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DIFFERENTIAL GROUND MOTION ESTIMATION USING A TIME-SPACE STOCHASTIC PROCESS MODEL

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A practical procedure is proposed for estimating the maximum relative displacement between two points on the ground surface. A simple variant of the time-space ground motion spectral models and a Poisson model of extremes are employed as theoretical bases. The proposed procedure comprises the estimation of RMS displacement, temporal and spatial correlation functions of ground displacement. An empirical attenuation equation of RMS displacement is presented by a multiple regression analysis of independent strong motion observation data, and the temporal correlation function is calibrated from the same data set. The spatial correlation function is calibrated from array observation records. Maximum relative displacement is estimated for a combination of earthquake magnitude, epicentral distance and soil condition.

Keywords: *ground motion, spatial variation, differential motion, stochastic process, RMS displacement, array observation*

1. INTRODUCTION

Numerous studies on the spatial variation of ground motions have been conducted with the development of closely spaced arrays of accelerographs. Particularly, a number of studies[1)-5)] focus on the differential ground motion characteristics, which may have significant influence on the dynamic behavior of buried lifeline facilities and large structures during earthquakes. However, it may not be suitable to apply these research results directly to engineering practice, because most of the array data obtained up to present were recorded from relatively small events.

A practical method to estimate the maximum relative displacement between two points on the ground surface is here established. The time-space separable correlation model[4),5)] is adopted as a time-space stochastic process model of ground displacement. In that model, the time-space covariance function is expressed as the product of mean square value of displacement, the temporal correlation function at any fixed spatial location and the spatial correlation function at any fixed time. An advantage of the proposed method is that conventional independent strong motion records can be utilized for estimating the mean square value and the temporal correlation function of ground displacement.

197 sets of two horizontal strong motion records obtained in Japan are employed to assess root mean square (RMS) displacement and the temporal correlation function. With use of a multiple regression analysis, attenuation equations of RMS displacement in terms of earthquake magnitude and epicentral distance are presented for three soil conditions classified from an engineering point of view. The temporal correlation function of displacement is calibrated from the same data set. The spatial correlation function of displacement is calibrated from 8 events data recorded at 4 array observation sites in Shizuoka Prefecture, Japan.

Maximum relative displacement is estimated by using a Poisson process model of extremes. Results are compared with those calculated from array observation data, and examples of predicted maximum relative displacement are shown for several combinations of earthquake magnitude and epicentral distance.

2. STOCHASTIC FORMULATION OF RELATIVE DISPLACEMENT STATISTICS

One approach to study the time-space variation of ground motion is to model the time-space covariance function of ground motion. Suppose that the earthquake ground motion displacement $u(t, x)$ at time t and location x results from a stationary Gaussian process with zero mean, the time-space covariance function $C(\tau, \eta)$ is defined as

$$C(\tau, \eta) = E[u(t+\tau,\ x+\eta)u(t, x)] \quad \cdots\cdots\cdots (1)$$

where $E[\ \ ]$ represents the expectation operator. Assuming that $C(\tau, \eta)$ is a separable function of time and space, the time-space separable correlation model was proposed, and its validity was confirmed by comparing with array observation

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records[4),5)]. In that model, $C(\tau,\eta)$ is expressed as

$$C(\tau,\eta)=\sigma_u^2\rho_T(\tau)\rho_S(\eta) \cdots\cdots (2)$$

where σ_u represents the root mean square (RMS) value of displacement, $\rho_T(\tau)$ is the correlation function in time at any fixed spatial location, and $\rho_S(\eta)$ is the correlation function in space at any fixed time.

The relative displacement $d(t,x;\xi)$ between two points with a separation distance ξ is given by

$$d(t,x;\xi)=u(t,x+\xi)-u(t,x) \cdots\cdots (3)$$

The time-space covariance function of relative displacement $C_d(\tau,\eta;\xi)$ is expressed in terms of the corresponding displacement covariance function as

$$C_d(\tau,\eta;\xi)=2C(\tau,\eta)-C(\tau,\eta+\xi) -C(\tau,\eta-\xi) \cdots\cdots (4)$$

The mean square value of relative displacement can be obtained by setting $\tau=\eta=0$ in Eq.(4):

$$\sigma_d^2=C_d(0,0;\xi) =\sigma_u^2\{2-\rho_S(\xi)-\rho_S(-\xi)\} \cdots\cdots (5)$$

Assuming that the crossings of a specified threshold occur as a Poisson arrival process, the p-fractile of the maximum relative displacement $d_{\max}$ over a temporal interval B_T can be obtained as[4),6),7)]

$$\frac{d_{\max}}{\sigma_d}=\begin{cases}\sqrt{2\ln\left(-\dfrac{2B_T}{T_D\ln p}\right)} \cdots\cdots -\dfrac{2B_T}{T_D\ln p}\geqq e\\ \sqrt{2} \cdots\cdots \text{otherwise}\end{cases} \cdots\cdots (6)$$

where T_D is estimated as

$$T_D=2\pi\sqrt{\frac{C_d(\tau,\eta;\xi)}{-\partial^2C_d(\tau,\eta;\xi)/\partial\tau^2}}\Bigg|_{\tau=\eta=0} =2\pi\sqrt{\frac{1}{-\rho_T''(0)}} \cdots\cdots (7)$$

$2B_T/T_D$ in Eq.(6) signifies an average number of zero crossings during B_T. As seen from Eqs.(5), (6) and (7), $d_{\max}$ can be completely decided, once σ_u, $\rho_T(\tau)$ and $\rho_S(\eta)$ are estimated. Note that simultaneous array data at multiple spatial locations are indispensable to calibrate $\rho_S(\eta)$, however they are not always necessary to estimate σ_u or calibrate $\rho_T(\tau)$. A limitation of this study lies in its assumption of ground motion stationarity in time and space, and the presenting procedure may not be applicable to estimate relative ground motion characteristics where spatial heterogeneity, such as ground condition discontinuity exists.

3. RMS DISPLACEMENT AND STRONG MOTION DURATION

The RMS value σ_u of a stationary process with zero mean $u(t)$ may be estimated as

$$\sigma_u^2=\lim_{T\to\infty}\frac{1}{T}\int_{-\infty}^{\infty}u(t)^2dt \cdots\cdots (8)$$

where T is duration of process. When Eq.(8) is applied to a ground motion time history with finite duration, it is necessary to define strong motion duration for which stationarity is assumed and RMS is calculated. Various studies on the definition of strong motion duration have been made[6),8)-11)]. Among them, the definition by Trifunac et al.[9)] is adopted in this study. Their definition corresponds to the 5% to 95% of the integral of a squared time history. The lower and upper bounds of this integral seem to be somewhat arbitrary, however it may be acceptable at present to assume ground motion stationarity over certain strong motion duration[12)]. RMS value studied in the following analysis is calculated for this duration.

Table 1 Classification of soil condition

Soil Condition	Natural Period Range [sec]	Geological Feature	Number of Sites	Number of Records
Group-1	T_G<0.2	Tertiary or Older	19	46
Group-2	0.2≤T_G<0.6	Alluvium and Diluvium	31	107
Group-3	0.6≤T_G	Soft Alluvium	17	44

197 sets of two horizontal components of strong motion data are considered in this study[13)]. They were recorded at 67 free filed sites in Japan from earthquakes with magnitude 5.0 to 7.9. The soil conditions at recording sites are classified into three groups as shown in **Table 1**. This classification depends on the earthquake resistant design specifications for highway bridges in Japan[14)]. All the data analyzed were obtained by the SMAC accelerographs, which provide most of the Japanese strong motion records currently available for engineering analyses. The ground displacement is calculated by integrating acceleration record in the frequency domain[15)]. Considering the SMAC accelerograph record reliability[16)], the lower and upper cut-off frequencies for filtering are taken as 1/3Hz and 12Hz, respectively. This frequency range may be widened with the accumulation of strong motion data recorded by newly developed accelerographs, such as digital strong motion accelerographs.

Two horizontal orthogonal components of ground displacement are combined to give a motion along an arbitrary direction. RMS displacement is calculated for every 5 degrees, and the maximum RMS displacement on the horizontal plane and the correspondent strong motion duration is obtained. Besides RMS value and duration, the temporal correlation function which will be mentioned in the following section, is also calibrated for the same duration of ground motion along the same direction.

The following empirical equation[13)] is assumed to

Table 2 Coefficients of RMS displacement attenuation equation

Soil Condition	a	b	c	Correlation Coefficient	Standard Error
Group-1	7.394×10^{-2}	0.460	-1.314	0.814	0.247
Group-2	7.022×10^{-3}	0.545	-1.000	0.761	0.247
Group-3	5.935×10^{-3}	0.595	-1.027	0.652	0.256

represent the attenuation characteristics of RMS displacement σ_u[cm] :

$$\sigma_u = a(GC_i) \times 10^{b(GC_i)M} \times (\Delta + 30)^{c(GC_i)} \cdots\cdots (9)$$

where M and Δ[km] are earthquake magnitude and epicentral distance, respectively, and $GC_i (i=1 \sim 3)$ represents soil condition group. Coefficients $a(GC_i)$, $b(GC_i)$ and $c(GC_i)$ are constants to be determined for each soil condition.

The coefficients determined by a multiple regression analysis are shown in **Table 2**. The coefficient $b(GC_i)$, which represents the effect of earthquake magnitude on RMS displacement, varies from 0.46 to 0.60, and it becomes larger in the order soil condition group 1, 2 and 3. The coefficient $c(GC_i)$, representing the attenuation rate of RMS displacement with epicentral distance, is about -1.0 for groups 2 and 3. The coefficient $c(GC_i)$ for group 1 is a little smaller than those for groups 2 and 3.

Fig.1 compares the RMS displacement calculated from the observed data and estimated from the attenuation equation for $M=5$, 6, 7 and 8. RMS displacement for a combination of earthquake magnitude M and epicentral distance Δ can be read from **Fig.1**. For example, RMS displacement for $M=7$ and $\Delta=50$ [km] is 0.39 cm for group 1, 0.57 cm for group 2 and 0.96 cm for group 3.

4. TEMPORAL CORRELATION FUNCTION

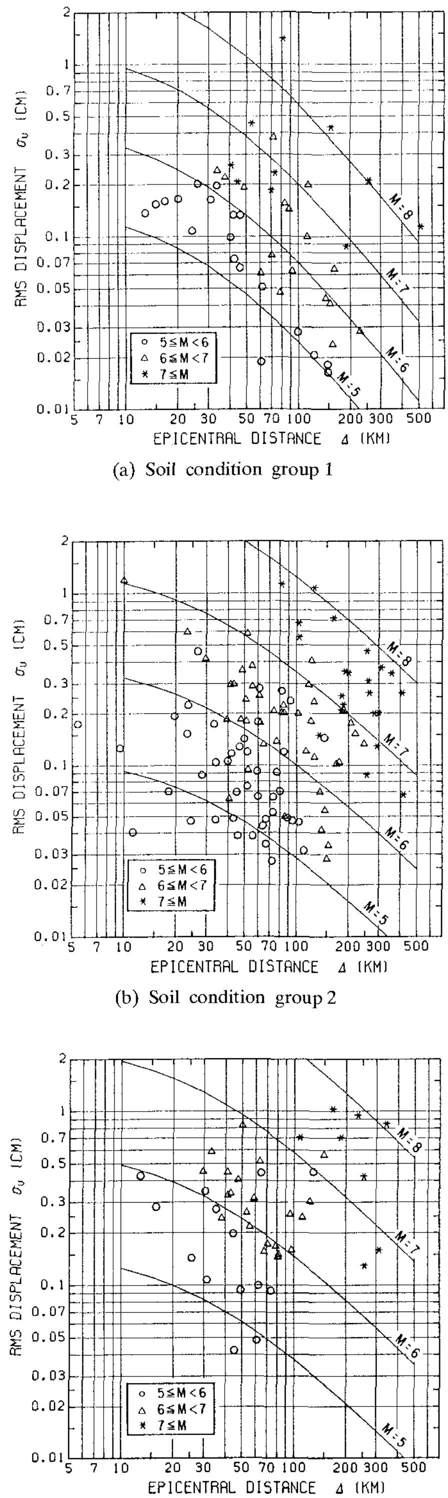
The following function is adopted as the temporal correlation (auto-correlation) function of ground displacement[4),5)] :

$$\rho_T(\tau) = \frac{\cos(2\pi\tau/T_0)}{(2\pi\alpha\tau/T_0)^2 + 1} \cdots\cdots\cdots\cdots (10)$$

where T_0 and α are determined by least squares fit to the observed data. Substituting Eq.(10) into Eq.(7) yields :

$$T_D = T_0/\sqrt{1+2\alpha^2} \cdots\cdots\cdots\cdots (11)$$

Eq.(10) is applied to the 197 sets of ground displacement records, and the parameters T_0 and α are determined for each data. **Fig.2** shows an example of least squares fits, where the solid and dashed lines correspond to temporal correlation functions calculated from the observed data and estimated by least squares fit, respectively. Strong



(a) Soil condition group 1

(b) Soil condition group 2

(c) Soil condition group 3

Fig.1 Attenuation of RMS displacement

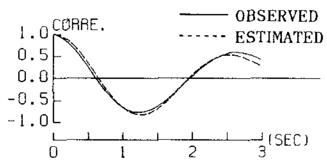


Fig.2 Temporal correlation function (Horoman Bridge, Aftershock of Tokachi-Oki earthquake, 1968)

Table 3 Mean and standard deviation of $\log(2B_T/T_D)$

Soil Condition	Mean	Standard Deviation
Group-1	1.092	0.255
Group-2	1.437	0.285
Group-3	1.393	0.268

motion duration B_T, T_D defined by Eq.(11) and an average number of zero crossings over the duration $2B_T/T_D$ are evaluated simultaneously with a RMS value. **Fig.3** shows $2B_T/T_D$ for each soil condition. Although systematic relationship between $2B_T/T_D$ and, M or Δ is hard to be found from **Fig.3**, the distribution of $\log(2B_T/T_D)$ may be regarded as a Gaussian distribution. **Table 3** shows the mean value and standard deviation of $\log(2B_T/T_D)$ for each soil condition. The mean values of $2B_T/T_D$ are 12.4, 27.4 and 24.7 for soil condition groups 1, 2 and 3, respectively.

5. SPATIAL CORRELATION FUNCTION

The following function is assumed to represent the spatial correlation function of ground displacement[4),5)] :

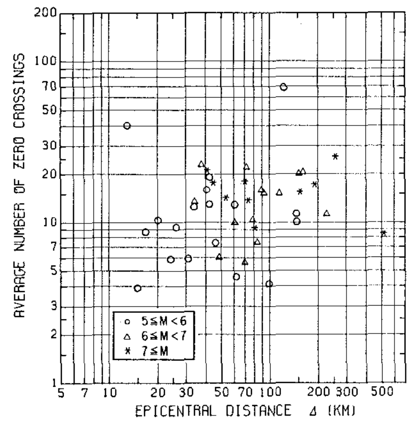
$$\rho_S(\eta)=\{1-(\eta/\xi_0)^2\}\exp\{-(\eta/\xi_0)^2\}\cdots\cdots(12)$$

where ξ_0 is determined by least squares fit to the observed array records. Substituting Eq.(12) into Eq.(5) gives σ_d for the time-space separable correlation model as

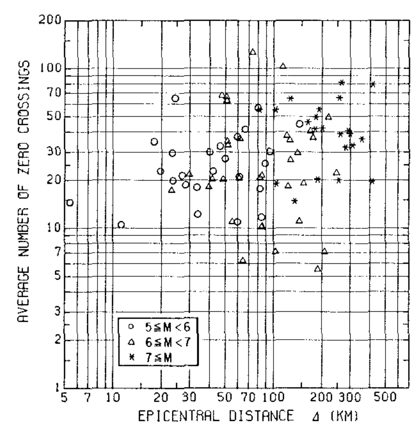
$$\sigma_d^2=2\sigma_u^2[1-\{1-(\xi/\xi_0)^2\}\exp\{-(\xi/\xi_0)^2\}]\cdots\cdots(13)$$

Eq.(12) is applied to 8 events data recorded at 4 array sites in Shizuoka Prefecture, Japan[17)]. Results of least squares fits are shown in **Fig.4**. Although the observed data show certain scatter, Eq.(12) coincides well with the trend of spatial correlation coefficients calculated from the observed data. ξ_0 is estimated for the radial and transverse components of ground displacement as shown in **Table 4**. ξ_0 varies from 300 m to 910 m among these 8 events.

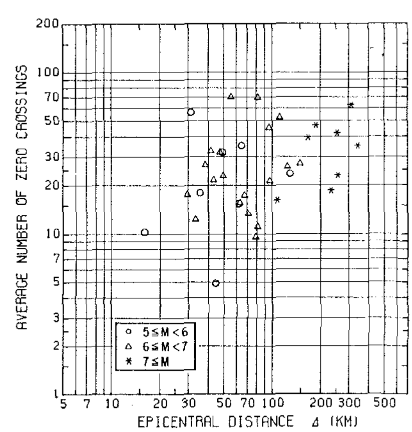
ξ_0 may be related to earthquake magnitude, epicentral distance, soil condition, and so on, however it is not practical to discuss this relationship with only 8 relatively small event records.



(a) Soil condition group 1



(b) Soil condition group 2



(c) Soil condition group 3

Fig.3 Relationship between $2B_T/T_D$, earthquake magnitude and epicentral distance

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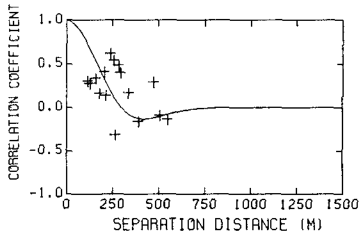
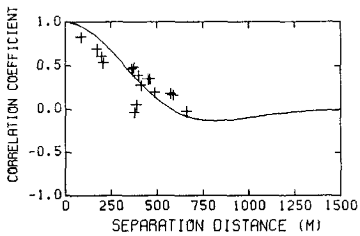


(a) Sagara, radial component, 3/16/83



(b) Numazu, transverse component, 9/5/88

Fig.4 Spatial correlation function

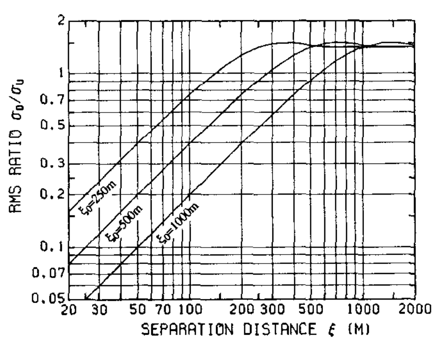


Fig.5 RMS displacement ratio σ_d/σ_u

Therefore, the effect of ξ_0 on the estimation of σ_d is studied by varying ξ_0 over a certain range. **Fig.5** shows the rate of σ_d/σ_u for $\xi_0=250$, 500 and 1000 m. As seen from Eq.(13) and **Fig.5**, σ_d/σ_u varies nearly as $2\xi/\xi_0$ up to $\xi=\xi_0$, and then it takes almost a constant value ($=\sqrt{2}$). Note that the shorter ξ_0 will give the larger σ_d for a given σ_u.

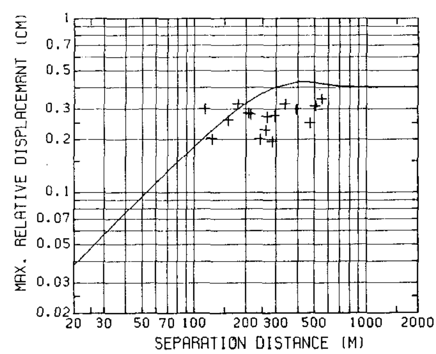
Combining either Eqs.(9) and (13), or **Figs.1** and **5**, σ_d can be estimated for an arbitrary earthquake magnitude and epicentral distance. Besides that Eq.(13) represents a deviation of a Gaussian process once σ_u is determined, it should be noted that the estimation of σ_u also involves scatter around the regression value as shown in **Fig.1**.

6. MAXIMUM RELATIVE DISPLACEMENT AND GROUND STRAIN

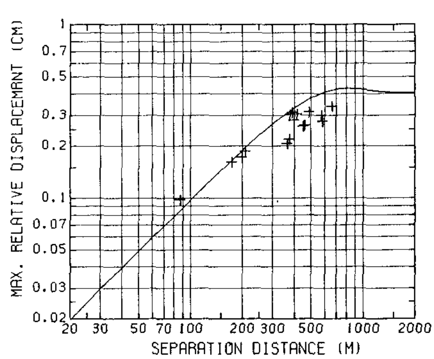
The maximum relative displacement $d_{\max}$ over a temporal interval B_T can be estimated by combining Eqs.(6), (9), (13), and either B_T and T_D defined Eq.(11) or $2B_T/T_D$. **Fig.6** compares the maximum relative displacement calculated from the observed array data and inferred from the presenting procedure for correspondent earthquake magnitude and epicentral distance. Results are shown for Sagara and Numazu sites. Considering ground conditions at these sites, the soil group 2 is chosen for both

Table 4 Array records analyzed and their spatial parameters

Site	Date	Epicentral Region	Magnitude	Epicentral Distance	Radial ξ_0[m]	Transverse ξ_0[m]
Sagara	3/16/83	S Coast of Chubu	5.7	55km	300	340
	11/24/83	S Coast of Chubu	5.0	45km	470	480
Yaizu	9/14/84	Central Chubu	6.8	126km	530	910
	9/15/84	Central Chubu	6.2	127km	620	650
Numazu	3/18/88	Tokyo Pref	6.0	96km	700	820
	9/5/88	E Yamanashi Pref	5.6	45km	550	580
Matsuzaki	11/22/86	Near Izu-Oshima Is.	6.0	72km	380	550
	12/17/87	Kujukuri Coast	6.7	171km	420	370



(a) Sagara, radial component, 3/16/83



(b) Numazu, transverse component, 9/5/88

Fig.6 Comparison of maximum relative displacement sites. $2B_T/T_D$ is set up as the mean value which is shown in **Table 3**, and the probability of not being exceeded p is taken as 0.5. As seen from **Fig.6**, the inferred $d_{\max}$ may be considered to be consistent with that calculated from the observed array records, allowing for the following facts ;

1) The SMAC accelerograph records are employed to estimate σ_u and $\rho_T(\tau)$ in the presenting procedure, consequently the maximum relative displacement is not directly approximated in **Fig.6**.

2) Although σ_u and $2B_T/T_D$ calculated from the observed records exhibit scatter as shown in **Figs.1** and **3**, only the average values of them are considered to estimate the maximum relative displacement for simplicity.

3) A resultant of two horizontal ground motion components is considered in the proposed method to give the maximum σ_u on the horizontal plane, however the radial and transverse components are analyzed independently for the array data.

Fig.7 shows the maximum relative displacement estimated for earthquake magnitude $M=6$, 7, 8 and epicentral distance $\Delta=50$[km], where $p=0.5$ and $2B_T/T_D$ is put as the mean value for each soil condition. ξ_0 is assumed to be 500[m] for all soil groups, considering the average value of ξ_0 presented in **Table 4**. As mentioned in the previous section, further studies will be required to make clear ξ_0 dependence on event and site characteristics, therefore the presenting result should be considered as an example of estimation.

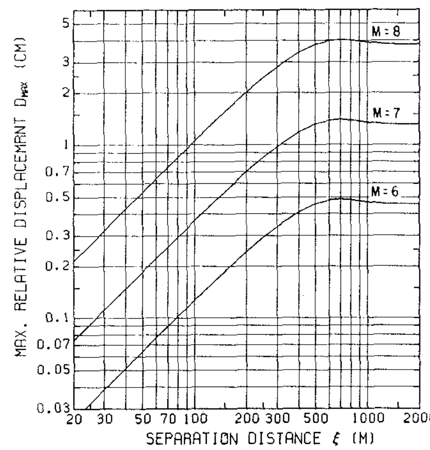
The maximum ground strain between two points can be described as

$$\varepsilon_{\max}=d_{\max}/\xi \cdots\cdots\cdots\cdots (14)$$

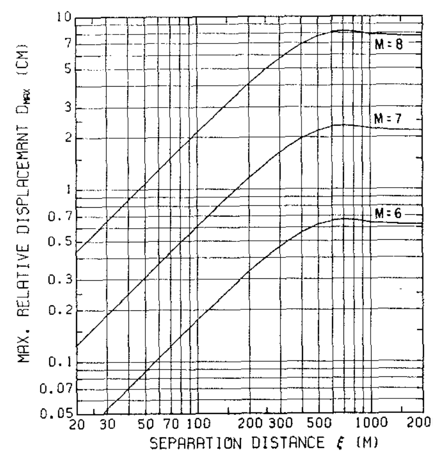
where ξ is a separation distance between two points. From the definition by Eq.(14), the maximum ground strain can be read from axes which go up from left to right with 45 degrees in **Fig.7**. When the combination of $M=7$ and $\Delta=50$ [km] is considered, the maximum ground strains over an any fixed distance are about 40×10^{-6} for soil condition group 1, 60×10^{-6} for group 2 and 100×10^{-6} for group 3.

7. CONCLUSIONS

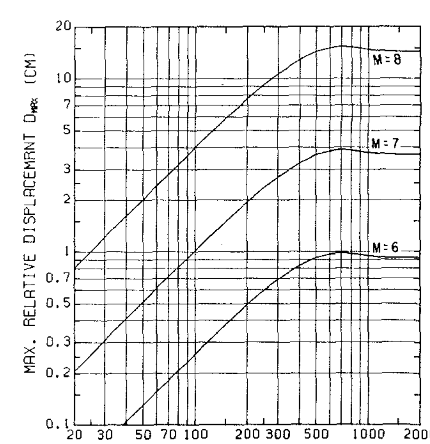
A practical method to estimate the maximum relative displacement between two points on the ground surface is proposed. It basically depends on a random process theory and consists of the estimation of RMS displacement, temporal and spatial correlation functions of displacement. RMS displacement and the temporal correlation function are estimated from conventional independent strong motion observation data. The spatial correlation function is calibrated from the array observation records. The following conclusions



(a) Soil condition group 1



(b) Soil condition group 2



(c) Soil condition group 3

Fig.7 Estimated maximum relative displacement for $M=6$, 7, 8 and $\Delta=50$ km may be deduced from this study.

1) Attenuation equations of RMS displacement in terms of earthquake magnitude and epicentral distance are proposed for three soil conditions.

2) Both the proposed temporal and spatial correlation function models agree well with the correlation functions calculated from the observed data.

3) An average number of zero crossings during strong motion duration estimated from the temporal correlation function may not be very sensitive to earthquake magnitude or epicentral distance.

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時間-空間確率過程モデルを用いた相対地震動の評価

田村敬一・相沢　興

本研究は，地中構造物の耐震設計上重要な地震時の地盤ひずみを評価することを目的として，地震動の簡便な時間-空間確率過程モデルを用いて，2地点間の最大相対変位の推定を試みたものである．用いた確率過程モデルの構築には，現時点では大きな地震によるデータが得られていない同時多点強震記録を補足するために従来の独立した観測点による強震記録を用いることが可能であり，最終的に最大相対変位を地震のマグニチュードおよび震央距離から推定する手法を提案した．