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THE EXTENDED QUASI-THREE-DIMENSIONAL GROUND MODEL FOR IRREGULARLY BOUNDED SURFACE GROUND

Takeyasu SUZUKI* and Kunitaka UNAMI**

For the purpose of simulating acceleration response as well as displacement of irregularly bounded surface ground with three-dimensional geological structure, the extended quasi-three-dimensional ground model is proposed. The new model presented here treats fundamental through N-th mode of shear vibrations of a surface layer, while the conventional one takes only fundamental shear mode into consideration. The equation of motion for the model is induced and examples of earthquake response analyses using the proposed model are introduced as well as those by finite element procedure and its applicability is verified.

Keywords : *three-dimensional, ground model, earthquake response analysis*

1. INTRODUCTION

The quasi-three-dimensional ground model was proposed by Tamura and Suzuki[1] in 1987 for the earthquake response analysis of surface ground with three-dimensional geological structure. The model was verified by model vibration tests on a shaking table[2],[3]. It was applied to the simulation of the actual behavior of a shield-driven tunnel and surface ground at the earthquake observation site[4] and its validity was corroborated. The model was also applied to the simulation of dynamic behavior of a submerged tunnel during earthquakes[5].

It is considered that the quasi-three-dimensional ground model will take an important role for evaluation of earthquake resistance of underground structures. In other words, however, it can be said that its usage is limited to the earthquake response analysis of surface ground to obtain ground displacement as the input data for earthquake response analysis of underground tunnels, whose force of inertia is negligible in the analysis. That is because the model takes only fundamental shear vibration into consideration : In general, the influence of the vibrations due to higher modes on the displacement response is relatively small in natural soil deposits, compared with that of fundamental mode of shear vibration in general cases. Even in the analysis of ground displacement responce during earthquakes, however, the model can not necessarily be applied to the ground in which second or third shear vibration mode is highly predominant. If it is possible to take fundamental through N-th mode of shear vibrations into consideration, the model can be extended to the model which analyzes not only ground displacement but also ground velocity and acceleration of irregularly bounded and complicated three-dimensional surface ground due to earthquakes.

The greatest merit of the quasi-three-dimensional ground model is the simplicity in modeling : The model enables us to simulate earthquake ground motions of irregularly bounded 3-D surface ground, the discretization of which by finite solid elements is extremely difficult. To make the most of the merit and widen the scope of its applicability, in this paper, the model is further extended so that it can deal with fundamental through N-th mode of shear vibrations. The theory is presented in both two-dimensional and three-dimensional cases and the validity of the extended model is demonstrated by the comparison of the results analyzed by the proposed method and finite element procedure.

2. EXTENDED QUASI-TWO-DIMENSIONAL GROUND MODEL (EXQ2D MODEL)

Prior to the description of the extended quasi-three-dimensional ground model, the theory proposed in this paper is introduced on the two-dimensional case for the sake of easier understanding. In this paper, the model proposed by Tamura et al.[6], which is composed of multiple sets of single degree of freedom systems for the earthquake response analysis of two-dimensional surface ground, is named as the quasi-two-dimensional ground model (Q2D model), while the new model which takes fundamental through N-th mode of

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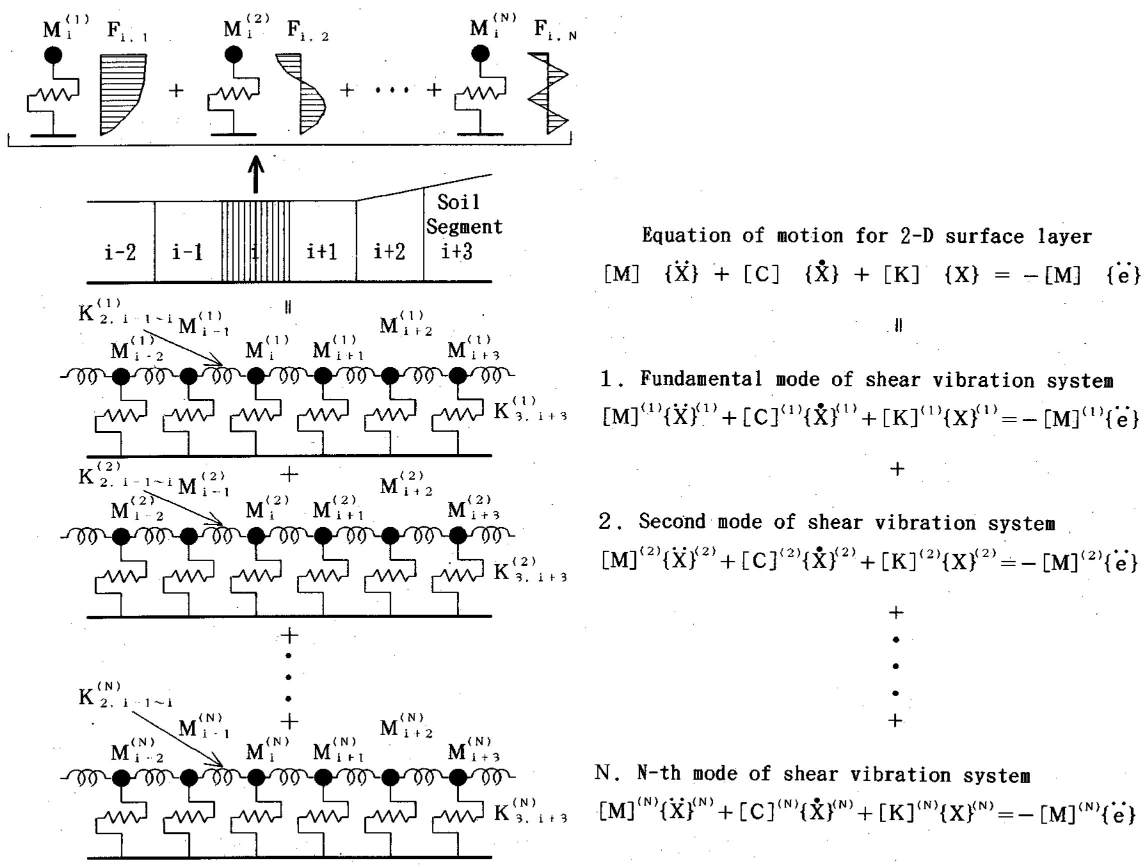


Fig.1 A schematic diagram of the extended quasi-two-dimensional ground model

shear vlbrations into consideration, is named as the extended quasi-two-dimensional ground model (EXQ2D model).

Fig.1 illustrates the schematic diagram to show how to model two-dimensional surface ground by the proposed method. The model is constituted based on the following assumption : The ground is sliced into M pieces of soil segments ; Each soil segment has fundamental through N-th shear vibration modes ; N set of M degree of freedom systems can be constituted independently, each of which is composed of M set of single degree of freedom systems of soil segments which represent same order of shear vibration mode ; An earthquake response alalysis is carried out for each system independently ; Finally, the response of a total system can be given by superposing the response results of fundamental through N-th mode of shear vibration systems. This assumption based on the incremental mode-superposition technique is fully applicable to a layer with uniform thickness and no local irregularity. However, a large amount of error may generate, if such technique is applied to the ground of irregular formation, where the frequency of fundamental shear vibration mode of a soil segment and that of the second mode of the neighboring segment are almost identical. Strictly speaking, there is a limitation in its applicable ground condition. However, we take it for granted that the error will be negligible in a wide range of natural surface ground existing actually.

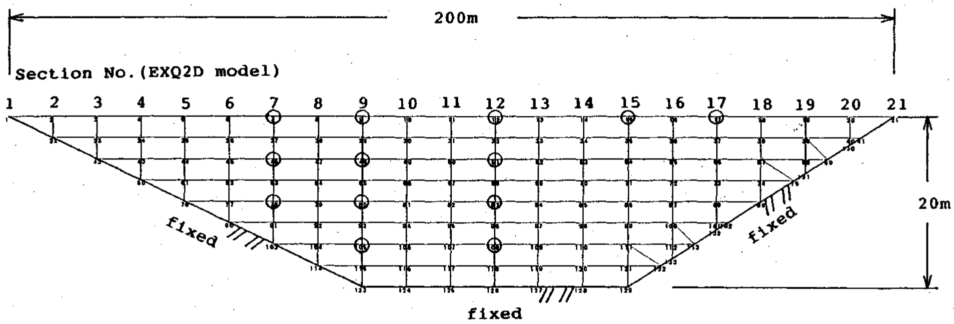
The ground is divided into a number of soil segments and each segment is replaced by N set of single degree of freedom systems. The masses and spring constants of each system are determined by the technique used in a modal analysis as follows :

$$M_i^{(n)}=\frac{\left(\int_0^{H_i} m_i(z)f_{i,n}(z)\,dz\right)^2}{\int_0^{H_i} m_i(z)f_{i,n}^2(z)\,dz}=\lambda_{i,n}\int_0^{H_i} m_i(z)\,dz \quad \cdots\cdots(1)$$

$$K_{3,i}^{(n)}=\omega_{i,n}^2 M_i^{(n)} \quad \cdots\cdots(2)$$

in which, $M_i^{(n)}$: effective mass of soil segment i on n-th shear vibration mode,

$K_{3,i}^{(n)}$: constant of K_3 spring connecting $M_i^{(n)}$ with base ground,



Soil Properties

γ_t (t/m³)	V_s (m/sec)	ν
1.5	60	0.495
1.6	80	0.495
1.8	210	0.470
1.9	270	0.450

Fig.2 Two-dimensional model ground used in the analyses by EXQ2D and FEM2

$m_i(z)$: mass of soil segment i at depth z,
$f_{i,n}(z)$: normalized modal vector on n-th shear vibration mode of soil segment i,
$\lambda_{i,n}$: effective mass ratio on n-th shear vibration mode of soil segment i,
$\omega_{i,n}$: circular frequency on n-th shear vibration mode of soil segment i,
H_i : thickness of soil segment i,
z : depth below ground surface.

In a single degree of freedom system which represents n-th mode of shear vibration of soil segment i, its earthquake responses at arbitrary depth z and time t are calculated as the product of the response of the mass, i.e. $\ddot{X}_{i,n}(t)$, $\dot{X}_{i,n}(t)$ and $X_{i,n}(t)$, and the modal function, $F_{i,n}(z)$, as follows :

$$\ddot{x}_{i,n}(z,t)=\ddot{X}_{i,n}(t)F_{i,n}(z)+\ddot{e}_i(t) \quad \cdots\cdots (3)$$

$$\dot{x}_{i,n}(z,t)=\dot{X}_{i,n}(t)F_{i,n}(z) \quad \cdots\cdots (4)$$

$$x_{i,n}(z,t)=X_{i,n}(t)F_{i,n}(z) \quad \cdots\cdots (5)$$

$$F_{i,n}(z)=\beta_{i,n}f_{i,n}(z) \quad \cdots\cdots (6)$$

in which : $\ddot{x}_{i,n}(z,t)$, $\dot{x}_{i,n}(z,t)$ and $x_{i,n}(z,t)$ denote absolute acceleration, relative velocity and displacement of soil segment i, respectively, at depth z and at time t ; $F_{i,n}(z)$ means modal function of n-th mode when the mass $M_i^{(n)}$ causes unit displacement ; $\beta_{i,n}$ denotes participation factor of n-th shear vibration mode for soil segment i ; and $\ddot{e}(t)$ denotes input acceleration.

In two-dimensional analysis, each soil mass is connected with each other by K_2 spring. In Q2D model proposed by Tamura et al.[6], the constant of K_2 spring laterally connecting mass i with mass i-1 is determined as follows :

$$K_{2,i\sim i-1}=\frac{1}{l_{i\sim i-1}}\int_0^{H_i}E_i(z)dz \quad \cdots\cdots (7)$$

in which, $E_i(z)$ means the vertical distribution of elastic modulus of segment i, and $l_{i,i\sim1}$ denotes the distance between the centers of i-th and $(i-1)$-th segments.

Extending this manner to n-th mode of shear vidration system, equation (7) is necessary to be modified, replacing the modal function into absolute form of $|F_{i,n}(z)|$, because the modal function has negative components of vector in higher vibration modes than fundamental. Even in such cases, from the aforemensioned assumption, modal shapes of two adjacent soil segments are considered almost coincident with. In consideration of plane strain condition of ground segment i, the effect of Poisson's ratio, $\nu_i(z)$, should be also added to equation (7). In addition, in the case that the depths and soil profiles of two adjacent segments differ relatively from each other, the constant of K_2 spring should be determined using the average property of the two adjacent soil layers. Then, the following stiffness is induced.

$$R_{i,n}=\int_0^{H_i}\frac{E_i(z)\cdot|F_{i,n}(z)|}{1-\nu_i^2(z)}dz \quad \cdots\cdots (8)$$

$R_{i,n}$ means the concentrated stiffness when the mass causes unit deformation based on n-th mode of shear vibration of segment i. $R_{i-1,n}$ is also given by equation (8) and the constant of K_2 spring in the system of n-th mode of shear vibration, $K_{2,i\sim i-1}^{(n)}$, is defined as the average of concentrated stiffnesses of the two adjacent segments devided by the distance between mass points i and $i-1$, i.e. $l_{i\sim i-1}$.

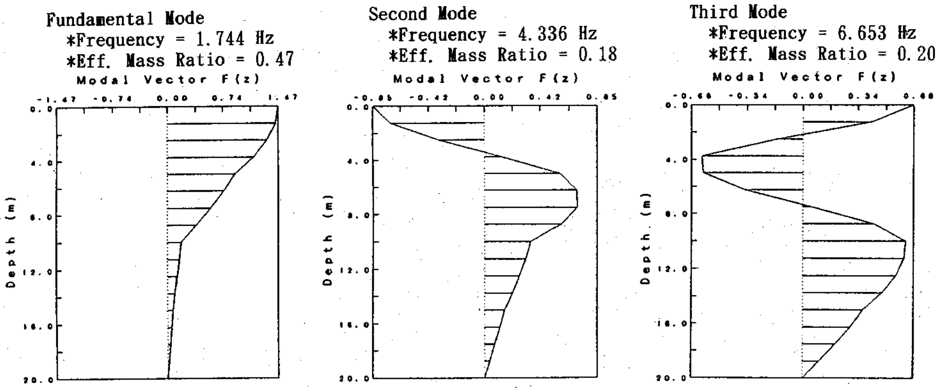
$$K_{2,i\sim i-1}^{(n)}=\frac{1}{2l_{i\sim i-1}}(R_{i,n}+R_{i-1,n}) \quad \cdots\cdots (9)$$

The method of notation for K_2 and K_3 springs is coincident with that made by Tamura et al.[6], while K_1, which is not used in this paper, is used to notate the spring connecting a soil mass with a tunnel element in the reference paper[6].

The equation of motion for n-th mode of shear vibration system is formed as follows :

$$[M]^{(n)}\{\ddot{X}\}^{(n)}+[C]^{(n)}\{\dot{X}\}^{(n)}+[K]^{(n)}\{X\}^{(n)}=-[M]^{(n)}\{\ddot{e}\} \quad \cdots\cdots (10)$$

When fundamental through N-th mode of shear vibrations are taken into consideration in the analysis, N set of the above equations exist. The



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1.9	270	0.450

Fig.3 Fundamental through third shear vibration modes of the soil layer in the central region of the model ground

Table 1 Comparison of the results of acceleration and displacement responses between EXQ2D model and finite element model

Location		Max. Acceleration (gal)				Max. Displacement (cm)			
Section No.	Depth (m)	EXQ2D (1)	EXQ2D (2)	EXQ2D (3)	FEM	EXQ2D (1)	EXQ2D (2)	EXQ2D (3)	FEM
7	0.0	211	233	222	255	1.48	1.43	1.44	1.45
7	-5.0	132	195	194	164	0.86	0.97	1.01	0.83
7	-10.0	148	156	158	164	0.11	0.14	0.15	0.16
9	0.0	198	276	264	269	1.87	1.77	1.80	1.68
9	-5.0	155	250	259	242	1.13	1.32	1.29	1.10
9	-10.0	149	185	197	194	0.22	0.33	0.37	0.31
9	-15.0	149	156	171	171	0.08	0.13	0.16	0.13
12	0.0	231	307	307	297	2.03	1.94	1.97	1.82
12	-5.0	170	268	276	288	1.23	1.41	1.38	1.31
12	-10.0	146	188	197	214	0.24	0.35	0.39	0.40
12	-15.0	148	154	170	182	0.09	0.14	0.16	0.18
15	0.0	191	283	268	284	1.83	1.73	1.75	1.62
17	0.0	223	214	214	216	1.37	1.34	1.35	1.22

numerical integration is conducted N times to solve the earthquake responses of N systems. Then the earthquake responses of soil segment i at arbitrary depth z, i.e. $\ddot{x}_i(z,t)$, $\dot{x}_i(z,t)$ and $x_i(z,t)$, are calculated by superposing the responses of N set of vibration systems :

$$\ddot{x}_i(z,t)=\sum_{n=1}^{N}\ddot{X}_{i,n}(t)F_{i,n}(z)+\ddot{e}_i(t) \quad \cdots\cdots\cdots\cdots (11)$$

$$\dot{x}_i(z,t)=\sum_{n=1}^{N}\dot{X}_{i,n}(t)F_{i,n}(z) \quad \cdots\cdots\cdots\cdots (12)$$

$$x_i(z,t)=\sum_{n=1}^{N}X_{i,n}(t)F_{i,n}(z) \quad \cdots\cdots\cdots\cdots (13)$$

3. COMPARISON OF THE RESULTS WITH TWO-DIMENSTIONAL FINITE ELEMENT ANALYSIS

a) Model ground and modeling

In order to verify EXQ2D model and to show its effectiveness, earthquake response analyses were performed using the proposed method and two-dimensional finite element procedure (FEM2) on the two-dimensional ground shown in **Fig.2.** At the deepest section with uniform layer thickness in the figure, the soil profile is selected so that effective mass ratios of second and third shear vibration modes may become relatively large, i. e., 0.18 and 0.20 as shown in **Fig.3**. In general cases, the deeper the depth is, the higher the shear velocity of soil material becomes. Therefore, the soil profile abopted in this model ground is regarded as one of the representative profiles of a natural surface ground.

In **Fig.2**, the discretization of soil segments in EXQ2D model is shown in comparison with the finite element mesh. As shown in the figure, the interval of vertical discretization in both models are coincident with each other, then thier conformity is given. Rayleigh damping is employed in both models as shown in equations (14) and (15), respectively. Equation (14) indicates that every system of EXQ2D model adopts a fundamental

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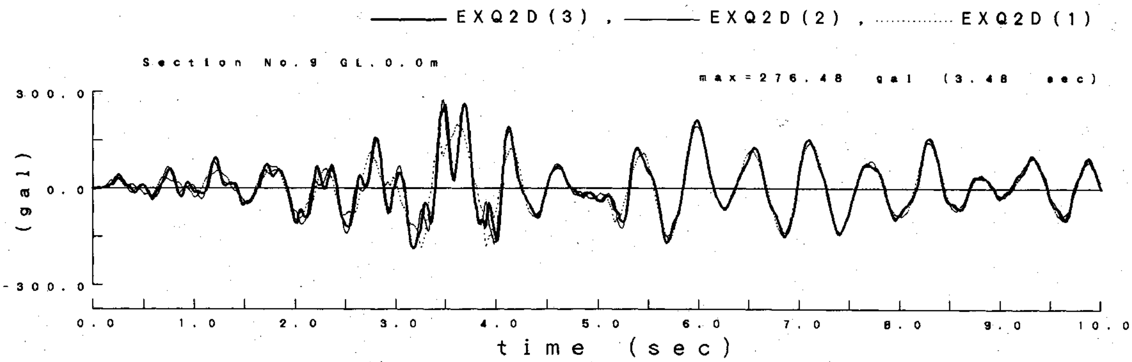
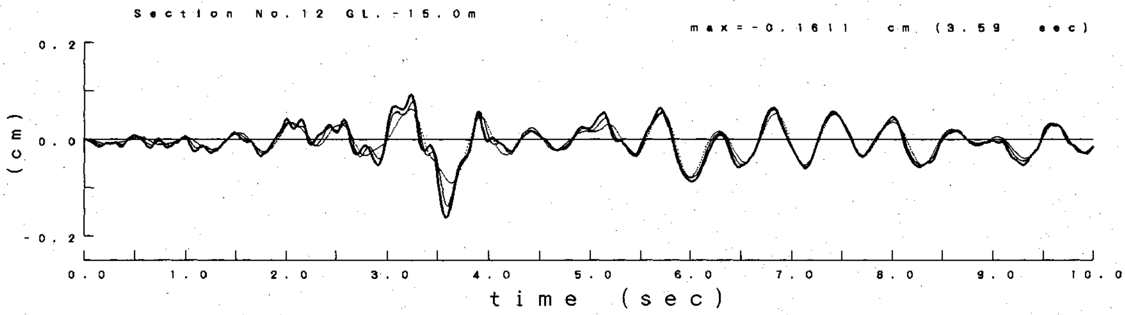


(a) An example of comparison of ground acceleration response



(b) An example of comparison of ground displacement response

Fig.4 Examples of the comparison of ground responses among three cases of earthquake response analyses using EXQ2D model

natural circular frequency of fundamental mode of shear vibration system, in order to conform damping effect of EXQ2D model with that of finite element procedure shown in equation (15).

$$[C]^{(n)}=h\omega_{1,1}[M]^{(n)}+\frac{h}{\omega_{1,1}}[K]^{(n)} \quad \cdots\cdots\cdots\cdots (14)$$

$$[C]^{f}=h\omega_{1}[M]^{f}+\frac{h}{\omega_{1}}[K]^{f} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (15)$$

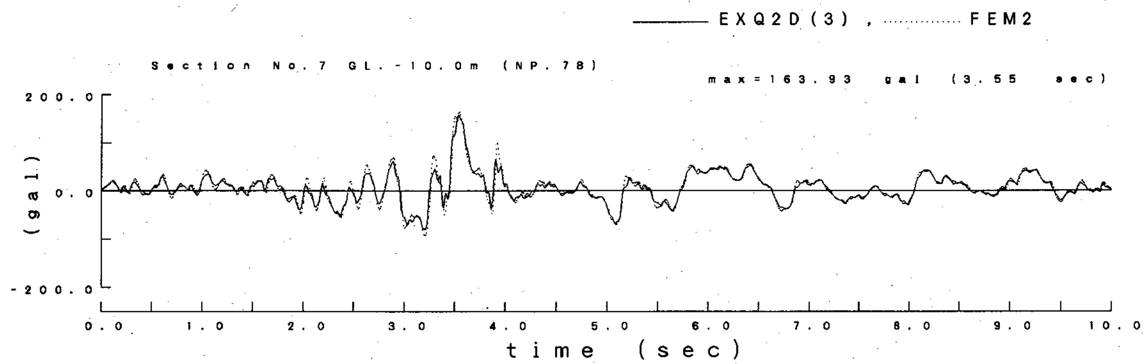
in which,

- $[C]^{(n)}$: Rayleigh damping matrix for the system of n-th mode of shear vibration in EXQ2D model,
- h : damping factor
- $\omega_{1,1}$: fundamental circular frequency of the system of fundamental mode of shear vibration in EXQ2D model,
- $[M]^{(n)}$: effective mass matrix for the system of n-th mode of shear vibration in EXQ2D model,
- $[K]^{(n)}$: stiffness matrix for the system of n-th mode of shear vibration in EXQ2D model,
- $[C]^{f}$: Rayleigh damping matrix for finite element model,
- ω_1 : fundamental circular frequency of finite element model,
- $[M]^{f}$: consistent mass matrix for finite element model,
- $[K]^{f}$: stiffness matrix for finite element model.

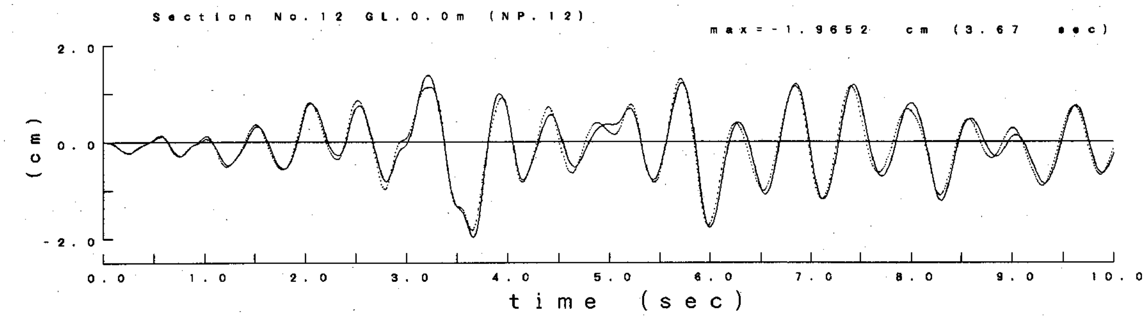
Damping factors adopted in both models are 0.1 and input acceleration used is NS component of Hachinohe wave in Tokachi-oki Earthquake of 1968 with maximum acceleration of 150 gals.

b) Comparison of the results

The comparison was carried out on the acceleration and displacement responses of selected points of both models in the horizontal direction. The points selected for comparison are marked by circles in **Fig.2** and also listed in **Table 1**. Three cases of the earthquake response analyses using EXQ2D model were conducted, which are the cases of (1) fundamental mode of shear vibration system, (2) superposition of fundamental and second mode of shear vibration systems and (3) superposition of fundamental through third mode of shear vibration systems, respectively. In **Table 1**, the values of the maximum acceleration and displacement obtained by the analyses of EXQ2D model are summarized as well as those obtained by finite element analysis. In terms of maximum



(a) An example of comparison of ground acceleration response



(b) An example of comparison of ground displacement response

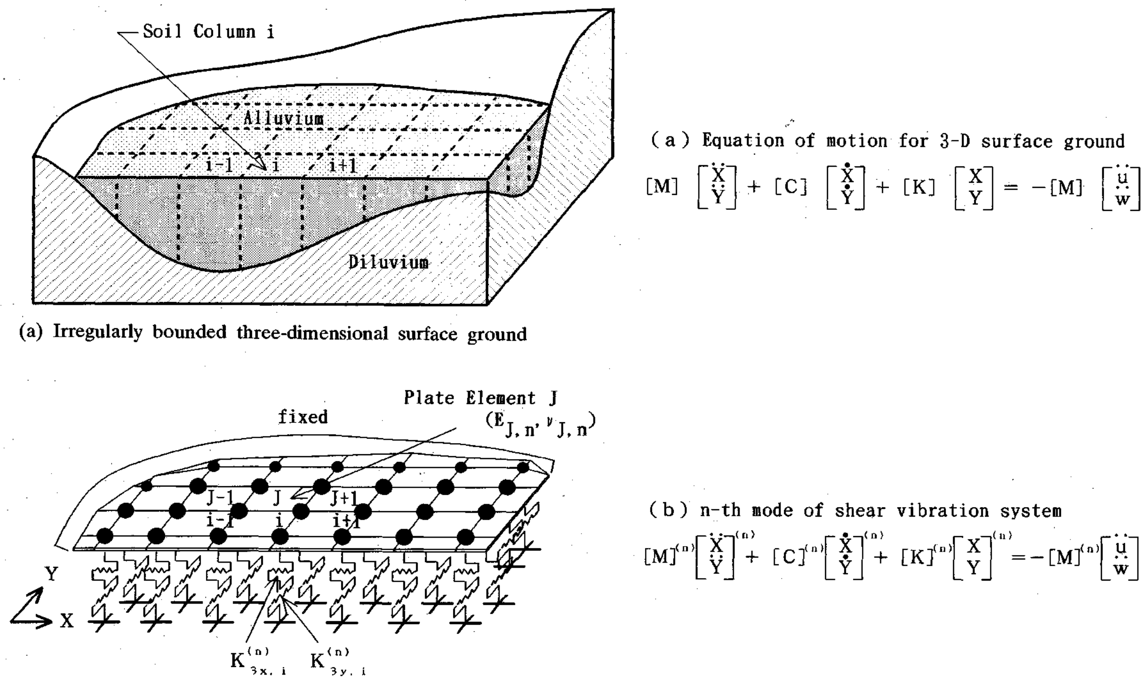
Fig.5 Examples of the comparison of the results between EXQ2D model and finite element model (FEM2)

values on response acceleration and displacement in **Table 1**, the values in case (2) and (3) obviously give better approximations of the results by FEM2 than those in case (1) do. The difference in maximum values of response between case (2) and case (3) is not so clear at the points in **Table 1**. However, a conspicuous difference among case (1) through (3) can be recognized in time-dependent charateristics of ground responses. **Fig.4** (a) shows an example of the comparative plotting of ground acceleration responses at the ground surface of section 9 which were obtained through 3 analyses using EXQ2D model. In the acceleration response, second and third mode of shear vibrations affect greately the response at shallow part of ground as shown in the figure. **Fig.4** (b) also shows the comparative plotting of the ground displacement responses at the depth of 15.0 m of section 12. In the displacement response, on the other hand, they affect the response at deeper part as shown in the figure. The superposition up to 4th shear vibration system was conducted by way of trial, but it did not bring about any remarkable result which differs from those of case (3) in maximum values of response and time-dependednt characteristics.

Fig.5 illustrates the examples of comparative plotting between the analyses of case (3) in EXQ2D model and finite element model on ground acceleration and displacement responses, respectively. Their time-dependent characteristics of the response is sufficiently coincident with each other, which proves that EXQ2D model can be applied to the evaluation of the seismic motion of surface ground, not only on ground displacement response but on ground acceleration response. Thus, the validity of the method which introduces higher modes of shear vibration than fundamental one was satisfactorily verified.

4. EXTENDED QUASI-THREE-DIMENSIONAL GROUND MODEL (EXQ3D MODEL)

Fig.6 illustrates the schematic diagram of the extended quasi-three-dimensional ground model. Same as the two-dimensional case, the extended quasi-three-dimensional ground model (EXQ3D model) named here is the extended one of the quasi-three-dimensional ground model proposed by Tamura and Suzuki[1] (Q3D model) so that the model can take fundamental through N-th mode of shear vibrations into consideration. Alluvial sur-



(a) Irregularly bounded three-dimensional surface ground

(b) n-th mode of shear vibration system in EXQ3D model

Fig.6 A schematic diagram to represent the extended quasi-three-dimensional ground model

face ground is divided into a number of soil columns as shown in **Fig.6** (a). Then, each column is represented by N set of two degree of freedom systems in the horizontal directions, X and Y. The mass $M_i^{(n)}$, which is the effective mass of soil column i in n-th mode of shear vibration system, is determined using equation (1). It should be noted that $m_{i,n}(z)$ in EXQ3D model denotes the mass of soil column i at depth z instead of a soil segment. $K_{3x,i}^{(n)}$ and $K_{3y,i}^{(n)}$, which are the constants of K_3 springs of n-th mode of shear vibration system connecting mass i with base ground in X and Y directions respectively, coincide with each other and can be given by equation (2).

With respect to the horizontal connection of ground masses adjoining each other, finite plate elements with unit thickness are introduced[1]. Ground deformation in X direction is accompanied by that in Y direction, and vice versa, because there is a volumetric change originated from Poisson's ratio of the ground. Taking this effect into account, finite element approach like this is indispensible. In n-th mode of shear vibration system, then, the elastic modulus of plate element J, $E_{J,n}$, is given by averaging the concentrated stiffnesses of the soil columns constituting the element. In case of a rectangular plate element, $E_{J,n}$ is determined as follows :

$$E_{J,n}=\frac{1}{4}\sum_{i=1}^{4}R_{i,n},\quad R_{i,n}=\int_0^{H_i}E_i(z)\cdot|F_{i,n}(z)|dz \quad\cdots\cdots(16)$$

Poisson's ratio at nodal point i, $V_{i,n}$ is given by weighting Poisson's ratio of soil column i, $v_i(z)$ with the modal function at nodal point i, $F_{i,n}(z)$. Then, Poisson's ratio of element J is defined by equations (17) and (18) :

$$\nu_{J,n}=\frac{1}{4}\sum_{i=1}^{4}V_{i,n} \quad\cdots\cdots(17)$$

$$V_{i,n}=\frac{\int_0^{H_i}v_i(z)\cdot|F_{i,n}(z)|dz}{\int_0^{H_i}|F_{i,n}(z)|dz} \quad\cdots\cdots(18)$$

Thus, the element stiffness matrix of plate element J, $[K_2]_J^{(n)}$ is given, using finite element procedure under plane stress condition of a plate element with unit thickness :

$$[K_2]_J^{(n)}=\int_{vol}B^TDBdv \quad\cdots\cdots(19)$$

in which, B and D denote a strain martrix and a two-dimensional elasticity matirix under plane stress condition, respectively. Each component of the matrix in equation (19) has a physical meaning of a sort of K_2 spring connecting 2 soil masses mutually. In this sence, equations (9) and (19) can be regarded as identical, though there is a

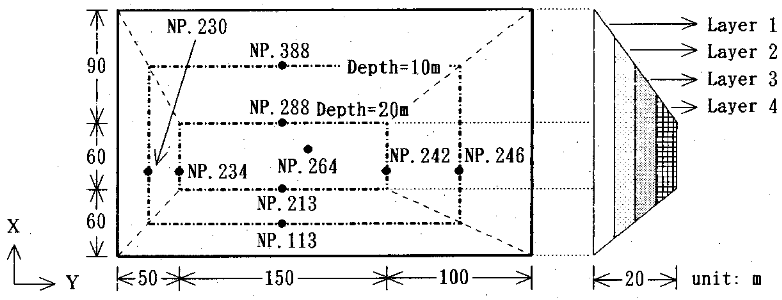


Fig.7 Configuration of the model ground to be analyzed by EXQ3D model

difference in dimension.

Then, total element stiffness matrix of plate elements in n-th mode of shear vibration system can be given as follows:

$$[K_2]^{(n)} = \sum_{J=1}^{N_e} [K_2]_J^{(n)} \cdots\cdots (20)$$

in which, N_e denotes the total number of plate elements in the model. Finally, the total stiffness matrix of n-th mode of shear vibration system is induced as follows:

$$[K]^{(n)} = [K_2]^{(n)} + [K_3]^{(n)} \cdots\cdots (21)$$

in which, $[K_3]^{(n)}$ denotes the stiffness matrix composed of $K_{3x}^{(n)}$ and $K_{3y}^{(n)}$.

Fig.6 (b) illustrates the system of n-th mode of shear vibration in EXQ3D model for three-dimensional surface ground. The equation of motion for this system is written as follows:

$$[M]^{(n)}\begin{bmatrix}\ddot{X}\\ \ddot{Y}\end{bmatrix}^{(n)} + [C]^{(n)}\begin{bmatrix}\dot{X}\\ \dot{Y}\end{bmatrix}^{(n)} + [K]^{(n)}\begin{bmatrix}X\\ Y\end{bmatrix}^{(n)} = -[M]^{(n)}\begin{bmatrix}\ddot{u}\\ \ddot{w}\end{bmatrix} \cdots\cdots (22)$$

in which,

$[M]^{(n)}$: effective mass matrix of n-th mode of shear vibration system,
$[C]^{(n)}$: damping matrix of n-th mode of shear vibration system,
$[K]^{(n)}$: stiffness matrix of n-th mode of shear vibration system,
X, Y : horizontal displacement in X and Y directions respectively,
$\ddot{u}, \ddot{w}$: input horizontal acceleration in X and Y directions respectively.

N set of the above equations also exist when fundamental through N-th mode of shear vibrations are dealt with in the analysis using EXQ3D model. Then, the earthquake responses of nodal point i or soil column i at arbitrary depth z and time t are given by the following equations:

$$\left.\begin{aligned}\ddot{x}_i(z,t) &= \sum_{n=1}^{N}\ddot{X}_{i,n}(t)F_{i,n}(z) + \ddot{u}_i(t),\\ \ddot{y}_i(z,t) &= \sum_{n=1}^{N}\ddot{Y}_{i,n}(t)F_{i,n}(z) + \ddot{w}_i(t)\end{aligned}\right\} \cdots\cdots (23)$$

$$\left.\begin{aligned}\dot{x}_i(z,t) &= \sum_{n=1}^{N}\dot{X}_{i,n}(t)F_{i,n}(z),\\ \dot{y}_i(z,t) &= \sum_{n=1}^{N}\dot{Y}_{i,n}(t)F_{i,n}(z)\end{aligned}\right\} \cdots\cdots (24)$$

$$\left.\begin{aligned}x_i(z,t) &= \sum_{n=1}^{N}X_{i,n}(t)F_{i,n}(z),\\ y_i(z,t) &= \sum_{n=1}^{N}Y_{i,n}(t)F_{i,n}(z)\end{aligned}\right\} \cdots\cdots (25)$$

in which : $\ddot{x}_i(z,t)$, $\dot{x}_i(z,t)$ and $x_i(z,t)$ are absolute acceleration, relative velocity and displacement, respectively, of nodal point i in the horizontal X direction at depth z and at time t ; $\ddot{y}_i(z,t)$, $\dot{y}_i(z,t)$ and $y_i(z,t)$ are absolute acceleration, relative velocity and displacement, respectively, of nodal point i in the horizontal Y direction at depth z and at time t, respectively ; $\ddot{X}_{i,n}(t)$, $\dot{X}_{i,n}(t)$ and $X_{i,n}(t)$ are relative acceleration, velocity and displacement, respectively, of mass i in X direction at time t in n-th mode of shear vibration system ; $\ddot{Y}_{i,n}(t)$, $\dot{Y}_{i,n}(t)$ and $Y_{i,n}(t)$ are relative acceleration, velocity and displacement of mass i in Y direction at time t in n-th mode of shear vibration system ; $\ddot{u}_i(t)$ and $\ddot{w}_i(t)$ are input ground motions acting on soil column i in X and Y directions, respectively, at time t.

In EXQ3D model, variable ground motions can easily be input to deal with the propagation of earthquake waves, by the modification of the right term of equation (22). In addition, it will be possible to carry out non-linear analysis, by the adoption of equi-linearization as performed by Suzuki[7]. Furthermore, the change in soil stiffness due to liquefaction will be applied to this type of ground model as conducted by Towhata and Islam[8]. The data required for EXQ3D analysis are limited to boundary information and soil profiles at

Table 2 Comparison of the results of acceleration and displacement responses among three analyses by EXQ3D model

Location		Max. Acceleration (gal)						Max. Displacement (cm)					
NP.	Depth	X-direction			Y-direction			X-direction			Y-direction		
(EXQ3D)	(m)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
264	0.0	286	325	313	217	248	242	2.51	2.49	2.50	1.99	1.87	1.90
264	-5.0	195	204	204	160	220	217	1.52	1.58	1.55	1.21	1.34	1.31
264	-10.0	147	153	164	146	167	184	0.30	0.33	0.36	0.24	0.32	0.37
264	-15.0	149	151	156	148	153	164	0.11	0.12	0.14	0.09	0.12	0.15
288	0.0	298	337	325	214	242	237	2.53	2.55	2.55	1.97	1.84	1.87
288	-10.0	146	153	163	147	167	184	0.30	0.32	0.35	0.23	0.31	0.36
242	0.0	316	350	339	208	230	221	2.57	2.58	2.59	1.93	1.78	1.81
242	-10.0	147	153	163	148	168	186	0.30	0.31	0.34	0.23	0.31	0.36
213	0.0	319	355	344	210	251	214	2.61	2.62	2.62	1.74	1.61	1.64
213	-10.0	146	152	163	148	165	185	0.31	0.31	0.34	0.21	0.29	0.34
234	0.0	308	328	317	201	226	221	2.39	2.39	2.40	1.82	1.70	1.73
234	-10.0	149	157	166	146	166	184	0.28	0.30	0.33	0.21	0.30	0.35
388	0.0	270	270	265	192	198	193	1.81	1.79	1.79	1.25	1.23	1.23
246	0.0	273	267	267	174	180	179	1.75	1.72	1.72	1.31	1.26	1.27
113	0.0	272	266	266	223	233	233	1.73	1.71	1.71	1.32	1.29	1.29
230	0.0	242	238	240	179	198	193	1.56	1.54	1.54	1.19	1.18	1.19

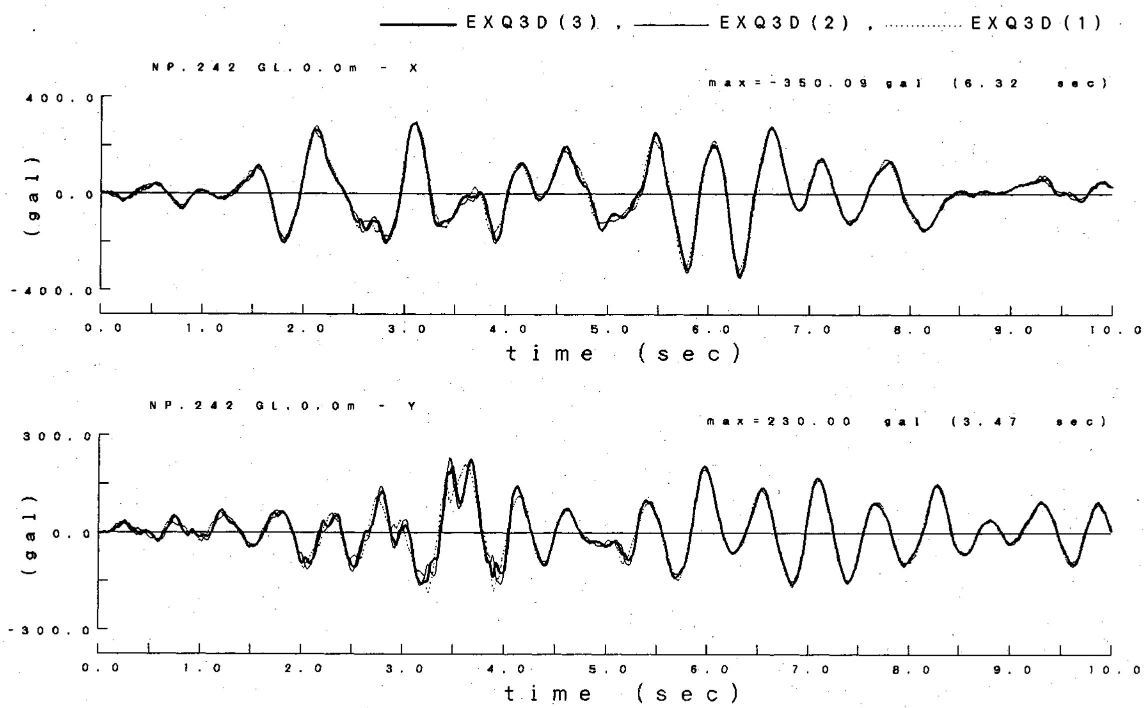


Fig.8 Influence of modal orders considered in the analyses on acceleration response (EXQ3D model)

each nodal point, and the number of freedom required in this model is much less than that of 3-D finite element method. Therefore, this method will enable to model the ground where finite element procedure can not be applied and will take an important role in evaluating the effect of surface geology on seismic motion and seismic zonation.

5. EARTHQUAKE RESPONSE ANALYSES BY EXQ3D MODEL

a) Model ground and modeling by EXQ3D model

In order to demonstrate the effectiveness of EXQ3D model, earthquake response analyses were conducted using a proposed method on the three-dimensional model ground as shown in **Fig.7**. The soil profile of this model ground at the

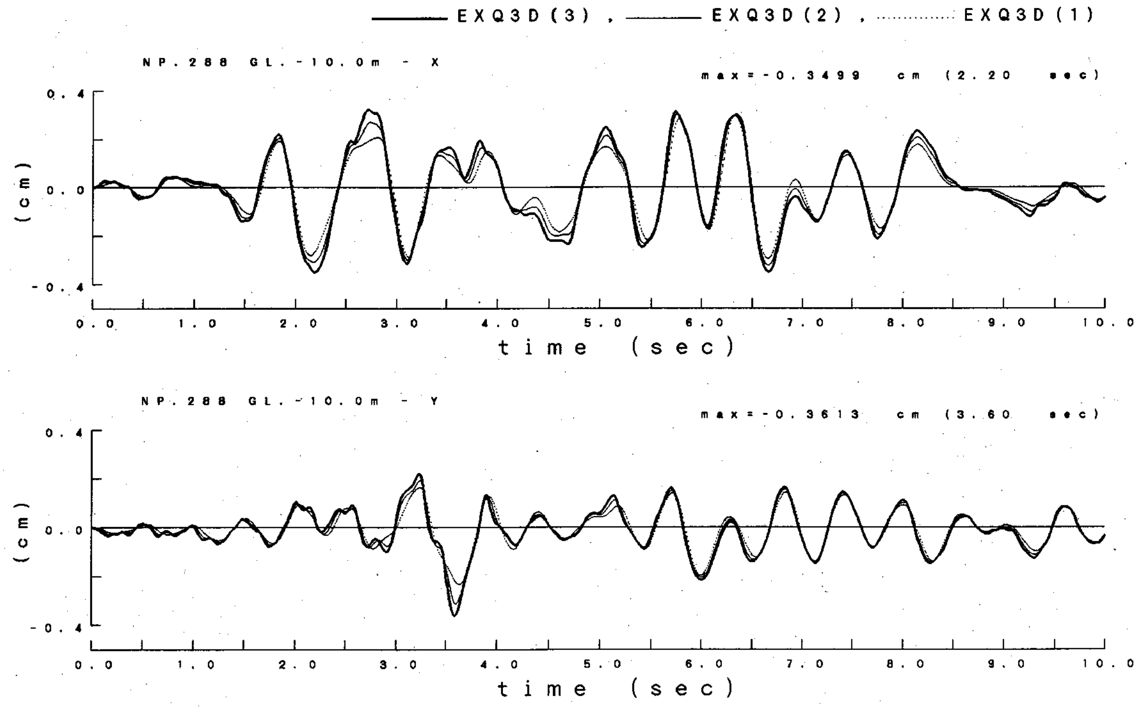


Fig.9 Influence of modal orders considered in the analyses on displacement response (EXQ3D model)

deepest point is the same to that used in two-dimensional analyses. The cross section along the Y-axis in the central area of the ground is just the same to the two-dimensional model ground illustrated in **Fig.2**. The discretization by EXQ3D model was conducted with 500 nodal points, the number of freedom in the analysis of which being 912. Damping matrix was given by equation (14), while damping factors used in the analyses are 0.1. The EW and NS components of Hachinohe wave in Tokachi-oki Earthquake were used as input ground motions in the horizontal X and Y directions, respectively. Both of these input accelerograms are adjusted to the maximm value of 150 gals. In this chapter, three cases of response analyses were performed. These cases were classified by the number of superposed systems of vibration modes : (1) fundamental ; (2) fundamental and second ; (3) fundamental through third, which correspond to the cases set up for the EXQ2D analyses discussed in Chapter 3.

b) Influence of modal orders on seismic motions

The maximum values of acceleration and displacement through the analyses are listed in **Table 2** for comparison. The locations where comparison was made are numbered by nodal points in **Fig.7**. A comparison of the maximum values in **Table 2** shows that the differences in both maximum acceleration and displacement among three cases are not so large as those in the two-dimensional analyses shown in **Table 1**. In the time-dependent characteristics of acceleration, notable differences can be seen in the main part of the time history at some shallow points in the central region of the ground (**Fig.8**). With respect to the time-dependent characteristics of displacement responses, on the other hand, the differences due to the number of superposed modes are clearly demonstrated in the whole part of the time history, especially at deep points of the ground as shown in **Fig.9**.

6. CONCLUDING REMARKS

The authors proposed the extended quasi-three-dimensional ground model (EXQ3D model) for the earthquake response simulation of the surface ground under the three-dimensional geological condition. The validity of this new method which introduces higher shear vibration modes was verified, based on the comparison between the results analyzed by the proposed method and those obtained by the finite element model in two dimensions.

It was clarified, through the analyses, that the introduction of the shear vibration modes, that are higher than fundamental one, to the quasi-three-dimensional ground model enables us to simulate the acceleration response of a three-dimensional surface ground. In addition, even in the displacement response, the higher shear vibration modes should be dealt with in the analysis for the ground composed of soil deposits having large effective mass ratios in higher shear vibration modes.

Since EXQ3D model is based on the aforementioned assumption, its application is limited. Even though the number of systems superposed increases, higher accuracy of a response result cannot be necessarily guaranteed, due to the fact that the error caused by the discordance between actual behavior and the assumption is also accumulated as the number of superposed modes increases. In order for the model to be applied adequately, the following 3 points should be taken into consideration to overcome this problem :

(1) The discritization of a mesh should not be excessively rough. The difference in thickness of adjoining soil columns should be kept at the level less than 50% ;

(2) The model can not be applied to the surface ground with excessively sharp slope of base rock. The slope angle should not exceed 45 degress at maximum ; and

(3) The fifth mode will be the highest mode desirable to be superposed. Modes higher than the fifth mode do not predominate in natural soil deposits in general. In such cases, the extent of the abovementioned error becomes compatible to or larger than increase in accuracy due to an incremental mode-superposition.

The surface ground under the condition stated in (2) is rarely seen ordinarily. Therefore, when the discretization is made according to the condition (1) and when predominant shear vibration modes out of lower 5 modes are exclusively superposed, EXQ3D model can widely be applied for evaluation of the effect of surface geology on seismic motion for engineering purposes.

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不整形表層地盤のための拡張擬似３次元地盤モデル

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不整形３次元表層地盤の地震時地盤変位応答のみならず，加速度，速度応答をシミュレートすることを目的として，擬似３次元地盤モデルを基本せん断振動から n 次のせん断振動まで取り扱えるように拡張した．本論文では拡張擬似３次元地盤モデルに関する定式化を行い，まず２次元問題で FEM 解析結果との比較によって定式化の妥当性を示し，さらに２次元および３次元地震応答解析を行ってモデルの特徴と有効性を説明している．