber bearing, which is called as HDR in this paper. This was made by laminating 37 high energy absorbing rubber plates with a thickness of 0.5 mm each and 36 steel plates with a thickness of 0.3 mm each. The total thickness of high energy absorbing rubber is 18.5 mm.

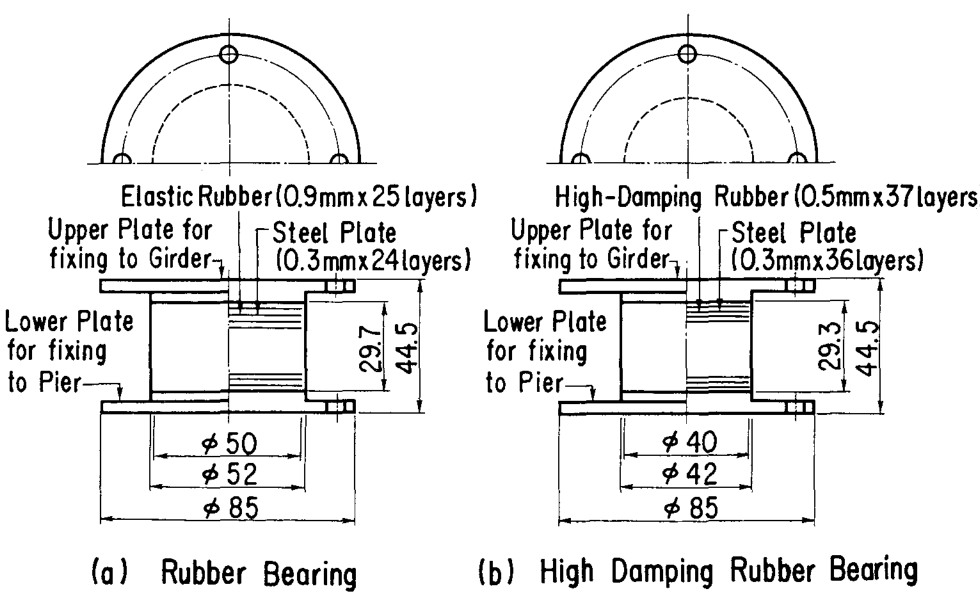


**Fig. 4** Model Bearings for Free Oscillation Test

In the model bridge, energy dissipation is developed at bearings and columns. Therefore, bearings and columns are regarded as substructures for evaluating the modal damping ratio of the model bridge by Eq.(11). Because the deck is rigid, no energy dissipation at the deck is considered in the analysis. Only the modal damping ratio for the 1st mode is evaluated in the test in order to make the excitation simple and easy. It should be noted that the frequency dependence of the danping ratios is small for steel, natural rubber and the high damping rubber [5) ].

For evaluating the damping ratio of each substructure, free oscillation tests were conducted individually for two types of bearings (RB and HDR) and two columns (Column A and B). Free oscillation tests of the model bridge were also conducted for evaluating the modal damping ratio of the entire structure.

In the free oscillation test of the model bridge, the deck was displaced in the longitudinal direction, and by releasing it from the displaced position, free oscillation was developed. The 1st mode shape of the model bridge in the longitudinal direction is of lateral movement of the deck with flexural deformation of cantilevered columns. Similarly the free oscillation test was conducted individually for the columns A and B considering the possible difference in the energy dissipation capabilities of two columns. For the bearings, the free oscillation test was conducted simultaneously for four bearings of a type. This was due to difficulty in conducting the test individually for each bearing. In the free oscillaiton test, four bearings of the same type were directly placed on a rigid steel frame and the deck was placed on the bearings. Then the deck was displaced laterally and released smoothly so that shear deformation was developed in the four bearings. Since the steel frame and deck are rigid, energy dissipation developed in this test represents that developed in the bearings. The tests were made individually for RB and HDR.

Because the stiffness of the bearing depends on temperature, all the tests were conducted in a room with a constant temperature of 20 ℃. Lateral displacements of the deck and columns were measured. Accelerations were measured so that the inertia force of the deck be evaluated. They will be used to evaluate the stiffness of bearings. Each test was made twice to secure the stability of the test results.

## 4 FREE OSCILLATION OF MODEL BRIDGE

**Fig. 5** shows the decay of free oscillation of the bridge model. It is apparent that the free oscillation decays much faster in the model supported by HDR than in the model supported by RB. The logarithmic damping ratio $\delta$ can be evaluated from the free oscillation as

$$\delta = log_e\left(\frac{a_m}{a_{m+1}}\right) \tag{12}$$

where

$\delta$ : logarithmic damping ratio

$a_m$ : displacement amplitude of $m$-th peak of free oscillation

The damping ratio $h$ may be evaluated from the logarithmic damping ratio as

$$\delta = \frac{2\pi h}{\sqrt{1-h^2}} \tag{13}$$

Damping ratios have been obtained for both plus and minus displacements in the free oscillation.

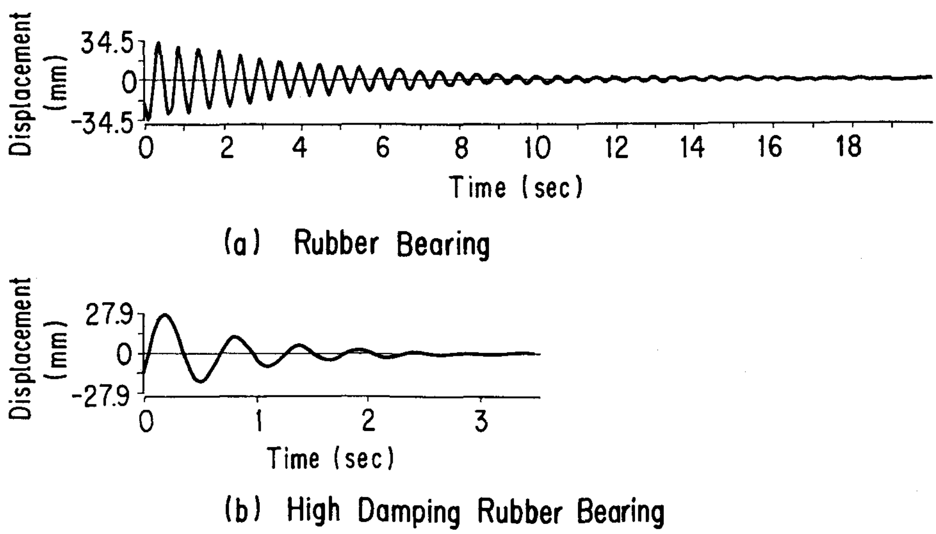


**Fig. 5** Decay of Free Oscillation of Bridge Model Deck

**Fig. 6** shows the dependence of the thus obtained 1st-mode damping ratio of the entire model bridge on the displacement of the deck. Some scatter is seen at small displacements in the data for the bridge model supported by HDR. Considering that the data with large displacements have more significance for the seismic design, the damping ratio was analyzed for the 1st to 4th cycles of the free oscillation. The displacement ranges of the deck are 6.3 mm – 27.9 mm for the model supported by HDR and 23.8 mm–33.0 mm for the model supported by RB. The averaged damping ratios over the above mentioned ranges are 11.4% for the model supported by HDR and 1.69% for the model supported by RB.

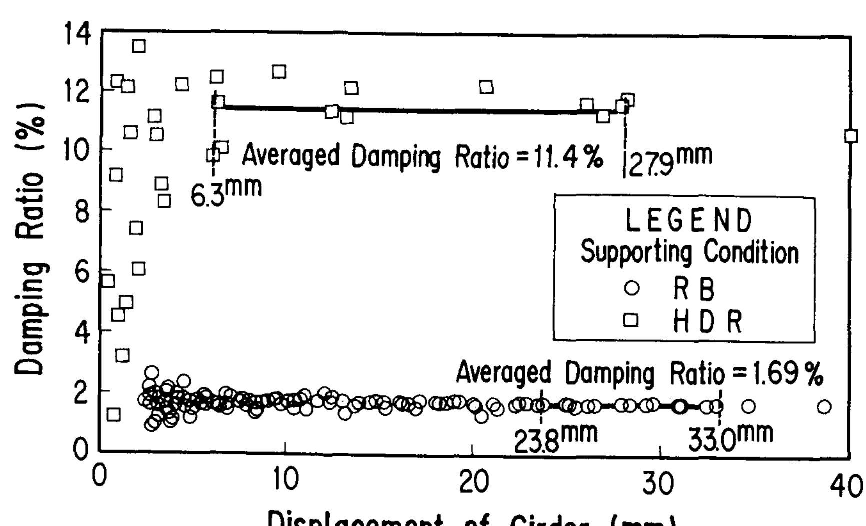


**Fig. 6** Damping Ratio of Model Bridge vs. Oscillation Amplitude

**Fig. 7** compares the decay of free oscillation of RB and HDR themselves. It is apparent that the oscillation decreases faster in HDR than in RB. **Fig. 8** shows the dependence of the bearing's damping ratios on the lateral displacement developed in the bearings. A decrease in the damping ratio at small displacements is observed, especially for HDR. This would be the reason for the decrease of the damping ratio observed for the bridge model supported by HDR, which is shown in **Fig. 6**. The damping ratios of both RB and HDR have little displacement dependence over the displacement range observed during the above mentioned entire bridge oscillation test. The averaged damping ratios are 3.27% for RB and 25.4% for HDR.

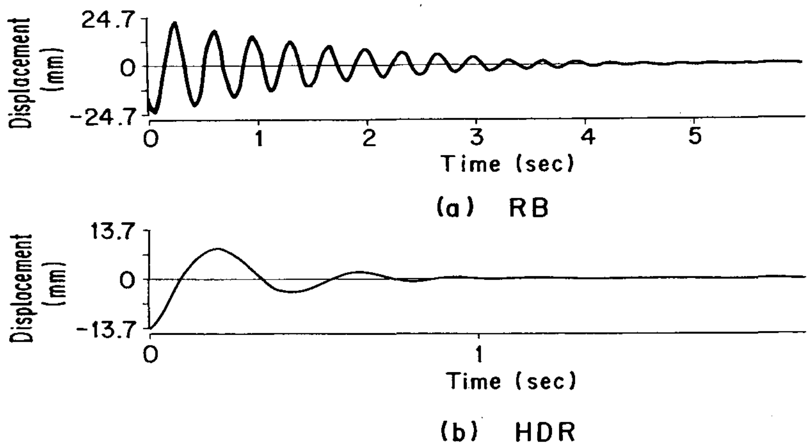


**Fig. 7** Decay of Free Oscillation of Bearing Displacement

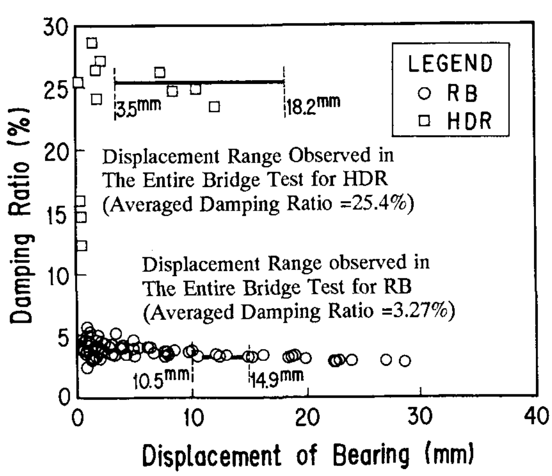


**Fig. 8** Damping Ratio of Bearing vs. Shear Displacement

**Fig. 9** shows the decay of free oscillation at the top of a column. **Fig. 10** shows the displacement dependence of the column's damping ratios. It is interesting to note that the scatter in the damping ratios is larger for Column B than for Column A. This would be due to inevitable difference in the rigidity of the connection between the columns and the steel frame. The displacement observed at the column top during the free oscillation test of the entire bridge model is 13.4 mm–18.2 mm when the deck is supported by RB and 3.0 mm–10.9 mm when the deck is supported by HDR. For the former displacement range, the damping ratios of the columns A and B are 0.708% and 0.954%, respectively. For the latter, the damping ratios of columns A and B are 0.459% and 0.544%, respectively.

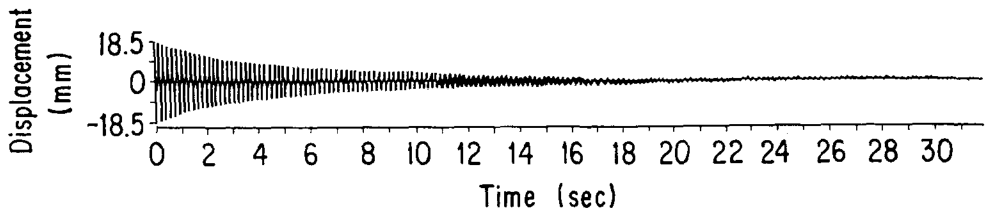


**Fig. 9** Decay of Free Oscillation of Column

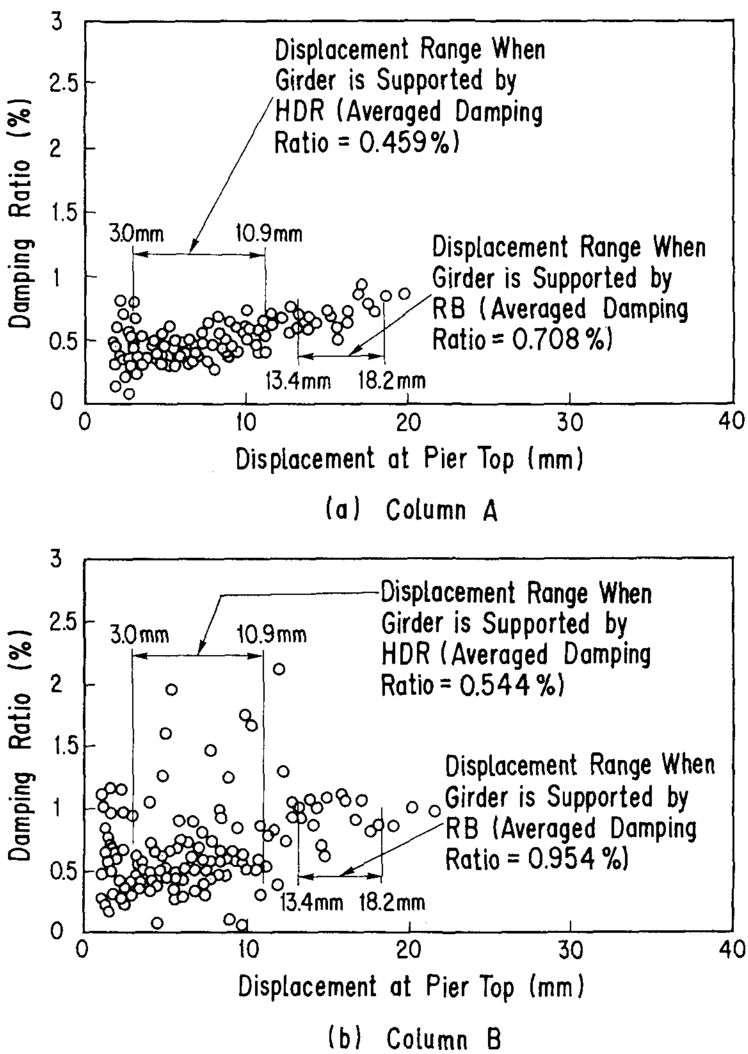


**Fig. 10** Damping Ratio of Columns vs. Displacement Amplitude at Column Top

## 5 ACCURACY OF STRAIN ENERGY PROPORTIONAL DAMPING METHOD

Now that the damping ratio of each substructure has been obtained, the damping ratio $h_1$ of the entire bridge model for the 1st mode may be evaluated using Eq.(10) as

$$h_1 = \frac{w_{1CA} h_{1CA} + W_{1CB} h_{1CB} + W_{1b} h_{1b}}{W_{1CA} + W_{1CB} + W_{1b}} \tag{14}$$

$W_{1CA}$ , $W_{1CB}$ : strain energy of column $A$ and $B$ for 1st mode

$W_{1b}$ : strain energy of bearings for 1st mode

$h_{1CA}$ , $h_{1CB}$ : damping ratio of column $A$ and $B$

$h_{1b}$ : damping ratio of bearings

Since the damping ratio of the entire bridge model was obtained from the test, accuracy of Eq.(10) can be examined by comparing the damping ratio predicted by Eq.(14) with the one evaluated from the test.

Strain energy of the columns and bearings may be evaluated as

$$W_{1CN} = \frac{1}{2} k_{CN} u_{CN}^2 \quad (N = A, B) \tag{15}$$

$$W_{1b} = \frac{1}{2} k_b u_b^2 \tag{16}$$

where

$W_{1CA}$ ,$W_{1CB}$ : strain energy of column $A$ and $B$ for 1st mode

$W_{1b}$ : strain energy of bearings for 1st mode

$k_{CA}$ ,$k_{CB}$ : stiffness of column $A$ and $B$

$k_b$ : stiffness of bearings

$u_{CA}$ ,$u_{CB}$ : lateral displacement at top of column $A$ and $B$

$u_b$ : shear displacement of bearings

The stiffness of the columns $k_{CA}$ and $k_{CB}$ can be evaluated based on the dimensions of the columns and elastic modulus. The stiffness of the bearings was evaluated as an equivalent stiffness from the hysteresis loop of the force vs. displacement relation. **Fig. 11** shows the relation between the thus evaluated stiffness of the bearings and their shear displacements. As the stiffness of the bearings shows some displacement dependency, it was approximated as

$$\begin{aligned} k_b &= 22.7 - 0.2 u_b && \text{for RB} \\ k_b &= 8.0 - 0.2 u_b && \text{for HDR} \end{aligned} \tag{17}$$

In the free oscillation test of the entire bridge model, the deck displacement can be evaluated as

$$u_d = u_c + u_b \tag{18}$$

where

$u_d$ : deck displacement

$u_c$ : displacement at column top

$u_b$ : shear displacement of bearings

Since $u_d$ and $u_c$ were measured during the test, $u_b$ can be calculated from Eq.(18). Then the stiffness of the bearings can be estimated by Eq.(17).

Thus the damping ratio of the model bridge was evaluated from Eq.(14) for several peaks of free oscillations as shown in **Table 1**. **Fig. 12** compares the predicted and measured damping ratios of the bridge model. The predicted values are very close to the measured ones. They are within $\pm$ 20% range of the measured damping ratio.

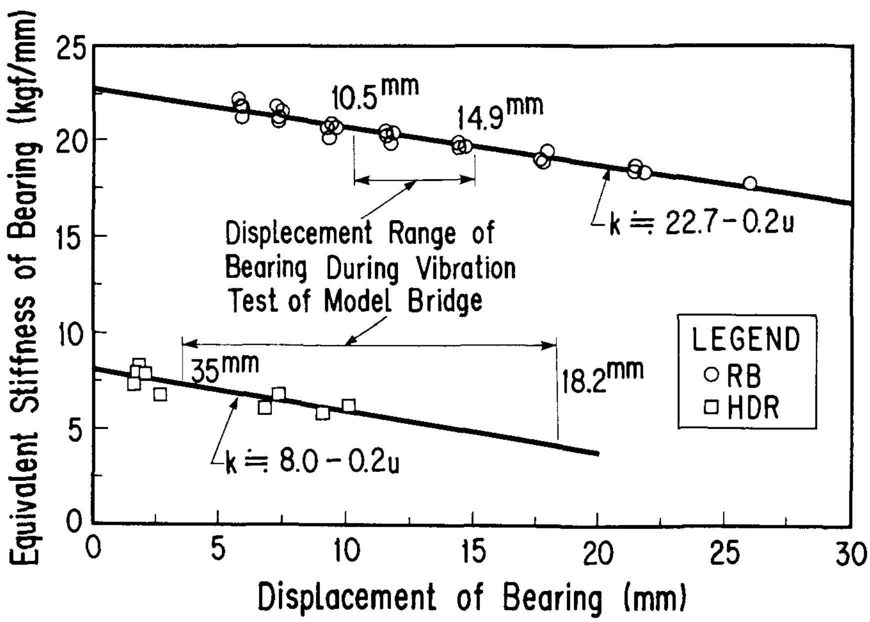


**Fig. 11** Displacement Dependence of Stiffness of Bearings

**Table 1** Comparison of Predicted and Measured Damping Ratio of Model Bridge

| Type of Bearing | Number of Free Oscillation | Displacement of Bridge Model | | | Damping Ratio and Strain Energy of Structural Members | | | | | | Modal Damping of Bridge | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Displacement at Girder $d_d$ (mm) | Displacement at Pier Top $d_p$ (mm) | Displacement at Bearing $d_b$ (mm) | Pier A | | Pier B | | Bearing | | Estimated by Eq.(11) $h_A$ | Measured $h_E$ |
| | | | | | $h_{pA}$(%) | $W_{pA}$ | $h_{pB}$(%) | $W_{pB}$ | $h_b$ (%) | $W_b$ | | |
| Rubber Bearing (RB) | 1 | 33.0 | 18.2 | 14.9 | 0.708 | 1.437 | 0.954 | 1.424 | 3.273 | 2.192 | 1.890 | 1.690 |
| | 2 | 29.6 | 16.5 | 13.3 | 0.708 | 1.181 | 0.954 | 1.170 | 3.273 | 1.775 | 1.881 | 1.690 |
| | 3 | 26.5 | 14.8 | 11.8 | 0.708 | 0.950 | 0.954 | 0.944 | 3.273 | 1.418 | 1.877 | 1.690 |
| | 4 | 23.8 | 13.4 | 10.5 | 0.708 | 0.779 | 0.954 | 0.772 | 3.273 | 1.137 | 1.864 | 1.690 |
| High-Damping Rubber Bearing (HDR) | 1 | 27.9 | 10.9 | 18.2 | 0.459 | 0.516 | 0.544 | 0.511 | 25.4 | 0.710 | 10.7 | 11.4 |
| | 2 | 13.4 | 5.7 | 8.2 | 0.459 | 0.141 | 0.544 | 0.140 | 25.4 | 0.213 | 11.2 | 11.4 |
| | 3 | 6.3 | 3.0 | 3.5 | 0.459 | 0.039 | 0.544 | 0.039 | 25.4 | 0.045 | 9.6 | 11.4 |

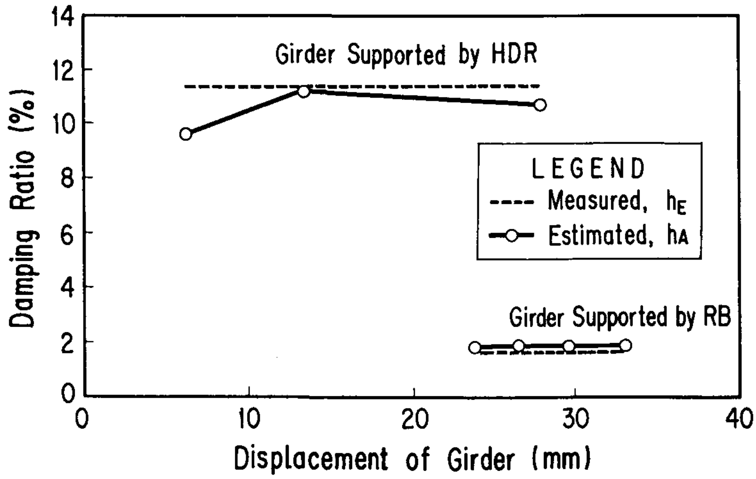


**Fig. 12** Accuracy of Predicted Damping Ratio

## 6 CONCLUSIONS

The effectiveness of the strain energy proportional damping method has been verified through the free oscillation test of a model bridge. Only the damping ratio for the 1st mode was analyzed because of the simplicity of the free oscillation test. It has been made apparent that the damping ratio of the bridge can be predicted with enough accuracy by the strain energy proportional damping method when the damping ratios of substructures are correctly evaluated.

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