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ENERGY-BASED FACTOR OF SAFETY AGAINST LIQUEFACTION

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The following expression of the factor of safety against liquefaction is found useful:

$$F_{le}=\sqrt{\frac{W_{l0}}{W_{e0}}}$$

where W_{l0} is the energy accumulated in the pore-water from the initial state until liquefaction; W_{e0} is the work done to the pore-water by the ground motion. F_{le} can be directly related to the pore-water pressure ratio. From the historical data of liquefaction sites and earthquakes, compiled by Kuribayashi and Tatsuoka, the effect of the ground motion is found to be captured by the quantity $X=SA_c$, in which S is the slip displacement of the unit mass with critical acceleration of sliding A_c. The procedure for evaluating F_{le} statistically from the JMA magnitude, epicentral distance, effective compressibility of the pore-water and critical angle of dislocation is presented.
Keywords: *liquefaction, safety factor, energy, dislocation, pore-pressure*

1. INTRODUCTION

Most of the recent structural damage, occurring during and after earthquakes in urban districts, is associated with the liquefaction of the soil. One can point out two reasons: the economic pressure toward population concentration urged the development of the bay areas where the soil is easily liquefied; improvement of the construction technology has produced safe structures, provided that the ground does not weaken during earthquakes. Thus, the assessment of the liquefaction hazard has become one of major significance in urban development.

The factor of safety against liquefaction has been defined as the ratio between the amplitude of the sinusoidal shear stress that causes liquefaction of the soil specimen in a certain, usually 20, number of cycles, and that of the ground acceleration (Seed and Idriss, 1967). For design purposes this has been related statistically to fundamental quantities of soil and ground motions, for example, SPT N value, relative density, magnitude, peak ground acceleration (e.g. Seed, 1979; Iwasaki, Tatsuoka, Tokida and Yasuda, 1978; Ishihara and Perlea, 1984; Tokimatu and Yoshimi, 1983; Poulos, Castro, and France, 1985).

The conventional factor of safety has been tested through field observations and laboratory data to determine whether the factor can distinguish the liquefied cases from the unliquefied, in other words, whether the excess pore-pressure ratio is greater or less than unity. Either theoretically or experimentally, the conventional factor of safety is not directly related to the absolute value of the pore-pressure ratio. As a result of the demand for a more accurate safety assessment of recent complex structures, evaluation has been required to determine not only whether the ground will liquefy but also the degree of liquefaction so that the stress state and residual strength of the ground can be estimated. Moreover, detailed information about ground motions has become available. For example, given a specific site, the power spectrum of the ground motion can be predicted. This will enable us to rationalize the safety assessment in terms of the seismic loading. To make use of previously mentioned advances in both the demand for safety assessments and better technology, a new framework can be tailored.

In this paper, the factor based on the energy of soil and ground motion will be introduced, which can be related to the magnitude of the pore-pressure through its strain energy. Since energy is the fundamental physical quantity, the proposed framework can be applied to any case where a mechanism is known or assumed.

2. ENERGY-BASED FACTOR OF SAFETY

The generic factor of safety for soil and constructed facilities against a critical state due to the earthquake loading can be defined

$$F_{ce} = \sqrt{\frac{W_{c0}}{W_{e0}}} \quad\cdots\cdots(1)$$

in which W_{c0} is the external work necessary to bring the soil and/or constructed facilities to the critical state c from that at rest 0; W_{e0} is the part of the seismic energy

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that is used to change the state of the structure from that at rest 0 to the design state e. F_{ce} becomes unity if the structure absorbs exactly as much energy as the design earthquake provides, until it reaches the critical state. The square root is adopted considering a relation to the conventional safety factors defined by the amplitudes of the resistance R and load L:

$$F_s = \frac{R}{L} \quad \cdots\cdots(2)$$

A variety of definitions have been possible for a single problem depending upon how we measure them in time and space. For instance, in the slope stability analysis there have been several definitions of factor of safety according to the methods of slices (Fredlund, and Krahn, 1977). This is a practical aspect of the conventional definitions of the safety factor, but contains the danger of making the problem more complicated than it actually is and allowing the research to diverge in many directions. In the general formulation (1), we can derive the factor of safety against liquefaction by defining the state of liquefaction. This means literally to substitute l, denoting liquefaction, for the subscript c.

During earthquakes, the pore-water is held and is gradually compressed by the bulk of soil. Hence, we shall use the pore-pressure as the index to represent the state of the structure concerning liquefaction. Assuming the pore-water is an elastic material with compressibility C, the elastic strain energy density at pressure p can be

$$E=\frac{1}{2}nCp^2 \quad \cdots\cdots(3)$$

where n is the porosity. The work done by the bulk of soil to pressurize the pore-water from p_0 to p_v can be measured by the increment of its elastic strain energy,

$$W_{v0}'=\Delta E=\frac{1}{2}nC(p_v^2-p_0^2) \cdots\cdots(4)$$

where the prime denotes W_{v0}' as a density, i.e. the quantity per unit volume with the subscript v standing for either l or e. We define the critical state of soil liquefaction as the pore-water pressure p reaching a specific value, for example,

$$p_l = \sigma_{v0}'+p_0 \cdots\cdots(4a)$$

where σ_{v0}' is the initial effective vertical stress. Provided that the pore-water pressures are known, the external works W_{v0} can be calculated by integrating Eq. (4) for the volume V of the structure.

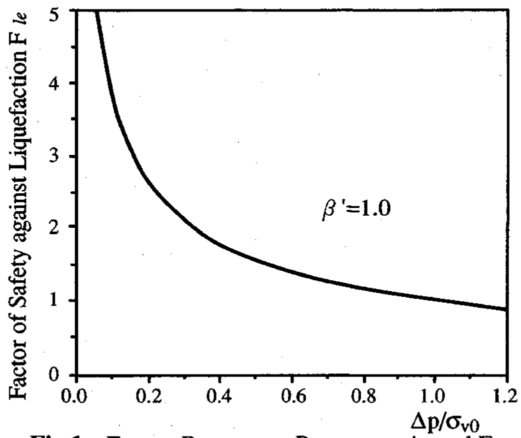

Fig.1 Excess Pore-water Pressure ratio and F_{le}

The factor of safety against liquefaction of a generic point in the soil can be calculated if the initial pore-pressure p_0, initial vertical effective stress of the bulk of soil σ_{v0}' and the increment of the pore-pressure Δp are known:

$$\beta' = \frac{p_0}{\sigma_{v0}'}, \quad r_u = \frac{\Delta p}{\sigma_{v0}'} \quad \cdots\cdots(5)$$

Then Eqs. (1), (3), (4) and (5) are combined to form

$$F_{le} = \sqrt{\zeta_p \frac{1+2\beta'}{r_u(r_u+2\beta')}} \quad \cdots\cdots(6)$$

where ζ_p is the ratio of n and C evaluated from the initial state 0 to liquefaction l and to the current state e:

$$\zeta_p = \frac{(nC)_{l0}}{(nC)_{e0}} \approx 1 \cdots\cdots(7)$$

This value can be regarded as varying from unity only to the second order. **Fig.1** plots this curve for $\beta'=1.0$, $\zeta_p=1$. The practical range of the initial pore-pressure ratio β' is about 0.5 to 2.0, where F_{le} deviates very little.

Conversely, the excess pore-pressure can be evaluated from the factor of safety;

$$\frac{\Delta p}{\sigma_{v0}'} = \sqrt{F_{le}^{-2}(1+2\beta')+\beta'^2} - \beta' \cdots\cdots(8)$$

3. EVALUATION OF F_{le} BY THE DISLOCATION ENERGY CONCEPT

There are various ways to evaluate the energy-based factor of safety; for example, one can do it experimentally, in the scaled model on the shaking table, by simulating the effect of the design ground motion and measuring the excess pore-water pressure ratios in the structure. The simulation can be replaced by a numerical one provided that the behavior of the saturated sand under a generic excitation is properly realized as the digital sequence in a computer. This seems possible in the future when we consider the remarkable progress made, to date, in this direction. However, the randomness of the earthquake and soil, stemming from the intrinsic uncertainty of the space and time, still remains untouched by these types of deterministic approaches; we cannot know the exact time history of an upcoming earthquake; we cannot obtain the material properties of every section of soil. We have to resort to the statistical approach to evaluate the factor of safety. This does not weaken the necessity of understanding the mechanism of liquefaction, but instead enhance it, because physical laws are among the few things which are certain. Otherwise, the applicability of the result could be limited within the narrow space of similar conditions where the statistical or experimental relations are originally obtained. In what follows, a model of liquefaction will be introduced.

Imagine the particles of the saturated sand which make up the ground and earth-structures. Most of the particles are in contact with each other. The seismic waves propagate through them by their contact forces. During ground motion, the pore-water can be regarded to be held by the particles that form the skeleton of the bulk. Assume that when the external acceleration exceeds the critical value A_c, a part of the particles, η being the volume ratio, dislocates from the bulk of the soil and compresses the pore-water at a steady acceleration A_c. During this process, the dislocating particles, $\eta\rho$ being the mass density, are assumed to receive the constant reaction $\eta\rho A_c$ from the pore-water. The work done to the pore-water can be written

$$W_{e0}' = 2\eta \rho A_c S \quad \cdots\cdots(9)$$

in which the prime indicates that W_{e0}' is a density; 2S is the relative displacement of the dislocating particles to the pore-water, i.e. to the bulk.

During the above process, the dislocating particles carry the same kinematic energy density as the bulk when they start dislocating from it and consume the energy by moving through the pore. Therefore, the work done to the pore-water can be related to the mean kinematic energy density K with a non-dimensional function f, i.e.

$$W_{e0}' = 2\eta f K \quad \cdots\cdots(10)$$

where

$$K = \rho v_r^2 N_1 \quad \cdots\cdots(11)$$

in which ρ and v_r is the mass density and the RMS velocity of the ground motion. N_1 is the effective number of cycles for the central frequency ω_1 and the duration s_0;

$$N_1 = \frac{s_0 \omega_1}{2\pi} \quad \cdots\cdots(11a)$$

In the above formula, s_0 is the denominator of the RMS computation. This can be selected arbitrarily without infringing on Parseval's theorem. By following Vanmarcke and Lai's definition (1980), s_0 is determined so that the record peak acceleration is observed once on the average in the s_0 seconds of the stationary Gaussian process with the same RMS values.

In the above model, the motion of the dislocating particles begins when the acceleration of the bulk exceeds the threshold Ac and continues at the steady rate Ac until it stops with respect to the bulk. This can be idealized as the motion of a mass on a rough boundary which has been called the Newmark model. For the f-function in Eq. (10), the analytical solution obtained by Igarashi and Hakuno (1987) can be used:

$$f = \exp\left(-\frac{1}{2}\left(\frac{A_c}{a_r}\right)^2\right)\left(\alpha_1 + \frac{\pi}{2}\sqrt{1-\alpha_1^2}\right) \quad \cdots\cdots(12)$$

where a_r is the RMS amplitude of the ground acceleration, and α_1 is the bandwidth index of the ground velocity, equalling unity and zero for sinusoid and white noise, respectively.

For a sinusoidal shear wave, it can be shown that the mean kinematic energy K is equal to the mean strain energy U:

$$K = U = \frac{1}{2G}\tau_d^2 N \quad \cdots\cdots(13)$$

in which τ_d is the amplitude of the shear stress, N is the number of cycles, and G is the mean shear modulus.

The dislocation of a particle is assumed to occur when the local shear force between the particle and the bulk or the other particles exceeds the Coulomb frictional limit.

Table 1 Parameters to Evaluate F_{le} by the Dislocation Energy Concept

Name	Sym.	Unit	Mean *
Dislocation Angle	ϕ_c	deg	13.8
Effective Compressibility	C_e	Pa^{-1}	6.10E-8
Porosity	n		
Bulk Density	ρ	kg/m^3	
Saturated Bulk Density	ρ'	kg/m^3	
Initial Pore-Pressure	p_0	Pa	
Initial vertical effect. str.	σ_{v0}'	Pa	
Initial mean effect. str.	σ_{m0}'	Pa	
RMS Velocity	v_r	m/sec	0.079
RMS Acceleration	a_r	m/sec^2	0.674
Strong Motion Duration	s_0	sec	11.88
Central Frequency	ω_1	rad/sec	12.17
Bandwidth Index	α_1		0.410

Note: The mean values are obtained from 6 cases for C_e and ϕ_c; from 104 components for v_r through α_1.

It is difficult to measure the local contact forces which vary randomly particle by particle. Especially, as the pore-pressure increases, the forces are expected to decrease and eventually reaches zero at liquefaction. The critical acceleration has been introduced to represent the criteria of dislocation on the average in the space and time. The averaging volume considered is large enough to contain many particles and is small enough to validate the differential calculus in the scale of the structures. On the time axis, the volume lies from the beginning until the end of the loading. Similarly, the criteria of dislocation can be written in terms of the friction angle ϕ_c between the representative values of the contact forces as

$$\tau_c = \sigma_{m0}' \tan \phi_c \quad \cdots\cdots (14)$$

where σ_{m0}' is the initial effective confining stress of the bulk, τ_c is the critical shear stress of dislocation, which is the average of the actual shear stresses between the dislocating particles and the bulk. The averaging volume is the same as that is used to define the critical acceleration. The critical acceleration of dislocation can be written by the critical shear stress if the relationship between the acceleration and the shear stress of the bulk is known. For example, for the soil at depth H of the free ground surface with mass density ρ and accelerated by a uniform horizontal acceleration:

$$A_c = \frac{\tau_c}{\rho H} = \frac{\sigma_{m0}'}{\rho H} \tan \phi_c \quad \cdots\cdots (15)$$

Similar proportionality of the shear stress and acceleration has been assumed for ground motions, and we have applied the stress ratio instead of acceleration to the specimen in the cyclic loading test. Using this proportionality, the exponent of Eq. (12) or the critical acceleration A_c normalized by the RMS value of the input acceleration a_r can be replaced by the stress ratio $\tau_c/\tau_d/\sqrt{2}$, where $\sqrt{2}$ is the ratio of the RMS value and the peak value of a sinusoid. Noting α_1 is unity for the sinusoidal waves, the f function takes the simple form:

$$f = \exp\left(-\frac{\kappa^2 \tan^2 \phi_c}{R_l^2}\right) \quad \cdots\cdots (16)$$

where

$$R_l = \frac{\tau_d}{\sigma_{v0}'}, \quad \kappa = \frac{\sigma_{m0}'}{\sigma_{v0}'} \quad \cdots\cdots (16a)$$

is the stress ratio of loading and of initial confining pressure, respectively. This is the dislocation energy concept of liquefaction. It introduces three new material constants: the volume ratio of the dislocating particle η, the compressibility of the pore-water C, and the critical angle of dislocation ϕ_c. We will compress the parameters into a pair by defining the effective compressibility:

$$C_e = \frac{C}{\eta} \quad \cdots\cdots (17)$$

From the undrained cyclic shearing test results for sands with medium relative density, reported by Ishihara et al. (1977) and Toki et al. (1986), using the above formula, ϕ_c and Ce is evaluated about 15 degrees and 5.0×10^{-8} Pa^{-1}, respectively.

The factor of safety against liquefaction can be evaluated by the following procedure:

1) Determine the critical angle of dislocation ϕ_c (deg) and effective compressibility of the pore-water C_e (P_a) by the undrained cyclic shearing test.
2) Specify the design earthquake by RMS velocity v_r (m/sec), RMS acceleration a_r (m/sec^2), strong motion duration s_0 (sec), central frequency ω_1 (rad/sec) and bandwidth index α_1 of the ground velocity .
3) Compute the external work necessary to liquefaction W_{l0} by Eq. (4).
4) Estimate the seismic energy used to dislocate the soil particle and increase the pore-water pressure W_{e0} by Eqs. (9), (10), (12) and (15).
5) Evaluate the factor of safety against liquefaction by Eq. (1); put c=l and substitute the above calculated values of W_{e0} and W_{l0}.

This procedure uses a total of 12 parameters, 7 for representing the strength and condition of the soil, 5 for characterizing the energy content of the ground motion. **Table 1** lists them.

4. EXCESS PORE-WATER PRESSURE RATIO AND CONVENTIONAL F_l

Recently, it has been required to estimate the excess pore-water pressure ratios in the design practices, for example, in the stability analysis of the underground utility ducts (JRA,1986) and pile foundations (JRA,1990). Current formulae have been derived from the experimental relationships obtained in the undrained cyclic loading test using the conventional F_l value as the index to represent the degree of liquefaction. However, the correlation does not appear sufficiently strong; Tokida et al. compiled the excess pore-water pressure ratios r_u obtained by the large-scale shaking table test (1981). **Fig. 2** plots their relationships.

Yasuda(1986) has compiled a number of cyclic undrained shear test results for undisturbed samples and has obtained statistical relations of the form:

$$r_u = a(F_l)^b \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (18)$$

In his book (1988), Yasuda pointed out that previous formulae can be summarized as above; for instance, the Japan Road Association (1986) specified a=1, b=-7 in the design guideline of utility ducts.

He concluded that it is necessary to include other parameters into the regression to reduce the error of estimate, namely the stress ratio for liquefaction R_l and the type of the sand. The exponent depends on the former:

$$b=-0.163(R_l)^{-2.42} \cdots\cdots\cdots\cdots\cdots\cdots (18a)$$

The coefficient a is defined with both of them (**Table 2**).

We can see in Yasuda's equation that for the same F_l, r_u can vary almost from zero to unity by changing the stress ratio R_l and the type of soil. From above observations, it can be concluded that by itself the conventional F_l value has little resolution in predicting the excess pore-water pressure. Let us analyze this fact by the dislocation energy concept.

The conventional factor of safety of the specimen is

$$F_l = \frac{R_{l,20}}{R_{,20}} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (19)$$

Table 2 parameter a of Eq. (18)

R_l	Type of soil	a
$R_l \le 0.35$		1.0
$R_l \le 0.35$	Fines Contained	$1.7-2R_l$
$R_l \le 0.35$	Dense	1.2

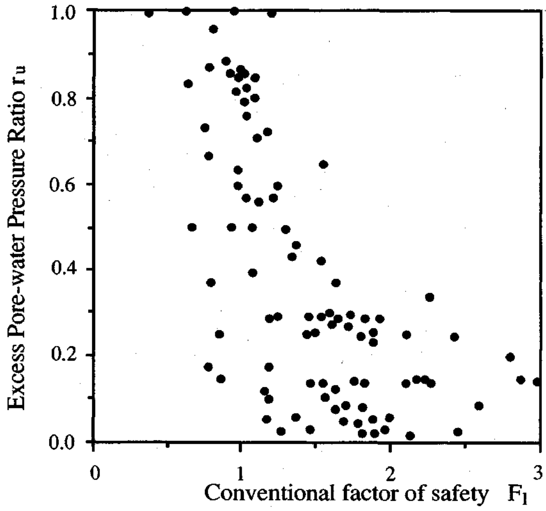

Fig. 2 Excess pore-water pressure ratio vs. Conventional F_l (Tokida et al.,1981)

in which $R_{l,20}$ is the stress ratio that causes liquefaction in 20 cycles; $R_{,20}$ is the working stress ratio, converted to the equivalent value for 20 cycles. In Eq. (4), the excess pore-water pressure ratio can be calculated by the external work W_{v0}' during the cyclic loading test, porosity n, compressibility C and initial pore-pressure ratio β' (Eq. (8));

$$r_u = \frac{\Delta p}{\sigma_{v0}'} = \sqrt{\frac{2W_{v0}'}{(nC)_{v0}\sigma_{v0}'^2} + \beta'^2} - \beta' \cdots (20)$$

Using the relations (10), (13), and (16), the external work is expressed by the stress ratio:

$$\frac{W_{v0}'}{\sigma_{v0}'^2} = \left(\frac{\eta}{G}\right)_{v0} R_v^2 \times 20 \times \exp\left(-\frac{\tan^2\phi_c}{R_v^2}\right) \cdots\cdots (21)$$

in which subscript v is either (l,20,) or (,20,). From Eqs. (19) through (21), noting that r_u is unity for v=l,20, the excess pore-water pressure ratio is related to F_l as

$$r_u = \sqrt{\zeta_p \zeta_b \varphi F_l^{-2}(1+2\beta')+\beta'^2} - \beta' \cdots\cdots (22)$$

This expression is similar to Eq. (8) which relates r_u with the energy-based factor of safety F_{le} except for

$$\zeta_b = \frac{\left(\frac{G}{\eta}\right)_{l,20,0}}{\left(\frac{G}{\eta}\right)_{,20,0}} \approx 1 \quad \cdots\cdots\cdots\cdots (22a)$$

and φ.

We have adopted the square root in the definition (1) to maintain this similarity that stems from the fact that the energy is proportional to the squared amplitude of the stress ratio as expressed in Eq. (21). The subscripts (l,20,0) and (,20,0) of Eq. (22a) denote respectively that G and η are evaluated between the initial state and liquefaction and from the initial state to the present when F_l is computed. The property of the bulk has entered into our expression in addition to that of the pore defined as ζ_p in Eq. (7). We assume that both ζ_p and ζ_b deviate from unity to the second order.

Consequently, the scatter of r_u is attributed to

$$\varphi = \frac{(f)_{,20,0}}{(f)_{l,20,0}} = \exp\left(-\frac{\tan^2\phi_c}{R_{l,20}^{2}}\left(F_l^{2}-1\right)\right) \quad \cdots\cdots\cdots (23)$$

where we assume that the deviation of tan ϕ_c is negligible. This result corresponds to Yasuda's observation, since tan ϕ_c and $R_{l,20}$ represent the type and strength of the soil. **Fig. 3 (a)** through **(c)** illustrates that Eq. (22) can be almost superposed on to Yasuda's relations by choosing the critical angle of dislocation between 10 to 20 degrees, which are ordinary values measured from sand with small to medium relative density.

The exponent of Yasuda's equation varies from minus infinity to zero, as R_l increases, equalling -1 at about R_l =0.45. Whereas, Eqs. (22) and (23) do not attribute the deviation to the exponent of the F_l value, but contain the strength $R_{l,20}$ and the type of sand ϕ_c explicitly. The derivation for the analytical expression has shown that, although it is a non-dimensional index, the exponent of the conventional F_l inherits the mechanism of liquefaction. It is difficult to imagine a model that yields an exponent far from -1. According to the dislocation energy concept, the factors to be supplemented with the conventional F_l, for predicting the excess pore-pressure ratio in the cyclic undrained test specimen, are identified as the stress ratio and the dislocation angle, which represent the strength and the type of the sand. On this account, the energy-based factor of safety is preferred because it is directly related to the pore-pressure.

The excitation is presumed to be a sinusoid in the above discussions. Therefore, the f function is simplified as in Eq. (16) and only the magnitude of the loading remained in Eq. (21). This analytical consequence is consistent with the previous

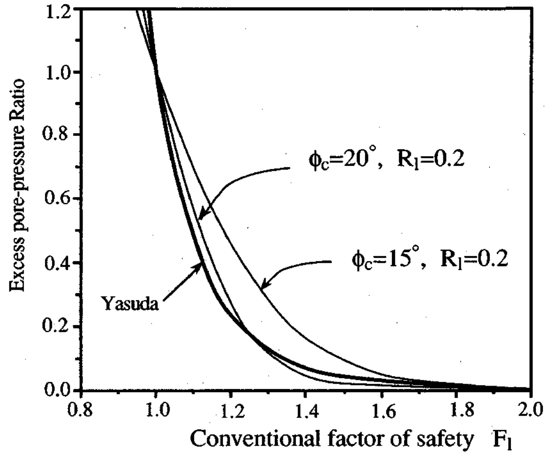

Fig. 3 (a) Analytical F_l vs. r_u and Yasuda's Relations (R_l=0.2)

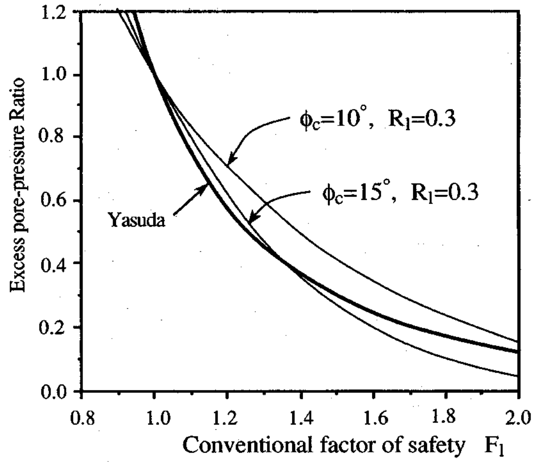

Fig. 3 (b) Analytical F_l vs. r_u and Yasuda's Relations (R_l=0.3)

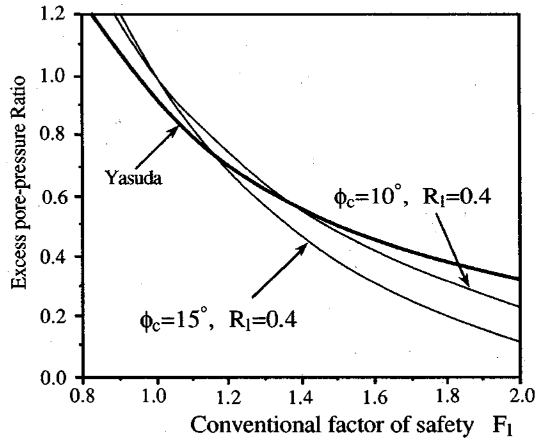

Fig. 3 (c) Analytical F_l vs. r_u and Yasuda's Relations (R_l=0.4, with fines)

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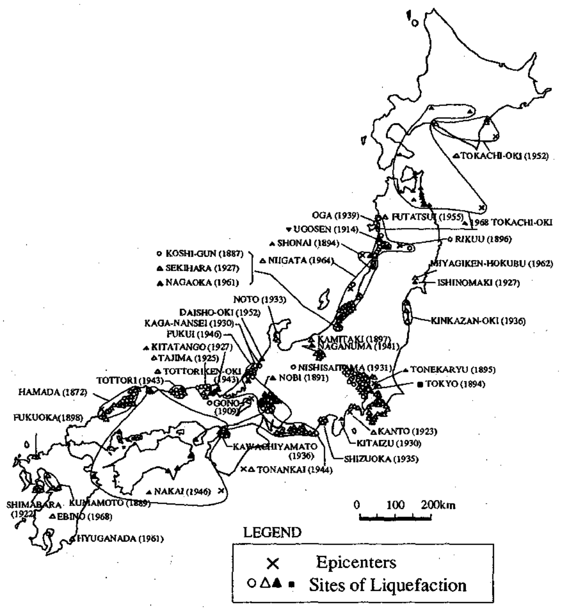

Fig. 4 Historical Liquefaction of Japan (reproduced from Kuribayashi & Tastuoka, 1975)

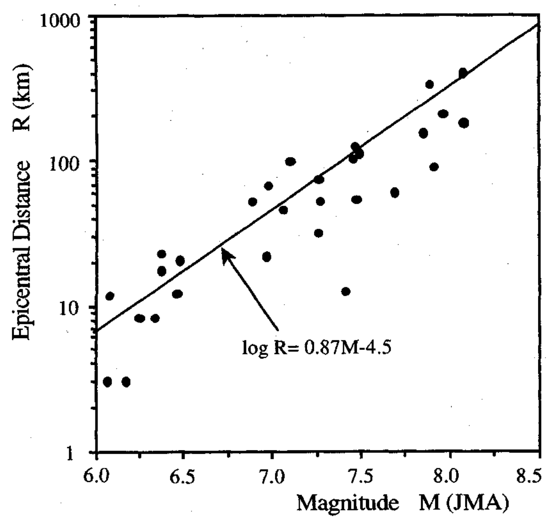

Fig. 5 Maximum Epicentral Distance of the Liquefied Sites (Kuribayashi & Tatsuoka, 1975)

experimental data which presents little bias toward the loading frequency (e.g. Tatsuoka et al., 1986). This fact stems from the very special nature of the sinusoid; the frequency and the form is maintained through its differential processes. This nature is expressed by the bandwidth parameters of the i'th derivatives α_i being unity.

Conventional methods appear to have assumed that the random loadings had this nature as well, for they depend mostly on the peaks of accelerograms, for instance, by counting all the peaks in them (e.g. Tatsuoka et al., 1980). A glance at Eqs. (10) through (12) will show that in the case of actual ground motion, even more factors have to be involved; frequency, duration, bandwidth and so forth. When detailed information on the ground motion is available, for example, in the form of a time history or a power spectrum, Eqs. (9), (10) and (12) give a simpler framework for evaluating the effect of the earthquake than the previous procedures.

Recent progress in the analysis and observation of strong motions has made it possible to predict the Power Spectrum Density Function of the ground motion at a specific site for design purposes, therefore, our procedure to deduce strong motion parameters from the PSDF has become a practical one. On the other hand, as is proposed by Seed and Idriss in 1967, the conventional methods have also provided the framework to assess the randomness of ground motion. They pointed out that the entire time history should be examined for this purpose (Seed and Idriss, 1971). Their definition of

$$\tau_{av} = 0.65\tau_{max} \cdots\cdots (24)$$

corresponds with our formula of RMS amplitudes and

$$N_c = 10, 20, 30 \text{ for } M = 7, 7\text{-}7.5, 8 \cdots (24a)$$

conforms with our N_l (Eq. (11a)). Although above formulae are very rough, they capture the essence concisely. Incidentally, Seed (1979) refined Eq. (24a) in the form of a chart.

In the following chapter, we will investigate another method without using the time history or PSDF but by focusing just on the global characteristics of the earthquake, i.e. magnitude and distance. For these are directly related to energy and are fundamental qualities of the earthquakes throughout history in all parts of the world.

5. CRITICAL DISLOCATION ENERGY OF GROUND MOTION

It is certain that the cause of the liquefaction is the ground motion. But it is not clear what type of seismic wave governs the phenomenon. It has been presumed

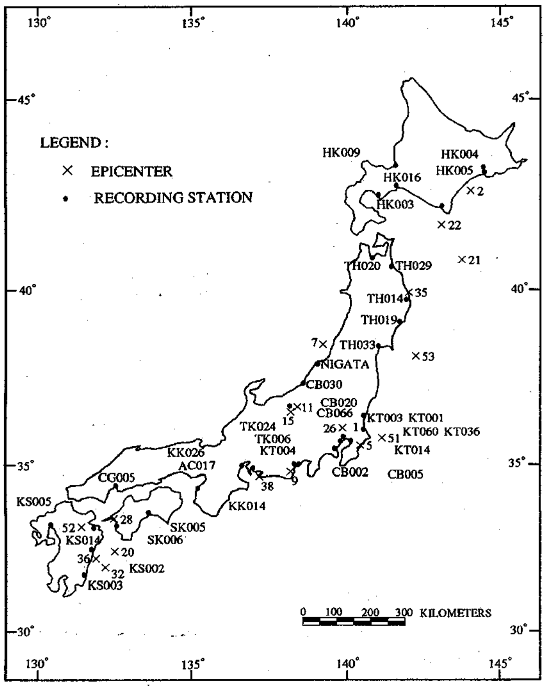

Fig. 6 Epicenters & Recorded Stations of Igarashi and Hakuno's Dataset

that acceleration, especially at its peak value, can represent the effect of the earthquakes. On the other hand, in the analytical expression, the velocity related parameters are dominant (**Table 1**).

Kuribayashi and Tatsuoka (1975) surveyed numerous cases of historical liquefactions in the literature, caused by the 44 major earthquakes in Japan, dated from the 1872 Hamada until the 1968 Tokachi-oki. In **Fig. 4** are plotted the locations of the sites and the epicenters of the causal earthquakes where liquefaction has been recorded. They have concluded with the following regression equation of the maximum epicentral distance R(km) of the sites where liquefaction was observed against the JMA magnitude M:

$$\log R = 0.87M - 4.5 \quad \cdots\cdots\cdots\cdots (25)$$

in which log is the common logarithm. In **Fig. 5** are plotted the data. They are almost uniformly distributed against both M and R, for which their data set can be evaluated as an unbiased one.

Suppose a seismic wave X attenuates like

$$\log X = a + bM - c\log R \quad \cdots\cdots\cdots\cdots (26)$$

and the soil does not liquefy with an amplitude of less than X_C, then the maximum distance Rx where the liquefaction is observed for the earthquake with the JMA magnitude M can be expressed by:

$$\log R_x = \frac{b}{c}M + \left(\frac{a}{c} - \frac{1}{c}\log X_c\right) \quad \cdots\cdots\cdots\cdots (27)$$

By comparing the coefficients of Eqs. (27) and (25), we can identify the type and threshold amplitude of the seismic wave that has governed the initiation of the historical liquefactions in Japan.

Their data set contains, for example, the Kanto earthquake,which occurred well before 1940 when the first accelerogram was recorded. Therefore, it is impossible to calculate the attenuation laws of this particular data set. Because the data set covers major earthquakes throughout Japan, the attenuation laws can be assumed similar to what are obtained from a relatively new data set which contains major events. On this account we use attenuation relations obtained from the 156 records of 20 earthquakes in the NOAA file (1981) dated from 1956 to the 1978 Miyagiken-Oki earthquake, which are originally selected by Igarashi and Hakuno (1987). In **Fig. 6** are plotted the epicenters and the recording sites. For the attenuation laws, specifically, the peak ground acceleration A (cm/sec^2) is

$$\log A = 1.11 + 0.28M - 0.47\log R \cdots\cdots (28a)$$

the peak ground velocity V (cm/sec) is

$$\log V = -1.00 + 0.46M - 0.58\log R \cdots\cdots (28b)$$

and the peak ground displacement D (cm) is

$$\log D = -2.86 + 0.63M - 0.57\log R \cdots\cdots (28c)$$

Igarashi and Hakuno(1987) computed residual displacements S (cm) of the Newmark's sliding block for various critical acceleration of sliding Ac (cm/sec^2). The slip displacements are also regressed against M and R:

$$\log S = -2.04 + 0.98M - 1.06\log R - 1.39\log A_c \quad \cdots\cdots (28d)$$

The regression coefficients for the magnitude and epicentral distances, b and c in Eq. (26), increase as the type of the wave changes from acceleration to slip displacement. Using this character, we can recognize, for instance, that the acceleration is a type of wave that attenuates like 0.3M and $R^{-0.5}$. The form of the attenuation function for R could be changed by adding terms to express, for example, viscous damping and near focal adjustment. We have adopted the simplest type of Eq. (26) while considering the consistency with Kuribayashi and Tatsuoka's equation.

The above relations are converted into the form of Eq. (27); The epicentral distance R_A where the peak acceleration is estimated as A for the given magnitude M is

$$\log R_A = 0.60M + 2.36 - 2.13\log A \quad \cdots\cdots(29)$$

substituting velocity V:

$$\log R_V = 0.79M - 1.72 - 1.72\log V \quad \cdots\cdots(30)$$

similarly for displacement D:

$$\log R_D = 1.11M - 5.02 - 1.75\log D \quad \cdots\cdots(31)$$

and finally with the slip displacement S:

$$\log R_S = 0.92M - 1.92 - \log S^{0.94} A_c^{1.31} \quad \cdots(32)$$

The coefficient for the magnitude increases from 0.60 for A up to 1.11 for D. The value for the slip displacement 0.92 in Eq. (32) is the closest and almost equal to the 0.87 of Eq. (25). It can be concluded that the numerous cases of the previous Japanese liquefactions verify the hypothesis that they have been governed by the type of seismic wave expressed:

$$X = S^{0.94} A_c^{1.31} \quad \cdots\cdots(33)$$

If the wave X is a physical existence, then it should have a divisible dimension. It can be assumed that Eq. (33) is a realization of

$$X = S\,A_c \quad \cdots\cdots(34)$$

which has the dimension of energy per unit mass. We will call this quantity the dislocation energy of ground motion, for it is related to the work done to dislocate the unit mass of the soil particle from the bulk (Eq. (9)).

The critical dislocation energy necessary to initiate the historical liquefactions can be calculated by comparing Eqs. (32), (25) and (27),

$$X_c = 380.2 \text{ cm}^2/\text{sec}^2 \quad \cdots\cdots(35)$$

The regression coefficients for M and for the intercept of Eq. (30) are similar to those of Eq. (32). Therefore, we can name the wave X which governs the liquefaction as $X=V^{1.72}$ or $X=V^2$ from a similar discussion. In either case, the wave X has the dimension of squared velocity or energy per unit mass. The threshold squared peak velocity of liquefaction is evaluated as $X=602.6$ cm^2/sec^2 from Eqs. (25) and (30). The dislocation energy obtained in Eq. (35) is 63% of this peak value, which is reasonable, since the former is related to the RMS value (Chap. 3).

6. EFFECTIVE COMPRESSIBILITY OF THE PORE-WATER IN THE GROUND

In Chapter 3, we have seen that according to the dislocation energy concept, the strength of the soil against liquefaction can be characterized by a pair of material constants. One is the critical angle of dislocation ϕ_c which is related to the critical acceleration of dislocation in Eq. (15). The other is the effective compressibility C_e, which is the compressibility of the pore-water divided by the volume ratio of the dislocating particles η (Eq. (17)). It has been mentioned that both of them can be computed from the undrained cyclic loading test results. Because they are defined with differential quantities based on mechanics, they are measured from other types of data on liquefaction.

We have found statistically that the previous liquefaction observed in Japan has been governed by the dislocation energy of the ground motion being more than the critical value of 380.2 cm^2/sec^2. Since the critical acceleration is already contained in the definition (34), the other parameter, namely the effective compressibility, should be determined from this value.

Kuribayashi and Tatsuoka's data set contains numerous sites of liquefaction. They reported that most of them are located in the alluvial near the original river course and in reclaimed lands. It is not clear how deep the liquefaction occurred for a site to be recognized as liquefied in the literature. If it were only 1m, it might not have been recorded. But, the depth need not to be as large as 10m, either. We assume that the criteria is from the ground surface to 5.0m deep; and the water-table is at the ground level for the worst. We will consider that the site is liquefied when the energy based factor of safety against liquefaction calculated in the soil column being 1m x 1m wide and 5.0m deep is equal to unity. Substituting $F_{le}=1.0$ into Eq. (1) yields

$$W_{e0} = W_{l0} \quad \cdots\cdots(36)$$

From Eq. (4), noting dV=dz and assuming the densities and compressibility are constant with respect to the depth z (m), the work necessary to liquefy the soil column is

$$W_{l0}=\int_0^5 \frac{1}{2}nC\left(P_l^2-P_0^2\right)dz$$
$$=\frac{1}{2}nC\left[\frac{1}{3}\left(\rho'^2g^2+2\rho'\rho_w g^2\right)z^3+\rho' g q_0 z^2\right]_0^5 \quad \cdots\cdots(37)$$

We will evaluate W_{l0} using the typical properties of the alluvial sand; the porosity n=0.5; the mass density of the soil in the water and that of the water $\rho'=\rho_w=1000$ kg/m^3; the atmospheric pressure $q_0=101300$ Pa; the gravity acceleration g=9.8 m/sec^2. In the sequel,

$$W_{l0} = 9.206\times10^9 C \cdots\cdots(38)$$

in Joule. By substituting the mass density $\rho=1900$ kg/m^3 and the dislocation energy SA_c=3.802E-02 m^2/sec^2 into Eq. (9), the work done by the seismic wave with that dislocation energy is evaluated as

$$W_{e0} = \eta\rho 2SA_cB^2H=7.224\times10^2\eta \cdots\cdots(39)$$

By equating Eqs. (38) and (39), the effective compressibility of the pore-water in the soil column of the typical site is estimated as

$$C_e=\frac{C}{\eta} = 7.847\times10^{-8}\left(P_a^{-1}\right) \cdots\cdots(40)$$

Incidentally, the compressibility of the pure water at ordinary temperature is 4.9x10^{-10} Pa^{-1}, and that of the pore of sand with medium relative density in a drained condition is about 1.8x10^{-6} Pa^{-1}. The computed value falls between the two. This is also comparable to what have been computed from the undrained cyclic loading test results in Chapter 3, which range from 2.69x10^{-8} to 8.38x10^{-8} Pa^{-1}. Their mean value is 6.1x10^{-8} (**Table 1**).

7. EVALUATION OF F_{le} FROM M, R, C_e AND ϕ_c

The energy-based factor of safety can be evaluated statistically by the M and R of the design earthquake and the C_e and ϕ_c of the soil by using the following procedure:

1) Compute the critical acceleration A_c of the soil by Eq. (15) from the critical dislocation angle ϕ_c, the bulk density, initial effective confining stress and the depth of the soil.

2) Estimate the slip displacement S (cm) by Eq. (28d) from the JMA magnitude M, epicentral distance R (km), and the critical acceleration A_c (cm/sec^2).

Table 3 Energy-based Factor of Safety estimated by M, R, C_e and ϕ_c

Parameter		Unit	Value
Bulk Density	ρ	kg/m^3	1900
Saturated B. D.	ρ'	kg/m^3	1000
Dislocation Angle	ϕ_c	deg	15
Effective Com.	C_e	Pa^{-1}	7.847E-8
dislocation volume ratio	η		0.62%
Critical Acc.	A_c	m/sec^2	1.382
Vol. of soil	V	m^3	5.0
Erg. to Liquefy	W_{l0}	J	4.51
Magnitude	M	JMA	7.5
Epicentral Dist. R (km)	30	70	150
S (cm)	6.15	2.50	1.11
$2A_cS$ (m^2/sec^2)	1.70E-1	6.90E-2	3.07E-2
W_{e0} (J)	10.0	4.06	1.80
F_{le}	0.67	1.05	1.57

3) Compute the dislocation energy $X=A_cS$ (cm^2/sec^2) using the above values of A_c and S.

4) Determine the volume to compute F_{le}. Calculate the energy W_{l0} necessary to liquefy the volume as illustrated in the previous chapter.

5) Compute the dislocation energy density W_{e0}' from the dislocation energy of the ground motion X, the mass density of the bulk ρ, and the volume ratio of the dislocating particle η. Integrate it for the volume and obtain the dislocation energy W_{e0} of the soil and/or structure.

6) Calculate F_{le} by Eq. (1) using the effective compressibility of the pore-water $C_e=C/\eta$.

For example, if the soil column of B=1m, and H=5m, used in the previous chapter, is located at the site when a big earthquake of JMA 7.5 occurred, whose epicenter is R=70 km apart, then its F_{le} is computed as 1.05. This implies that R=70 km is on average the maximum distance of liquefaction, which is consistent with Fig. 5.2. Table 7.1 lists the pertinent parameters and F_{le} for R=30, 70, and 150 km, where the effective compressibility in Eq. (40) is used; the angle of dislocation is presumed to be 15 degrees, which is a typical value for sands with medium relative density; the critical acceleration is computed as 1.382 m/sec^2.

8. CONCLUSIONS

A general framework for the assessment of the safety of structure and/or soil against liquefaction has been sought. The factor of safety against liquefaction F_{le} has been defined based on the energy of pore-water and earthquakes. The following can be concluded:

1) F_{le} inherits the generality of energy; it can be directly related to the pore-water pressure ratio; it can be evaluated for a finite volume utilizing the additivity of energy; it can be evaluated in many ways by computing or measuring the energy of pore-water and ground motions. These features are advantageous in a practical sense, when compared to the conventional F_l.

2) By using the dislocation energy concept, the parameters necessary to quantify the effect of the ground motion and the strength of the soil against liquefaction are identified in the physical dimension, namely, strong motion duration, RMS amplitude, bandwidth index, central frequency of the ground velocity, RMS amplitude of the ground acceleration, effective compressibility and critical dislocation angle of the soil. These parameters are useful when detailed information about the soil and ground motion is available.

3) By comparing the attenuation equations with the maximum distance of liquefaction against the JMA magnitude by Kuribayashi and Tatsuoka, it is found that the effect of ground motion can be expressed by the quantity $X=SA_c$ which is called the dislocation energy, where S is the slip displacement of a unit mass with critical acceleration A_c. This observation corresponds with the analytical consequence derived by the dislocation energy concept. The critical dislocation energy is evaluated as $X_c=380.2$ cm^2/sec^2. The effective compressibility of the pore-water is estimated as $C_e=7.85\text{x}10^{-8}$ Pa^{-1}.

4) The attenuation equation for the slip displacement can be used to estimate F_{le} statistically from JMA magnitude M, epicentral distance R, effective compressibility C_e, and the critical dislocation angle ϕ_c. This procedure is useful when the global evaluation of the liquefaction hazard is required.

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エネルギーによる液状化安全率

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液状化安全率を地震動が間隙水に及ぼすエネルギーと液状化に必要なエネルギーの比によって表す。これは、従来の応力比による定義を包含するものであり、理論的な取り扱いが出来、過剰間隙水圧比と直接関係づけられるなど、実用性も有る。過去の液状化事例から栗林・龍岡により求められた限界液状化距離とマグニチュードの関係から、液状化に必要な限界エネルギーを計算した。エネルギーによる液状化安全率をマグニチュードと震央距離及び間隙水の有効圧縮率と土の骨格の限界転位角から計算する式を求めた。