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THE RESPONSE OF MASS-ON-ROUGH-PLANE MODEL DUE TO EARTHQUAKES

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The response of mass-on-rough-plane model to both horizontal and vertical excitations is studied statistically. The critical acceleration envelope is defined for a potential sliding surface of a structure as the locus of input acceleration that mobilizes limit resistance against slippage. A method to determine the model parameters with critical acceleration envelope is proposed. Slip displacements are computed with 52 sets of strong motion records and regressed against JMA magnitude, focal distance, peak values and spectral moments of accelerograms. The analytical prediction by Igarashi is compared in a good agreement with the computed slip displacements.

Keywords : earthquake, mass-on-rough-plane, accelerograms, response, slip displacement

1. INTRODUCTION

Slip displacements in earth dams, rock slopes, facilities resting on direct foundations, simply supported bridge structures and other engineering facilites are a major cause of failure due to earthquakes. On the other hand, it has been recognized since the 1920's that the occurrence of slip or localized failure dissipates seismic energy and reduce seismic loads transmitted to the superstructure (Martel, 1929). Several design concepts using this phenomenon have been proposed (e. g. Arya et al. 1978, Varpasuo et al. 1980, Igarashi et al. 1985). In this context the prediction of slip displacement is an important part of the earthquake resistant design.

The mass-on-rough-plane model is the simplest structural model where the slip displacement is computed as the relative displacement of a rigid mass resting on a rough plane subject to excitations. The response of this system has been used as an index of structural behavior for seismic design of earth dams (Newmark 1965, Sarma 1975, Seed 1979), retaining walls (Richards and Elms 1979, Whitman and Liao 1985), and structures that allow sliding. The response analysis with the mass-on-rough-plane model has been called sliding block analysis, key elements of which include :

1) Determination of the model parameters so that the pertinent dynamic property of the real structure can be represented.
2) Prediction of the response of the nonlinear system due to random excitation.

A proper choice of the input to this model is the absolute acceleration averaged over the sliding portion of the structure, which has been called the effective acceleration (Seed, 1979). Several analytical and empirical studies have been conducted on the response of the model to some classes of the effective

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acceleration. Crandall et al. (1974) have obtained the non-stationary R. M. S. response under stationary white noise excitation of the mass-on-rough-plane model for two-sided resistance with the equivalent linearization technique (Caughey and Dienes, 1961). However, as has been shown in Igarashi et al. (1985), the response to the white noise excitation is rather different from that to real strong motions. Ambraseys (1973), Sarma (1975), Makdisi and Seed (1978) have simulated effective accelerations of earth dams under some simplifying assuptions and computed slip displacement normalized by the peak acceleration at the crest and natural period of the structure. Hynes-Griffin and Franklin (1984) have computed the empirical mean response and standard deviations with 348 horizontal components of natural earthquakes and 6 synthetic records. Lin (1982) has computed an empirical mean response from 140 real records and several simulated Gaussian excitations in terms of the central frequency, and strong motion durations and R. M. S. amplitude proposed by Vanmarcke and Lai (1980).

All the previous studies assume that the excitation is horizontal and the effect of vertical acceleration has been treated as an uncertainty of the analysis. An analytical solution is necessary to refine sliding block analysis for all the empirical predictions may have their applicability limited due to bias of their data sets. In this paper will be discussed the sliding block analysis with both vertical and horizontal excitations. Empirical conditional means of slip displacement will be obtained from an extensive regression analysis with 52 sets of strong motion records observed in Japan. The analytical mean slip displacement obtained by Igarashi (1986) will be compared with the empirical mean in a good agreement. the strong motion durations and R. M. S. amplitudes proposed by Vanmarcke and Lai (1980) are found to be effective in the analytical prediction with an adjustment allowing for the non-stationarity of real accelerograms.

2. MASS-ON-ROUGH-PLANE MODEL

(1) Critical Acceleration Envelope

Fig. 1 shows the configuration of the mass-on-rough-plane model. The mass O moves along the rough plane OH inclined by the angle β form the horizontal excited by the Coriolis translational acceleration OP of the prescribed displacement of the rough plane. ZOY is a stationary coordinate system and AO is the gravity acceleration. The external accelerations AO and OP are resolved in the tangential (HO and OT) and the normal (AH and HQ) to the rough plane. The Coulomb frictional resistance AQ tan ϕ=PQ with friction angle ϕ is the only internal force of this system. Clearly, if the input acceleration OP is on the envelope CAE in Fig. 1, the frictional resistance PQ is in equilibrium with the driving acceleration HT. The envelope CAE is called here the critical acceleration envelope, meaning that the input acceleration outside this envelope will cause slippage. The apex angle of CAE is 2 ϕ and the height from the rough plane is 1 G, ϕ and G being the friction angle and the gravity acceleration respectively.

The mass starts slipping if the input acceleration crosses the critical acceleration envelope, yielding the relative displacement x from a reference point on the rough plane. OP′ is such an acceleration whose net driving force per unit mass or the relative acceleration $\ddot{x}$ can be computed graphically in Fig. 1 as the distance of point P′ from the critical acceleration envelope :

$$\ddot{x}=Q'P'-Q'S'=S'P' \qquad (1)$$

The mass stpos when the relative velocity $\dot{x}$ equals zero. The relative displacement x in the parallel direction to the rough plane can be computed by integrating Eq. 1 twice while the mass is slipping. The critical acceleration envelope thus determines this system.

(2) Principal axes of the system

The computation of the slip displacement can be simplified by resolving the input acceleration a ($\ddot{Y}_g$, $\ddot{Z}_g$) into the perpendicular and parallel directions to the Critical Acceleration Envelope (see Fig. 2). There are two sets of such directions, one for uphill, the other for downhill movement. We refer to these directions as the principal axes of the mass-on-rough-plane model. In Fig. 2 is shown the downhill principal axes OX and OX′. By resolving the input acceleration into these directions ($\ddot{X}_g$, $\ddot{X}'_g$) and writing Eq. 1 with these

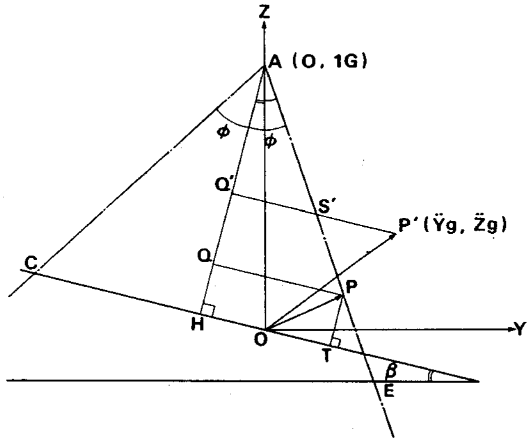

Fig. 1 Mass-on-rough-plane model.

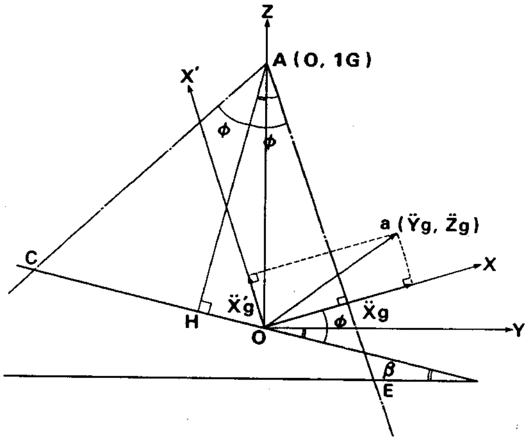

Fig. 2 Principal Axes of the Mass-on-rough-plane model.

components,

$$\ddot{x}=\frac{1}{\cos\phi}(\ddot{X}_g-G\sin\theta) \qquad (2)$$

in which

$$\ddot{X}_g=\ddot{Y}_g\cos\theta+\ddot{Z}_g\sin\theta \qquad (3)$$

and $\theta=\phi-\beta$. Eq. 2 can be simplified by using the slip displacement S in the principal direction,

$$S=x\cos\phi \qquad (4)$$

From Eqs. 2 and 4,

$$\ddot{S}=\ddot{X}_g-A_c \qquad (5)$$

where

$$A_c=G\sin\theta \qquad (6)$$

is the critical acceleration or the distance from the origin to the critical accleration envelope (See Fig. 2). The other component $\ddot{X}'_g$ has no contribution to the output. For so called one-sided case where the uphill movement is neglected, the slip displacement due to two-dimensional excitation can be computed as if it were a one-dimensional problem. This observation simplifies both the empirical and analytical response analyses that will be presented in the following chapters.

(3) Representation of real structures by mass-on-rough-plane model

Critical acceleration envelope for a real structure is defined here for each potential sliding surface as the locus of the Coriolis translational accelerations that mobilize the limit resistance to sliding. The potential sliding surfaces and associated critical acceleration envelopes may be obtained from the pseudo-static analysis with seismic coefficients K_h and K_v or the equivalent tilted problem of angle ψ :

$$\psi=\tan^{-1}(K_h/(1-K_v)) \qquad (7)$$

and the scaled gravity constant G' :

$$G'=((1-K_v)^2+K_h^2)^{1/2}G \qquad (8)$$

In Fig. 3 is illustrated a result of this tilting analysis with the critical tilting angle ψ and the force resultants R, W' and N acting on the sliding portion that is above the broken line. All the seismic coefficients that satisfy Eq. 7 are equivalent to this critical tilt angle ψ but to different gravity constants G'. Fig. 3(a) shows the locus of such seismic forces (K_hW, K_vW). Clearly, the critical acceleration envelope is linear if the critical tilt angle is independent of the magnitude of the scaled gravity constant G'. Tamura et al. (1985) have reported an experimental study on the response of a model earth dam of Onahama sand with both horizontal and vertical excitations. Their result may be interpreted as an example of an almost linear critical acceleration envelope.

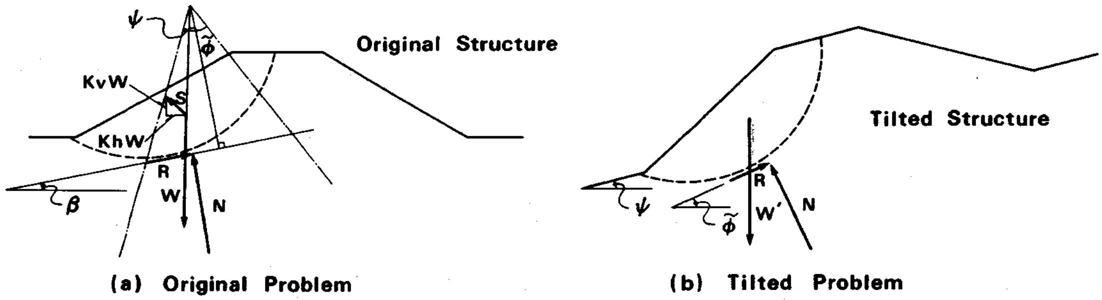

Fig. 3 Pseudo-static Analysis and Equivalent Tilted Problem.

In general the critical acceleration envelope of real structures may exhibit a curvature, the sliding portion of a real structure may experience non-elastic strains and local failures like cracks, therefore, the computation or even a measurement of slip displacement in a real structure may be difficult and controversial. However, as the first order approximation that neglects all these deformations of the sliding portion and consider it as a rigid body, the mass-on-rough-plane model may represent a real structure if it has an equivalent critical acceleration envelope. The following procedure is proposed:

1) Determine the potential sliding surfaces and associated critical acceleration envelope by pseudo-static analysis with both vertical and horizontal seismic coefficients.
2) Linearize the envelope and define a mass-on-rough-plane model with the linearized critical acceleration envelope. The linearization may be conducted on the point of the real critical acceleration envelope where the probability density of the input motion is maximum.
3) Study characteristics of the effective acceleration of the sliding portion and compute slip displacement as the residual movement of the mass to it. The effective acceleration may be computed from a linear analysis as the absolute acceleration averaged over the sliding portion assuming sliding does not occur (Lin, 1980).

3. EMPIRICAL RESPONSE ANALYSIS

(1) Slip displacement of 52 strong motion records

The correlation between the slip displacement and characteristics of input acceleration will be studied empirically with strong motion records. 52 sets of accelerograms, 78 components including 26 vertical ones, in the NOAA file (NOAA, 1981) are used. Statistics of pertinent characteristics are in Table 1 and epicenters and recorded stations are Igarashi and Hakuno (1987). Detailed informations about corrections are in NOAA (1981). A total of 21 recording stations are located on ground surface (12), on premises of buildings (7), on a bridge (1), and on a jetty (1).

The frequency content of the effective acceleration that is averaged over the sliding portion of the target structure may be different from that of these accelerograms due to its own amplification and averaging. Therefore, we have investigated regression equations that explain the simulated slip displacements with various characteristics of input motion such as peak values, spectral moments and durations in order to find general prediction equations that include the result of amplification and averaging effect explicitly. On the other hand, a global prediction equation against magnitude and focal distance of earthquakes can be useful to estimate slip displacement roughly before detailed information of the structure and the site is available.

For each pair of vertical and horizontal ground motion records, 20 slip displacements are computed with Eqs. 3 through 6 by varying the critical acceleration A_c from 5 % to 50 % of the absolute peak acceleration A of the horizontal component or by changing the principal direction accordingly:

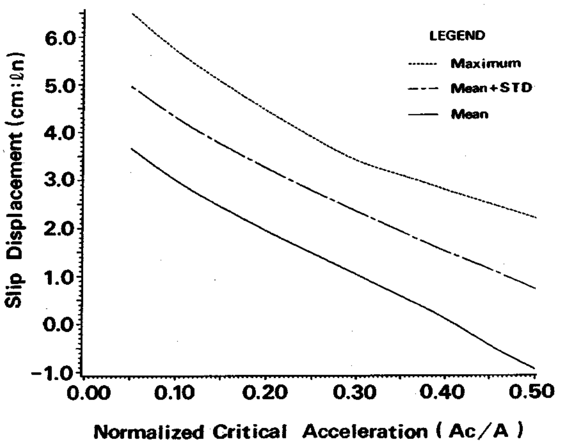

Fig. 4 Slip Displcement of 52 Sets of Strong Motion Records.

Table 1 List of Mean and S. D. of Selected Characteristics.

	Mean	S.D.
Slip Disp. (cm)	18.976	52.897
Number of Slippages	37.014	45.134
Magnitude (JMA)	6.685	0.895
Epicentral Dist. (km)	90.385	80.095
Site classes	2.538	0.888
Peak Acc. (gal)	176.474	100.520
Peak Acc. (Vertical, gal)	76.868	57.683
Peak Vel. (kine)	16.047	13.234
Peak Disp. (cm)	4.883	7.058
Central Freq. (rad/sec)	29.128	11.170
R.M.S. Acc. (gal)	82.299	70.943
Zero Cross Freq. (rad/sec)	25.448	11.344
Duration So (sec)	11.265	12.366

Note: Number of slippages is counted for each record. Average slip is computed for each record. Site classes are according to JSCE (class = 1, 2, 3, 4)

$$A_c/A=0.05\,k,\quad k=-10,-9,\cdots,9,10,\quad k\neq 0 \qquad (9)$$

The quantity A_c/A is selected as the control parameter of this simulation because it has been widely used in similar studies. Fig. 4 shows the mean, mean+ standard deviation, and maximum of the computed slip displacement plotted in the log scale against the control variable A_c/A.

Regression equations are nearly exhaustively searched among the log-linear models, i. e. the natural logarithms of the slip displacement are regressed against the logarithms of the selected characteristics. Slip displacements S are computed in centimeters and 6 data that are smaller than 10^{-2} cm are set to 10^{-2} cm in order to keep the logarithms in a finite range.

(2) Regression against global characteristics

Firstly, explanatory variables are chosen from the global characteristics, i. e. JMA magnitude M, focal distance R (km), and critical acceleration A_c (cm/s^2):

$$\ln S=-3.48+2.30\,M-1.29\ln R-1.47\,A_c \qquad (10)$$

in which the standard error of estimate is 1.722 and the coefficient of determination is 0.563. Eq. 10 may be interpreted as an attenuation equation of slip displacement for a given critical acceleration A_c. The first two terms are regarded as the magnitude of earthquakes in terms of slip displacement. The following term is regarded as geometrical and inelastic attenuations. The last term is the effect of the structural parameter A_c and close to other regressions that will be presented later. The horizontal peak acceleration A (cm/s^2), velocity V (cm/s) and displacement D (cm) were regressed in a similar form to Eq. 10:

$$\ln A=3.25+0.58\,M-0.50\ln R \qquad (11)$$

$$\ln V=-1.68+1.06\,M-0.69\ln R \qquad (12)$$

$$\ln D=-6.33+1.59\,M-0.82\ln R \qquad (13)$$

The coefficients of determination for Eqs. 11 through 13 are 0.365, 0.484, and 0.550, respectively. The attenuation equation of slip displacement has the highest coefficient of determination. This can be interpreted to indicate that the slip displacement is less sensitive to local site effect than other peak values. The regression coefficients of JMA magnitude and focal distance terms have a distinctive tendency from peak acceleration to peak displacement and slip displacement. As is seen in the analytical prediction and the detailed regression equation 15, the slip displacement is proportional to the total energy of the input acceleration. The peculiarity in the regression coefficients of Eq. 10 may be attributed to this characteristic of slip displacement.

(3) Regression against peak values

The peak acceleration A (cm/s^2), peak velocity V (cm/s) of the horizontal component and the normalized critical acceleration $N=A_c/A$ give,

$$S=0.315\,N^{-1.972}V^{1.595}A^{-0.877} \quad (14)$$

The logarithmic standard error of estimate is 0.969 and the coefficient of determination is 0.755. The quantity $V^{1.595}A^{-0.877}$ can be rounded to V^2/A that has the dimension of displacement. The regression coefficients in Eq. 14 are comparable to Newmark's prediction $S=0.5\,N^{-2}V^2/A$ and other authors' estimations reviewed in Whitman and Liao (1985).

(4) Regression against accelerogram characteristics

Thirdly, explanatory variables are selected from detailed characteristics of individual accelerograms. The following equation has achieved the lowest standard error of estimate of 0.441,

$$S=1.239\,s_0^{0.999}\sigma_2^{2.237}A_c^{-1.356}\omega_c^{-1.447}\exp\left(-0.792\,(A_c/\sigma_2)^2\right)F^{0.535} \quad (15)$$

in which s_0 (s), σ_2 (cm/s^2) are strong motion duration and R. M. S. amplitude for the equivalent stationary Gaussian excitation to the horizontal component of input acceleration. A_c (cm/s^2) is the critical acceleration and ω_c is the central frequency (rad/s). F is a function of the bandwidth index α_1 of the ground velocity

$$F=1+\frac{\pi}{2}\frac{(1-\alpha_1^2)^{1/2}}{\alpha_1} \quad (16)$$

$$\alpha_1=(\lambda_0/\lambda_{-2}\lambda_2)^{1/2} \quad (17)$$

in which λ_i is the i'th initial moment of the power spectral density function $S(\omega)$ of the equivalent stationary Gaussian excitation,

$$\lambda_i=\int_{-\infty}^{\infty}\omega^i S(\omega)d\omega \quad (18)$$

The power spectral density function $S(\omega)$ is estimated from the Fourier amplitude spectrum $A(\omega)$ of a segment of the horizontal accelerogram $\ddot{Y}_g(t)$ between time t_1 and t_2

$$A(\omega)=\left|\int_{t_1}^{t_2}\ddot{Y}_g(t)e^{-i\omega t}dt\right| \quad (19)$$

$$S(\omega)=\frac{1}{2\pi s_0}A^2(\omega) \quad (20)$$

where s_0 is the strong motion duration that will be determined so that the observed peak acceleration A will be found once on the average during s_0 in the equivalent Gaussian motion. When the bound $[t_1, t_2]$ is set to cover the entire record length, s_0 and σ_2 coincide with the strong motion parameters computed by Vanmarcke and Lai (1980). Several definitions of the sampling bound $[t_1, t_2]$ are tested in Eq. 19 and that between the first and last excursions of the half of the absolute peak acceleration (0.5 A) is found to give the equivalent stationary Gaussian parameters which explain the slip displacement best in the regression equation. The coefficient of determination is 0.889 and standard errors of regression coefficients are almost equal and about 0.05. In Fig. 5 are plotted the estimated values by Eq. 15 against the observed values computed by Eqs. 3 through 6. The regression equation pre-

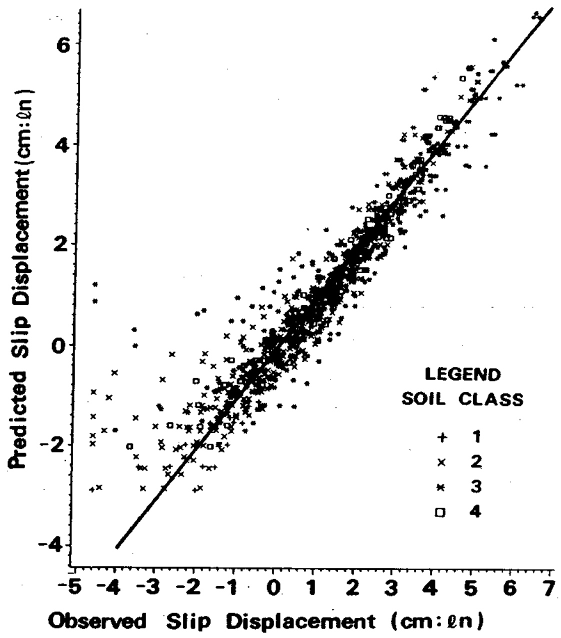

Fig. 5 Predicted Slip Displacement against Observed Slip Displacement.

dicts evenly well for all the ranges of the magnitude of slip displacement besides very small values that are less than $e^{-1}=0.37$ cm.

Three types of prediction equations are obtained from regression analysis. The global prediction equation 10 can be used to estimate slip displacement roughly and directly from M and R. Eq. 15 includes major frequency-related characteristics of input acceleration explicity and has very low standard error of estimate. Therefore, this can be used to compute slip displacement for structures with various response characteristics. Eq. 14 is a traditional one that can be placed between Eqs. 15 and 10 with respect to its error and utility.

4. ANALYTICAL PREDICTION OF SLIP DISPLACEMENT

Analytical form of the mean slip displacement has been obtained by Igarashi et al. (1985) for two sided case where the mass lies on a horizontal plane. Igarashi (1986) has computed the analytical mean rate of the slip displacement for one-sided case as a function of spectral moments of input acceleration. We will review his analytical solution so that we may compare his analytical prediction with the empirical mean response obtained here.

The principal component (see Eq. 3) of the ground motion is assumed to be a Gaussian process $\ddot{X}_g(t)$ and Eq. 5 is treated as a stochastic differential equation. The accumulatd slip displacement $S[t_1, t_2]$ for the duration $[t_1, t_2]$ can be written as an integral of a rate function $S_{Ac}(t)D_{Ac}(t)$,

$$S[t_1, t_2]=\int_{t_1}^{t_2} S_{Ac}(t)D_{Ac}(t)dt \qquad (21)$$

in which $S_{Ac}(t)$ is the magnitude of single slippage that starts at time t and $D_{Ac}(t)$ is a delta function that counts the occurrence of slippage. $D_{Ac}(t)$ may be written in a similar form to Middleton's rate function for the crossing problem (1960) if one tolerates a possible double counting while the mass is already slipping and counts all the up-crossings of the input acceleration $\ddot{X}_g(t)$ of the threshhold A_c.

$$D_{Ac}(t)=\begin{cases} \dddot{X}_g(t)\delta(\ddot{X}_g(t)-A_c), & \text{if } \dddot{X}_g(t)>0 \\ 0 & \text{, otherwise} \end{cases} \qquad (22)$$

where $\delta(\cdot)$ is the Dirac's delta function.

The mean of the slip displacement can be computed by taking the expectation of Eq. 21,

$$E[S[t_1, t_2]]=\int_{t_1}^{t_2} E[S_{Ac}(t)D_{Ac}(t)]dt \qquad (23)$$

The function $E[S_{Ac}(t)D_{Ac}(t)]$ is called here the mean rate of slip displacement and can be calculated if the probability density functions of the input process are known. By substituting Eq. 22 into the definition of mean process and using the conditional mean instead of $S_{Ac}(t)$ (Lin, 1960),

$$E[S_{Ac}(t)D_{Ac}(t)]=\int_0^{\infty}\int_{-\infty}^{\infty} E[S_{Ac}(t)\,|\,\ddot{X}_g(t)=\ddot{x}_g, \dddot{X}_g(t)=\dddot{x}_g]\dddot{x}_g\delta(\ddot{x}_g-A_c)f_{\ddot{x}_g(t),\dddot{x}_g(t)}(\ddot{x}_g, \dddot{x}_g)d\ddot{x}_g d\dddot{x}_g$$

$$=F_{\ddot{x}_g(t)}(A_c)\int_0^{\infty} E[S_{Ac}(t)\,|\,\ddot{X}_g(t)=A_c, \dddot{X}_g(t)=\dddot{x}_g]\dddot{x}_g d\dddot{x}_g \qquad (24)$$

where $f_{\ddot{x}_g(t),\dddot{x}_g(t)}(\ddot{x}_g, \dddot{x}_g)$ is the joint probability density function of the second and third derivatives of the input process $X_g(t)$, and $F_{\ddot{x}_g(t)}(A_c)$ is the cumulative distribution function of $\ddot{X}_g(t)$ evaluated at A_c. The second moment of the rate of slip displacement can be computed similarly (see Igarashi, 1986).

The conditional mean of the magnitude of single slippage $E[S_{Ac}(t)\,|\,\ddot{X}_g(t)=A_c, \dddot{X}_g(t)=\dddot{x}_g]$ in Eq. 24 can be computed by studying the behavior of the relative displacement process $S(\tau)$ between the mass and the rough plane,

$$S(\tau)=X_g(t+\tau)-X_g(t)-\frac{1}{2}A_c\tau^2-\dot{X}_g(t)\tau \qquad (25)$$

In Fig. 6 are shown the acceleration, velocity, and displacement of the mass and ground motion. The slip displacement of single sliding is the first local maximum of $S(\tau)$ in Eq. 25. Under some simplifying assumptions Eq. 24 has been evaluated for Gaussian excitations (Igarashi 1986),

$$E[S_{A_c}(t)D_{A_c}(t)]=\frac{\sigma_2^2}{2\pi A_c\omega_c}\exp\left(-\frac{1}{2}\left(\frac{A_c}{\sigma_2}\right)^2\right)\cdot\left(1+\frac{\pi}{2}\frac{(1-\alpha_1^2)^{1/2}}{\alpha_1}\right)\cdots\cdots\cdots\cdots (26)$$

in which σ_2 is the standard deviation of the input acceleration $\ddot{X}_g(t)$. $\omega_c=\sigma_3/\sigma_2$ is the central frequency of the input acceleration at time t and $\alpha_1=\sigma_2^2/\sigma_1\sigma_3$ is the bandwidth index of the input velocity, σ_i being the standard deviation of the i'th derivative of $X_g(t)$. The rate function in Eq. 26 may be used to compute the mean slip displacement for non-stationary Gaussian excitations if the rates of change in these parameters are small for a typical duration of slip events.

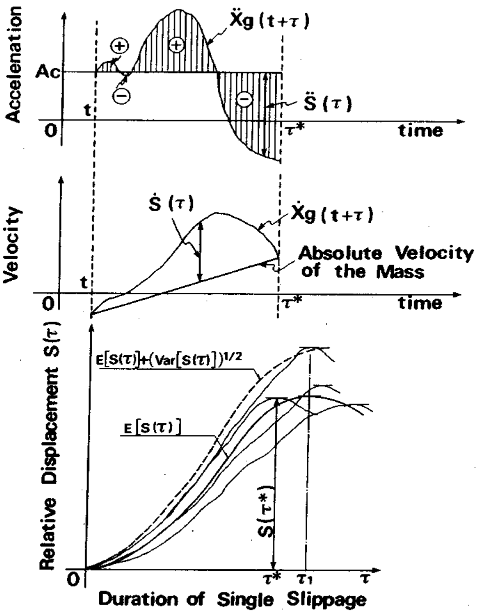

Fig. 6 Acceleration, Velocity and Displacement of Single Slip Event.

(2) Comparison with the empirical mean slip displacement

The comparison of the analytical prediction to the actual slip displacement that is computed with real strong motion records can be made in the following steps : 1) Define the equivalent Gaussian excitation and determine its pertinent parameters for strong motion records. 2) Compute the mean slip displacement for the equivalent Gaussian excitation by integrating the analytical rate function over the duration, and then compare it with the actual slip displacement. Several criteria for determining the equivalent Gaussian excitation and its parameters may be considered as to what quantity shall be preserved in the equivalent motion. Various analytical mean slip displacement may be computed according to what form of non-stationarity shall be assumed for the equivalent process.

A stationary Gaussian excitation for duration s_0 is chosen to be made equivalent to the principal component of accelerograms. The analytical mean slip displacement is computed by integrating the rate function obtained in the previous section,

$$E[S[0, s_0]]=\frac{s_0\sigma_2^2}{2\pi\omega_c A_c}F\exp\left(-\frac{1}{2}\left(\frac{A_c}{\sigma_2}\right)^2\right)\cdots\cdots\cdots\cdots (27)$$

in which $F=1+(\pi/2)(1-\alpha_1^2)^{1/2}\alpha_1^{-1}$ is called here the bandwidth parameter and α_1 is the bandwith index of the input velocity.

For each slip displacement simulated in the previous chapter, the power spectral density function and the strong motion parameters of the principal component are estimated in the same procedure as described in Sec. 3 (4). In Fig. 7 are the natural logarithms of the actual slip displacements computed in Sec. 3 and normalized by the quantity $\sigma_2 s_0/\omega_c$ and plotted against the normalized critical acceleration A_c/σ_2. The analytical prediction is computed with Eq. 27 and plotted in Fig. 7. Althogh a slight overshoot in the relatively large critical acceleration A_c/σ_2 was observed, the analytical prediction is found to fit well with the actual slip displacement.

The simulated slip displacements are regressed against the strong motion parameters and the critical acceleration in the principal direction with coefficient of determination of 0.885,

$$S=0.803\ S_0^{1.052}\sigma_2^{2.297}A_c^{-1.460}\omega_c^{-1.361}\exp(-0.785(A_c/\sigma_2)^2)F^{0.597}\cdots\cdots\cdots\cdots (28)$$

The analytical prediction in Eq. 27 will be compared with the empirical mean attained in Eq. 28 term by term. The difference of the contribution of the critical acceleration, A_c^{-1} in the analytical and $A_c^{-1.460}$ in the empirical, may be attributed to the nonlinearity in the conditional mean values of input process with respect to their control variables. The analytical dependence on the critical acceleration A_c^{-1} directly comes from

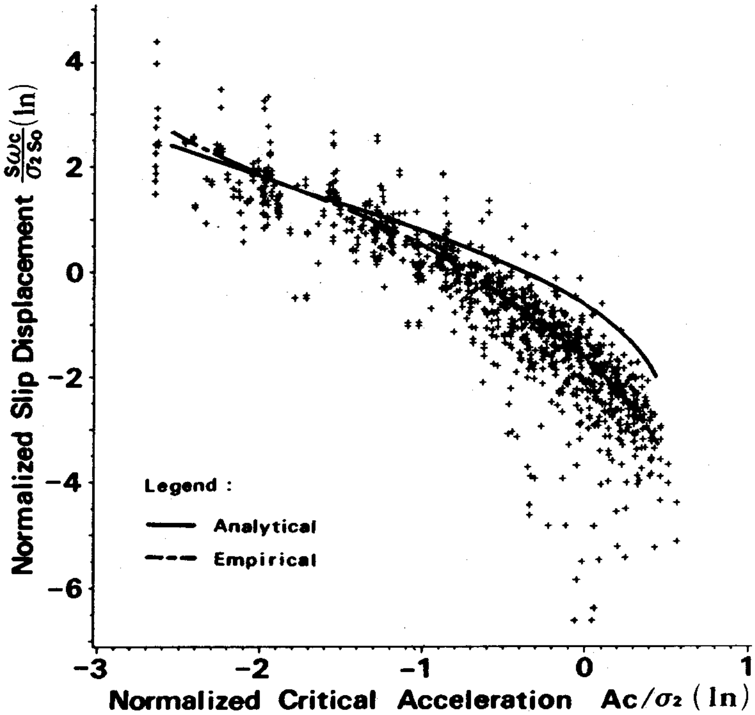

Fig. 7 Analytical Prediction and Actual Slip Displacement.

the linearity of the conditional mean that is a distinctive feature of Gaussian process. The non-stationarity of input motion will not affect the term A_c^{-1} in the analytical prediction. The term $A_c^{-1.460}$ produces the steeper decline of the empirical moving average in Fig. 7 for small A_c/σ_2.

The term exp $(-0.5(A_c/\sigma_2)^2)$ in the analytical solution reflects the form of the marginal probability density function of the excitation (see Eq. 24). The empirical term exp $(-0.785(A_c/\sigma_2)^2)$ corresponds to the sharper decline in the moving average in Fig. 7 for the larger value of A_c/σ_2. This difference can be adjusted by redefining the equivalent R. M. S. amplitude as

$$\hat{\sigma}_2 = 0.80\,\sigma_2 \quad (29)$$

This adjustment does not affect other terms in the analytical prediction as long as one determines the strong motion duration so as to preserve the total energy $I = s_0\sigma_2^2$ (see Eq. 27). Let us assume that the analytical mean slip displacement can be a valid criterion to determine the equivalent stationary Gaussian parameters and consider what is meant by Eq. 29. According to Vanmarcke and Lai's criteria, the R. M. S. amplitude σ_2 in the righthand-side of Eq. 29 is determined from the total energy and peak value of the actual record so that the observed peak factor, i. e. the ratio of peak value and R. M. S. amplitude, may be what is expected in the stationary Gaussian process (Vanmarcke and Lai 1980). This procedure seems to reflect the local R. M. S. amplitude around the observed peak. Usually, accelerograms exhibit a non-stationarity in the R. M. S. amplitude that gradually builds up and reaches its utmost around the peak value. Although the maximum single slippage that may occur around the peak acceleration may dominate in the slippages for a strong motion record, slip displacement can be regarded to reflect more global characteristic of the entire record than the magnitude of observed peak value. The R. M. S. amplitude estimated in the lefthand-side of Eq. 29 from the slip displacement is considered to be a global R. M. S. The 20 % difference observed in Eq. 29 between the local R. M. S. amplitude around the peak value and the global R. M. S. amplitude estimated from the slip displacement can be attributed to the non-stationarity of actual records.

It can be concluded from Fig. 7 and above-mentioned examinations that the analytical solution can give a good estimate of the mean slip displacement due to real strong motion records. The analytical solution consists of popular parameters that characterize frequency content of input motion. It is not derived from simulation but based on theory of Gaussian process. Therefore, it can be used as an element of sliding block analysis to compute slip displacement for structures with various response characteristics.

5. SUMMARY AND CONCLUSIONS

The usage and mechanical characteristics of mass-on-rough-plane model under two-dimensional excitations was studied. The response of the mass-on-rough-plane model to strong motion records is examined with 52 sets of accelerograms and compared in a good agreement with the analytical solution by Igarashi. The following are concluded :

(1) The critical acceleration envelope was defined for structures with potential sliding surfaces as the locus of Coriolis translational acceleration that mobilizes the limit resistance to slippage. The critical acceleration envelope of the mass-on-rough-plane model is linear and we can define a mass-on-rough-plane model for a real structure if we linearize its critical acceleration envelope.

(2) The response of mass-on-rough-plane model to both horizontal and vertical excitations can be computed using only the principal component of the gound motion that is perpendicular to the critical acceleration envelope, if one neglects the uphill movement. The effect of vertical ground motion increases as the principal direction itself becomes vertical.

(3) The response of mass-on-rough-plane model to 52 strong motion records was regressed against various characteristics of earthquakes and ground motions. The slip displacement was found to have a peculiar attenuation equation with higher coefficient of determination than peak acceleration.

(4) The analytical prediction derived by Igarashi was found to give a close prediction to actual response. The strong motion duration, R. M. S. amplitude, central frequency and bandwidth parameter are found to explain almost 90 % of the total variance of the 1 040 slip displacements.

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