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SURFACE STRAINS ASSOCIATED WITH STRONG EARTHQUAKE SHAKING

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Method of Lee and Trifunac (1985; 1987) for the synthetic generation of translational, torsional and rocking accelerograms has been generalized to enable computation of strains at ground surface during passage of strong earthquake ground waves. Variations of strain with time resemble ground velocity components in corresponding directions. The Fourier spectrum amplitudes of strain are proportional to the spectrum amplitudes of the corresponding velocity components divided by the representative phase velocities.

Keywords: strains, synthetic, rotational accelerograms, layers, surface waves

1. INTRODUCTION

With the passage of seismic waves, ground surface experiences three translations and three rotations. For a coordinate system chosen so that x_1 points in the radial and x_3 in the transverse direction, relative to earthquake source, and with ground assumed to consist of homogeneous parallel layers, the above six degrees of freedom can be reduced to five: u_1, u_2, u_3, ϕ_{12} and ϕ_{13} as shown in Fig. 1. Translational components of ground motion have been studied theoretically and experimentally and at present there are many recordings of strong motion acceleration ($\ddot{u}_1$, $\ddot{u}_2$, $\ddot{u}_3$) for various earthquake magnitudes, epicentral distances and local soil and geologic conditions.

Studies of the responses of long structures excited by propagating ground waves (Werner *et al.* 1979; Kojić *et al.* 1988; Todorovska and Trifunac 1989, 1990 a, b) have emphasized the need to describe the strong ground motion as a complete field surrounding the structure and considering all local amplification, scattering, diffraction and interference caused by local soil and geology and by the irregular surface topography. Response of a large foundation of a structure to ground waves can be described by the translation of the foundation only if η (ratio of the characteristic length of the foundation to the wave length of incident waves) is less than $\sim 1/1\,000$. For $1/1\,000 \leqq \eta \leqq 1/10$ the response of the foundation can be approximated by translation and rotation of ground motion and by consideration of the amplitudes of surface strains. For $\eta \geqq 1/10$ actual wave motion of the ground should be considered using datailed description of motion at various points of the foundation (and at various individual foundations) and replacing consideration of surface strains by relative displacements at various points of single foundation and of multiple foundations (Todorovska and Trifunac, 1990 b).

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The aim of the present work is to complete the description of strong ground motion by presenting a method for the simultaneous computation of surface strains to be associated with synthetic translational, torsional and rocking accelerograms in the algorithms which are analogous or equivalent to the computer program SYNACC and when $1/1\,000 \leqq \eta \leqq 1/10$ (Lee and Trifunac 1985, 87). By considering body and surface wave contributions, via exact theoretical wave propagation models in layered half space, it becomes possible to describe the three-dimensional features of surface strains. For the foundations located in more irregular soil and geologic site conditions, the method presented here can be generalized by using the approach presented by Vaziri and Trifunac (1988 a, b). Thus the objective of this work is to present a realistic procedure for three-dimensional description of strong ground motion near ground surface, in firm ground and in the linear response range. Nonlinear response of soft and water saturated soils then can be evaluated by using this procedure to describe "incident" ground excitation. As the displacements and the strains imposed on soft soil materials by the surrounding and underlying "rock" motions depend not only on the stress strain relationships in the soil and amplitudes of vertically incident portion of seismic wave energy, but on the wave length of excitation and focussing in the three-dimensional site materials, the examples presented will show that, except near source, during very large motions, and in very soft soil materials, response of ground surface will be in the linear range.

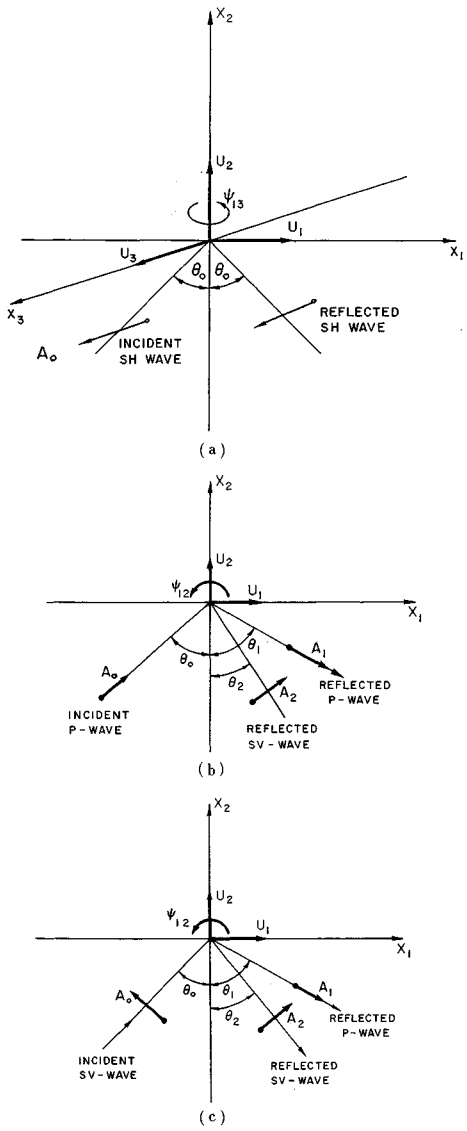

Fig. 1 (a) Displacement u_3 and torsion ϕ_{13} for incident plane SH-waves. (b) Displacement u_1 and u_2 and rocking ϕ_{12} for incident plane P-waves. (c) Displacements u_1 and u_2 and rocking ϕ_{12} for incident plane SV-waves.

2. THEORY

Trifunac (1979) has shown that surface strains ($x_2=0$) in the infinite half space, associated with incident plane P and SV waves are

$$\varepsilon_{x_1,x_1} = ik_0 \sin\theta_0 u_1 \quad \cdots\cdots (1\cdot a)$$

$$\varepsilon_{x_2,x_2} = ik_0 \left[\frac{2}{(\alpha/\beta)^2} - 1\right] \sin\theta_0 u_1 \quad \cdots\cdots (1\cdot b)$$

and

$$\varepsilon_{x_1,x_2} = 0. \quad \cdots\cdots (1\cdot c)$$

For incident plane SH waves,

$$\varepsilon_{x_1,x_1} = \varepsilon_{x_2,x_2} = 0 \quad \cdots\cdots (2\cdot a)$$

$$\varepsilon_{x_1,x_3} = ik_1 \sin\theta_0 u_3 \quad \cdots\cdots (2\cdot b)$$

where

$$k_0=\omega/\alpha, \quad k_1=\omega/\beta \quad \cdots (3\cdot a)$$

and

$$\frac{\alpha}{\beta}=\left[\frac{2(1-\nu)}{1-2\nu}\right]^{1/2} \quad \cdots (3\cdot b)$$

α is the velocity of P waves, $\alpha=[(2\mu+\lambda)/\rho]^{1/2}$, ν is the Poisson's ratio, and β is the velocity of shear waves.

By considering Rayleigh surface waves propagating in the x_1 direction, it can be shown that equations (1) describe the surface strains during the passage of Rayleigh waves also. Similarly equations (2) describe the strains associated with Love waves as well. Thus summarizing, at $x_2=0$

$$\varepsilon_{x_1,x_1}=u_{1,1}=iku_1 \quad \cdots (4\cdot a)$$

$$\varepsilon_{x_2,x_2}=u_{2,2}=ik\left[\frac{2}{(\alpha/\beta)^2}-1\right]u_1 \quad \cdots (4\cdot b)$$

$$\varepsilon_{x_3,x_3}=u_{3,3}=0 \quad \cdots (4\cdot c)$$

$$\varepsilon_{x_1,x_2}=\frac{1}{2}(u_{1,2}+u_{2,1})=0 \quad \cdots (4\cdot d)$$

$$\varepsilon_{x_2,x_3}=\frac{1}{2}(u_{2,3}+u_{3,2})=\frac{1}{2}u_{3,2}=0 \quad \cdots (4\cdot e)$$

$$\varepsilon_{x_1,x_3}=\frac{1}{2}(u_{1,3}+u_{3,1})=iku_3 \quad \cdots (4\cdot f)$$

where $k=\omega/c$ and c is the representative phase velocity in x_1 direction.

3. GENERATION OF SYNTHETIC STRAINS VERSUS TIME

We model the local soil and geologic conditions by equivalent parallel layers. For each of the layers $i=1, 2, 3, \cdots L$ the parameters h_i, α_i, β_i and ρ_i must be specified. h_i is thickness of i-th layer, α_i and β_i are the corresponding velocities of compressional and shear waves, and ρ_i is the material density. Through numerical analysis, the phase and group velocities $C_m(\omega)$ and $U_m(\omega)$ of m-th surface wave mode at frequent ω, for $m=1, 2, \cdots, M-1$ are then calculated. For simplicity in subsequent notation the incident body waves are "modeled" as "surface waves" with "constant" phase velocities $C_M(\omega)$. Then, the travel times to a station located at epicentral distance R, for energy centered at ω_n, are

$$t^*_{nm}=R/U_m(\omega_n). \quad \cdots (5)$$

The contribution of this energy to the Fourier spectrum of the complete time function, within frequency band $\omega_n\pm\Delta\omega_n$, is

$$A_{nm}(\omega)=\begin{cases}\frac{\pi}{2}A_{nm}\,e^{-i(\omega-\omega_n)t^*_{nm}+i\phi_n} & (\omega-\omega_n)\leqq\Delta\omega_n\\ 0 & \text{otherwise}\end{cases}$$

$$A_{nm}(-\omega)=A^*_{nm}(\omega). \quad \cdots (6)$$

In (6) ϕ_n represents a random phase near ω_n, and A_{nm} is the relative amplitude of m-th mode at ω_n.

The inverse transform of $A_{nm}(\omega)$ in equation (6) is

$$u_{nm}(t)=A_{nm}\frac{\sin\Delta\omega_n(t-t^*_{nm})}{t-t^*_{nm}}\cos(\omega_n t+\phi_n). \quad \cdots (7)$$

It represents the contribution of m-th mode, within $|\omega-\omega_n|\leqq\Delta\omega$ to $u(t)$. Multiplying $u_{nm}(t)$ in (7) by ik or by $ik[2/(\alpha/\beta)^2-1]$ as in equations (4·a) through (4·f) will give the contribution to surface strains resulting from the m-th mode near ω_n. The total contribution to ground motion from all surface wave modes and from body P and S waves, near $\omega=\omega_n$ is

$$u_n(t)=\sum_{m=1}^{M}\delta_n u_{nm}(t)=\sum_{m=1}^{M}\delta_n A_{nm}\frac{\sin\Delta\omega_n(t-t^*_{nm})}{t-t^*_{nm}}\cos(\omega_n t+\phi_n). \quad \cdots (8)$$

Relative amplitudes A_{nm} and overall empirical scaling parameters δ_n can be determined from theoretical source radiation, from some specified spectra of ground motion or from empirical equations on the

expected spectral amplitudes in terms of earthquake and local site parameters (Lee and Trifunac, 1985, 1987). A_{nm} will depend on the relative excitation of different surface waves.

The Fourier amplitudes of $u_n(t)$ is

$$|A_n(\omega)|=\begin{cases} \left|\sum_{m=1}^{M}\frac{\pi}{2}\delta_n A_{nm}\exp[-i\{(\omega-\omega_n)t^*_{nm}-\phi_n\}]\right| ; & |\omega-\omega_n|\leqq\Delta\omega_n \\ 0 & \text{otherwise}\end{cases} \quad \cdots\cdots (9)$$

for $0\leqq\omega\leqq\infty$ and for $|A_n(-\omega)|=|A_n(\omega)|$. Defining the mean amplitude of $A_n(\omega)$ within the narrow frequency band $2\Delta\omega_n$ as

$$|\overline{A_n(\omega)}|=\frac{1}{2\Delta\omega_n}\int_{\omega_n-\Delta\omega_n}^{\omega_n+\Delta\omega_n}|A_n(\omega)|\,d\omega, \cdots\cdots (10)$$

it is required here that this amplitude should be the same as some given or as empirically estimated (Trifunac 1976; 1989 a, b) Fourier spectrum amplitude $\widehat{FS}(\omega)$ of strong motion displacements at frequency ω_n,

$$|\overline{A_n(\omega)}|=\widehat{FS}(\omega_n). \cdots\cdots (11)$$

From this the scaling parameters δ_n can be computed as

$$\delta_n=\frac{2\Delta\omega_n\widehat{FS}(\omega_n)}{\frac{\pi}{2}\int_{\omega_n-A\omega_n}^{\omega_n+\Delta\omega_n}\left|\sum_{m=1}^{M}A_{nm}\exp\{-i\{(\omega-\omega_n)t^*_{nm}-\phi_n\}\}\right|d\omega}. \cdots\cdots (12)$$

The strains versus time are then

$$\varepsilon_{x_i,x_j}(t)=\sum_{n=1}^{M}\left(\sum_{m=1}^{M}A_{nm}i\frac{\omega_n}{C_m(\omega_n)}f_{i,j}\frac{\sin\Delta\omega_n(t-t^*_{nm})}{t-t^*_{nm}}\right)\delta_n\cos(\omega_n t+\phi_n) \cdots\cdots (13)$$

where

$$f_{i,j}=\begin{cases} 1 & \text{if } i=1,\ j=1,3 \\ \frac{2}{\alpha^2/\beta^2}-1 & \text{if } i=j=2 \\ 0 & \text{otherwise}\end{cases} \cdots\cdots (14)$$

and $\widehat{FS}(\omega)$ of strong motion displacement are used corresponding to u_1 for ε_{x_1,x_1} and ε_{x_2,x_2} and corresponding to u_3 for ε_{x_1,x_3}.

4. EXAMPLES OF COMPUTED SURFACE STRAINS

We consider the local site geology and dispersion of the associated surface waves for Westmoreland in Imperial Valley, California (Table 1, Figs. 2 and 3). To examine "large" strains we also modify this site

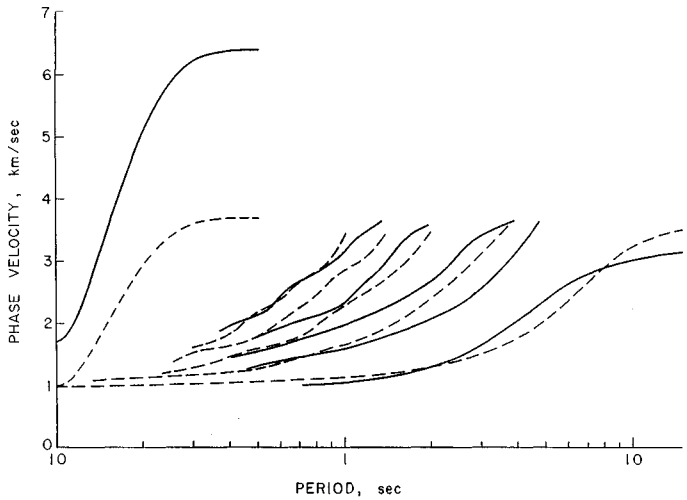

Fig. 2 Love (dashed lines) and Rayleigh (full lines) wave phase velocity curves for Westmoreland site, California (see Table 1).

Table 1 Assumed layer thicknesses (h_i), compressional (α_i) and shear (β_i) wave velocities and material densities (ρ_i) for site at Westmoreland, in Imperial Valley, California.

Layer	Depth h_i (km)	α_i (km/s)	β_i (km/s)	ρ_i (qm/ce)
1	0.18	1.70	0.98	1.28
2	0.55	1.96	1.13	1.36
3	0.98	2.71	1.57	1.59
4	1.19	3.76	2.17	1.91
5	2.68	4.69	2.71	2.19
6	∞	6.40	3.70	2.71

Table 2 Same as Table 1 except for a modification in top 180 m layer 1 to illustrate the effects for a "soft" soil site.

Layer	h_i (km)	α_i (km/s)	β_i (km/s)	ρ_i (qm/ce)
1 a	0.05	0.105	0.05	1.20
1 b	0.13	0.60	0.30	1.28
2	0.55	1.96	1.13	1.36
3	0.98	2.71	1.57	1.59
4	1.19	3.76	2.17	1.91
5	2.68	4.69	2.71	2.19
6	∞	6.40	3.70	2.71

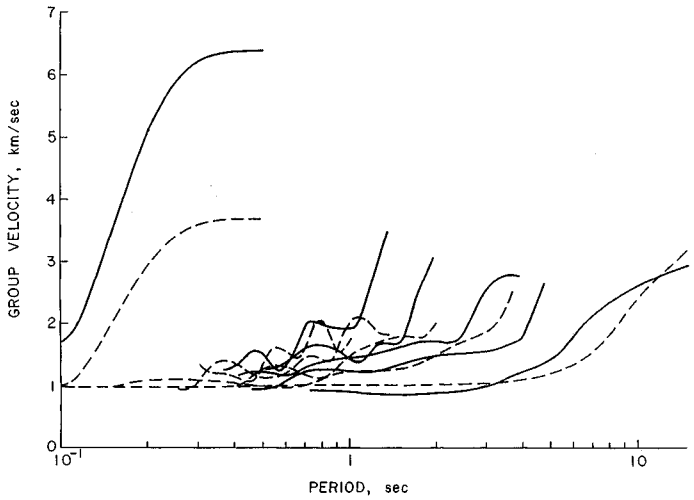

Fig. 3 Love (dashed lines) and Rayleigh (full lines) wave group velocity curves for Westmoreland site, California (see Table 1).

(Table 1) to have of a 50 m layer with shear wave velocity of 50 m/s and the second layer of thickness 130 m with shear wave velocity of 300 m/s (Table 2). For depths greater than 180 m we adopt the material properties an in Table 1. For long period motions dispression curves for this modified model would be identical to those in Figs. 2 and 3. For short periods and layer properties as in Table 2 the phase and group velocities would be reduced from 0.98 km/s to 0.05 km/s. The modified site will exemplify "soft" soil and geologic conditions in which, as implied by Eqs. (13) and (14) the strains could by "large". We also consider large amplitudes of strong shaking and in the immediate vicinity of the epicenter. Fig. 4 presents an example computed for the site intensity $MMI=XII$, source at epicentral distance of 5 km for $s=0$ (site on alluvium) and $s_L=2$ (seep soil site) and for $p=0.5$ where p is the probability of exceedance for the derived spectrum (Trifunac, 1989 a, b). These conditions result in peak accelerations (37.8 m/s^2 for radial, 33.9 m/s^2 for transverse and 18.0 m/s^2 for vertical ground motion) larger than what has been recorded so far and thus illustrate extreme response amplitudes and the associated strains. The resulting peak strains are respectively 0.04, 0.03 and 0.01. Because the strains are proportional to ku (see Eqs. (4)) the overall appearance of strain versus time is similar to that of the ground motion velocity. Fig. 5 shows another example of $M=7.5$, epicentral distance 1.5 km, $s=0$, $s_L=2$ and for $p=0.9$. The radial, transverse and vertical peak accelerations, respectively, 18.4 m/s^2, 19.3 m/s^2 and 13.7 m/s^2 lead to the corresponding peak strains equal to 0.026, 0.015 and 0.006 respectively. These examples have been chosen to illustrate extreme cases. Considering the local geologic cross section as given in Table 1 for $M=6.5$, $s=0$ and $s_L=2$ and for $p=0.5$ would result in peak accelerations equal to 7.85 m/s^2, 4.75 m/s^2 and 2.65 m/s^2 for example, and the associated strains only equal to 3.5×10^{-4}, 1.5×10^{-4} and 1.1×10^{-4} for ε_{x_1,x_1}, ε_{x_1,x_3} and ε_{x_2,x_2} respectively.

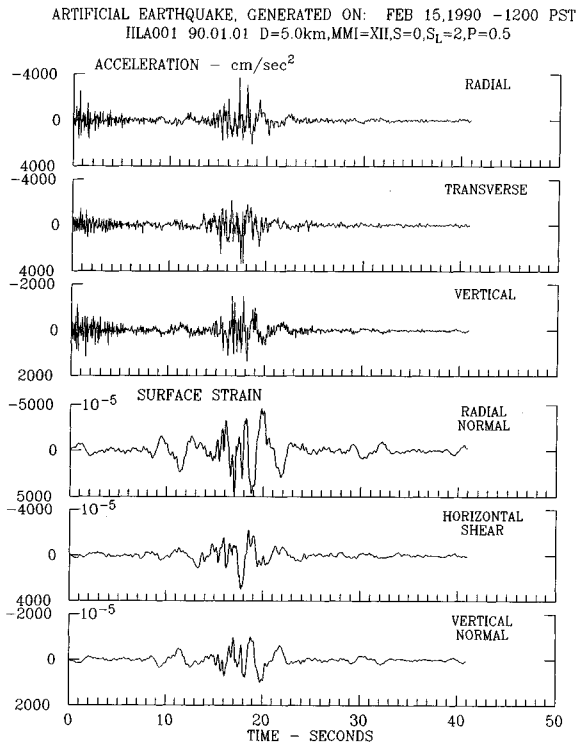

Fig. 4 Radial, transverse and vertical accelerations, and normal radial (ε_{x_1,x_1}), horizontal shear (ε_{x_1,x_3}) and normal vertical (ε_{x_2,x_2}) strains on ground surface for $MMI=XII$, epicentral distance 5 km, on deep soil site over alluvium.

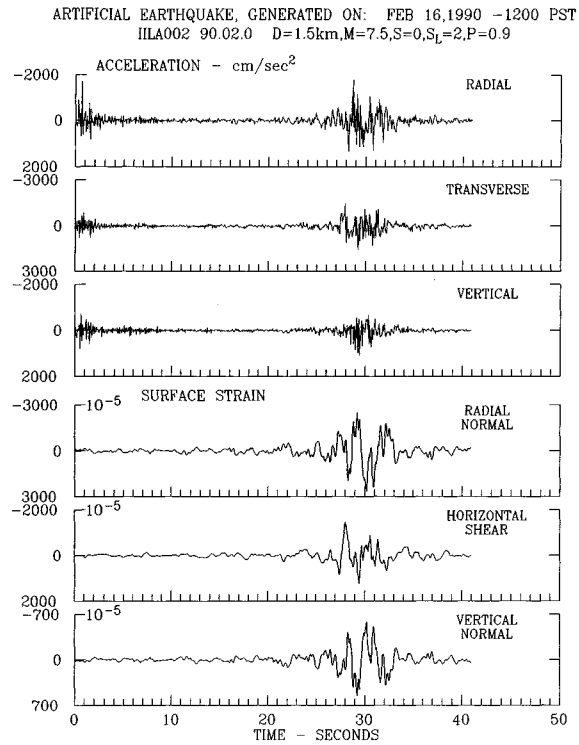

Fig. 5 Radial, transverse and vertical accelerations, and normal radial (ε_{x_1,x_1}), horizontal shear (ε_{x_1,x_3}) and normal vertical (ε_{x_2,x_2}) strains on ground surface for $M=7.5$, epicentral distance 1.5 km, on deep soil site over alluvium.

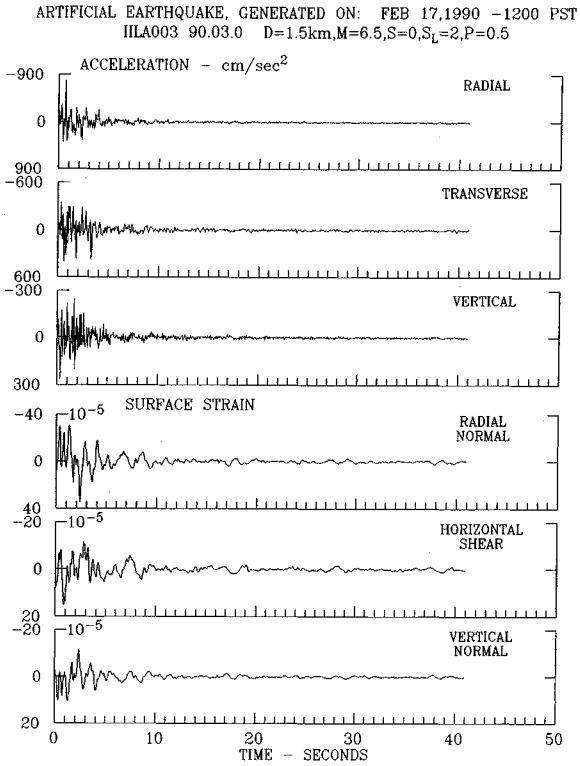

Fig. 6 Radial, transverse and vertical accelerations, and normal radial (ε_{x_1,x_1}), horizontal shear (ε_{x_1,x_3}) and normal vertical (ε_{x_2,x_2}) strains on ground surface for $M=6.5$, epicentral distance 1.5 km, on deep soil site over alluvium.

5. DISCUSSION AND CONCLUSIONS

The above examples illustrate the possible strain amplitudes implied by linear theory, but cannot be taken to be "average" or "typical" estimates, for magnitudes, site intensities or local conditions considered. In general the strain amplitudes will increase with overall increase in strong motion amplitudes and with decrease of shear wave velocities of soil and sedimentary layers near ground surface. Time dependence of strain components near ground surface is roughly proportional to the corresponding components of ground velocity and thus peak strains will also increase with peak ground velocity.

It is seen that by using the linear theory of wave propagation, the strain amplitudes can be evaluated exactly in three-dimensions. When local geologic conditions are too complex to model near surface motions by the equivalent parallel layer models the method presented here can be modified to give again the exact representation of near surface strains, but in terms of other than the rectangular Cartesian coordinate system (Vaziri and Trifunac 1988 a, b).

Many engineering inferences on linear and non-linear strains in rock and in soil, during passage of seismic waves are limited by the assumption that the incident energy arrives vertically in a one- dimensional wave propagation model. The method presented here should help in the development of input motions and input strains for modeling three-dimensional nonlinear response of soft and water saturated soils, and for full and direct estimation of strains in the linear response range.

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