

## A SIMPLIFIED ANALYSIS AND OPTIMALITY ON THE STEEL COLUMN BEHAVIOR WITH LOCAL BUCKLING

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A simplified analysis is given for the interactive steel column behavior with local buckling. The analysis utilizes the explicit solution of the elastic beam-column of Perry-Robertson type, newly incorporating the effective width concept of component plates. The results are proved to predict the available tests well with engineering accuracy. Optimality is examined from the views not only of ultimate strength but also of energy absorption to benefit on the earthquake-resistant design. The results indicate that allowing the occurrence of local buckling may not give the advantage much for the design of steel columns in ordinary civil engineering structures.

*Keyword*: interactive column strength, local buckling, effective width, optimization, energy absorption

### 1. INTRODUCTION

It has been common practice for the design of steel compression members that the occurrence of local buckling is restricted by the width-thickness ratio requirements of plate components, enabling the safety to be assured only against the overall column strength. There is a new trend, however, that the efficient design only be made possible by incorporating the interactive behavior of steel columns between overall and local bucklings, as exemplified by the appendix C of the AISC specification<sup>1)</sup>, which is based on the AISI specification<sup>2)</sup>, and the recent JRA specification<sup>3)</sup>. Although the reason for this trend seems to pursue more economical design by enlarging the freedom of design for the determination of geometries of structural components<sup>4)</sup>, the previous optimization study<sup>5)</sup> indicates that the inclusion of local buckling does not necessarily lead to efficient design, as far as the existing JRA and AISC interactive formulae are used. However, it is also said that both JRA and AISC formulae have been obtained on intuitive basis without any theoretical rationale.

This paper presents a simplified analysis of steel column strength, focusing on the interactive behavior with local buckling. The present analysis utilizes basically the explicit solution of elastic beam-columns called sometimes Perry-Robertson formula, incorporating collectively the influences of all the initial imperfections including residual stresses by the so-called 'equivalent initial deflection' as hinted before by Rondal and Maquoi<sup>6)</sup> and the influences of local buckling by the effective width concept for buckled plate

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components. The results are compared with the experimental data available and the aforementioned existing design formulae. Further, the optimality of steel compression members with local buckling is examined based on the present analysis, from the views not only of ultimate strength but also of energy absorption to benefit on the earthquake-resistant design.

## 2. SIMPLIFIED ANALYSIS

It seems complicated at present to evaluate the locally buckled column behavior with all the imperfections in inelastic finite displacements from the very rigorous theoretical standpoint<sup>7)</sup>. The use of effective width concept for buckled plate components can reduce the problem to the inelastic finite displacement column analysis which is rather tractable for numerical computations<sup>8),9)</sup> but still involves the tedious and cumbersome procedures with considerable nonlinearities.

To avoid those complexities for practical purposes, a simplified analysis is performed for the interactive column strength with local buckling, simply utilizing the well-known Perry Robertson formula<sup>10)</sup> which is obtainable explicitly from the initial-yield condition of elastic beam-columns. The influence of local buckling is reflected by the effective section comprising effective widths of buckled plate components, and that of all the initial imperfections including residual stresses is covered by the concept of 'equivalent initial deflections'.

### (1) Column analysis without local buckling

According to the beam-column theory, the maximum compressive stress  $\sigma_{max}$  of a simply supported column is given by

$$\sigma_{max} = \frac{P}{A} + \frac{P}{1 - P/P_e} \frac{\delta c}{I} \dots\dots\dots (1)$$

in which  $P$ =axial compressive force,  $A$ =cross sectional area,  $I$ =moment of inertia,  $c$ =distance between neutral axis and compressive fiber end,  $\delta$ =equivalent initial deflection defined later,  $P_e$ =Euler buckling load ( $=\pi^2 EI/L^2$ ),  $L$ =length of column, and  $E$ =Young's modulus. Column strength  $P_u$  is determined simply by the condition that  $\sigma_{max}$  reaches the yield stress  $\sigma_y$  of steel as

$$\sigma_{max} = \sigma_y = \frac{P_u}{A} + \frac{P_u}{1 - P_u/P_e} \frac{\delta c}{I} \dots\dots\dots (2)$$

Solving Eq. (2) with respect to  $P_u$  leads to

$$P_u = \frac{1}{2} A \left[ \sigma_y + \frac{\pi^2 EI}{AL^2} \left( 1 + \frac{Ac\delta}{I} \right) - \sqrt{\left\{ \sigma_y + \frac{\pi^2 EI}{AL^2} \left( 1 + \frac{Ac\delta}{I} \right) \right\}^2 - 4 \frac{\pi^2 EI}{AL^2} \sigma_y} \right] \dots\dots\dots (3)$$

which is well-known as the Perry-Roberston formula.

In order to reflect both the inelastic behavior after the initial yielding and the influences of all the imperfections present in actual steel columns in this very simplified formula, the initial deflection is replaced by the equivalent initial deflection in Eq. (3), the values of which are to be determined from the experimental and/or theoretically exact data regarding the ultimate column strength  $P_u$  without local buckling available in the literature.

### (2) Column analysis with local buckling

A similar treatment as for the column analysis without local buckling is made possible for the inclusion of the influence of local buckling by introducing the reduction of the original cross sectional shapes through the effective width concept of locally buckled plate components. For the simplicity of analysis, it is assumed that the effective area of mid-section is used throughout the column length and the neutral axis of the effective section does not shift from the original position. From this premise, the interactive column strength  $P_u$  with local buckling can be given in a similar way as Eq. (3) by

$$P_u = \frac{1}{2} A_e \left[ \sigma_y + \frac{\pi^2 EI_e}{A_e L^2} \left( 1 + \frac{A_e c \delta}{I_e} \right) - \sqrt{\left\{ \sigma_y + \frac{\pi^2 EI_e}{A_e L^2} \left( 1 + \frac{A_e c \delta}{I_e} \right) \right\}^2 - 4 \frac{\pi^2 EI_e}{A_e L^2} \sigma_y} \right] \dots\dots\dots (4)$$

in which  $A_e$  and  $I_e$  are effective cross sectional area and effective moment of inertia respectively. Since the effective area is evaluated by the effective widths of buckled plates which are functions of the stresses arising in the mid-section, depending on the axial compressive force  $P_u$ , an interactive procedure of successive substitutions is needed to solve Eq. (4).

### 3. COMPARISON OF TESTS WITH SIMPLIFIED ANALYSIS

#### (1) Test data

Test data of concern are available from Reference<sup>11)</sup>, which includes the experiments of the built-up box shaped columns of the HT 80 steel grade as shown in Fig. 1. From the test data excepting those for the stub columns and eccentrically loaded columns, the plate slenderness  $\lambda_t$  of compression flanges and the column slenderness  $\lambda_g$  (see Eqs. (5) and (11) for definitions) are given in Table 1 as well as their experimental ultimate strengths. The names of specimens indicate *S* and *R* for square and rectangular sections respectively followed consecutively by the values of column slenderness ratio  $L/r$  and flange width thickness ratio  $b/t$ . Yield stress  $\sigma_y=741$  MPa,

Young's modulus  $E=215\,000$  MPa, and Poisson's ratio  $\nu=0.24$  have been obtained from the tensile coupon tests. The welding residual stresses followed the well-known distributions, and the measured initial deflection was nearly half the sine wave with the average of the maximum magnitude being 2.6/10 000 of the column length. The local ultimate strength of plate components has been reported to be well approximated by

$$\frac{\sigma_{ul}}{\sigma_y} = \frac{0.75}{\lambda_t} \leq 1.0 \dots\dots\dots (5)$$

with

$$\lambda_t = \frac{1}{\pi} \sqrt{\frac{\sigma_y 12 (1 - \nu^2)}{E k}} \frac{b}{t}$$

in which  $k$  is buckling coefficient of plate component.

#### (2) Simplified analysis for test specimens

Eq. (4) is used to predict the theoretical interactive strength of specimen columns, incorporating the appropriate evaluations of the equivalent initial deflection and the effective width. The equivalent initial deflection is determined from the results of test columns of  $b/t=22$  whose ultimate strength is governed only by the overall buckling. On the other hand, the effective width of plate components is evaluated from the results of stub columns of  $L/r=10$  whose ultimate strength is governed only by the local buckling.

##### a) Equivalent Initial Deflection

By definition, equivalent initial deflection depends on all the compound factors which influence the overall column strength such as residual stresses, geometrical initial deflections and eccentricities, slenderness ratios and others. Noting that the residual stresses and geometrical crookedness did not vary

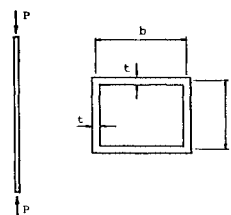


Fig. 1 A Centrally Loaded Box Column.

Table 1 Test Data<sup>11)</sup> and Present Analysis.

Specimen	$\lambda_g$	$\lambda_l$	$\frac{P_{uex}}{P_y}$	$\left(\frac{\delta}{L}\right)_e$	$\frac{P_{uth}}{P_y}$	$\frac{P_{uth}}{P_{uex}}$
S-35-22	0.640	0.686	0.852	0.00272	0.852	1.000
S-35-33	0.647	1.014	0.722	0.00272	0.680	0.942
S-35-38	0.648	1.177	0.621	0.00272	0.592	0.955
S-35-44	0.648	1.360	0.544	0.00272	0.514	0.947
S-50-22	0.948	0.683	0.740	0.00163	0.740	1.000
S-50-27	0.918	0.840	0.672	0.00163	0.745	1.109
S-50-33	0.927	1.026	0.670	0.00163	0.658	0.982
R-50-22	0.913	0.686	0.743	0.00173	0.743	1.000
R-50-27	0.924	0.840	0.731	0.00173	0.737	1.008
R-50-33	0.920	1.026	0.709	0.00173	0.694	0.979
R-50-38	0.923	1.180	0.639	0.00173	0.641	1.003
R-50-44	0.925	1.363	0.579	0.00173	0.558	0.964
R-65-22	1.186	0.683	0.593	0.00122	0.593	1.000
R-65-27	1.201	0.840	0.637	0.00122	0.564	0.885
R-65-33	1.203	1.026	0.585	0.00122	0.517	0.884

each other in the test specimens, the equivalent initial deflection  $(\delta/L)_e$  now is considered a function only of slenderness ratio, and is determined from Eq. (2) as

$$(\delta/L)_e = \left(1 - \frac{P_u}{A\sigma_y}\right) \left(1 - \frac{P_u}{P_e}\right) \frac{I\sigma_y}{cP_uL} \dots\dots\dots (6)$$

Substituting the test results of  $P_u$  and others for the columns of  $b/t=22$  into Eq. (6) yields the equivalent initial deflections of concern as given by Table 1.

b) Effective Width

The effective width of a purely compressed simply supported plate is introduced under an arbitrary stress  $\sigma$  between buckling and ultimate states of plate components as

$$b_e = C\pi \sqrt{\frac{k}{12(1-\nu^2)}} \sqrt{\frac{E}{\sigma}} t \leq b \dots\dots\dots (7)$$

which corresponds to

$$\frac{\sigma_{cr}}{\sigma_y} = \frac{C^2}{\lambda_i^2} = C^2 \frac{k\pi^2}{12(1-\nu^2)} \frac{E}{\sigma_y} \frac{t^2}{b^2} \leq 1.0, \quad \frac{\sigma_u}{\sigma_y} = \frac{C}{\lambda_i} \leq 1.0 \dots\dots\dots (8 \cdot a, b)$$

for the buckling and ultimate stress evaluations respectively<sup>12)</sup>. The effective width of Eq. (7) is a modified version from the original von Karman formula, in which the reduction factor  $C (<1)$  is introduced in order to reflect the initial imperfections present in actual steel plates, and further the yield stress  $\sigma_y$  is replaced to an arbitrary stress  $\sigma$  in Eq. (7) to account for the situation of plate components below the ultimate level.

Through the substitution of  $C=0.75$  from Eq. (5) and  $\nu=0.24$  relevant to the reported test results combined with  $k=4.0$ , the effective width of Eq. (7) is expressed as

$$b_e = 1.40 \sqrt{\frac{E}{\sigma}} t < b \dots\dots\dots (9)$$

For the ease of computations, the effective width of a purely compressed plate presented above is applied also for the web of a box column which is actually subject to bending and compression. The average compressive stress of the web is used for the evaluation of effective width Eq. (7), and moreover half the effective width is equally allocated to both sides of edges. Therefore, the effective widths of a box section as shown in Fig. 2 are given as

$$\left. \begin{aligned} b_{e,r} &= C\pi \sqrt{\frac{k}{12(1-\nu^2)}} \sqrt{\frac{E}{\sigma_r}} t_r \leq b_r \\ b_{e,r2} &= C\pi \sqrt{\frac{k}{12(1-\nu^2)}} \sqrt{\frac{E}{\sigma_{r2}}} t_{r2} \leq b_r \\ b_{ew} &= C\pi \sqrt{\frac{k}{12(1-\nu^2)}} \sqrt{\frac{E}{\sigma_w}} t_w \leq b_w \end{aligned} \right\} \dots\dots\dots (10)$$

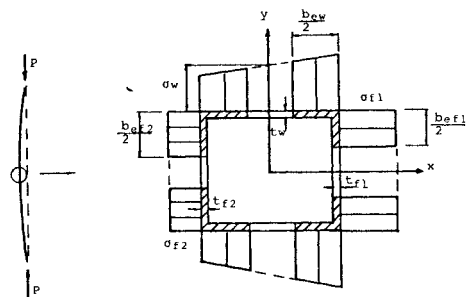


Fig.2 Effective Area of A Box Column.

c) Prediction and Comparison of Tests

Prediction of the interactive column strength with local buckling for test specimens is made possible through Eq. (4), using the equivalent initial deflection as tabulated in Table 1 and the effective width evaluation of Eq. (10) with  $C=0.75$ . Table 1 includes the theoretical predictions of  $P_{ult}$  non-dimensionalized by  $P_y = \sigma_y A$  and the ratio with experimental results  $P_{ult}/P_{uex}$ . It is found from the table that the present simplified analysis, as a whole, gives a safer approximation with engineering accuracy for the interactive strength of concern.

4. SIMPLIFIED ANALYSIS AND EXISTING DESIGN FORMULAE

In the previous chapter, the theoretical results have been obtained on the basis of the equivalent initial

deflection and the effective width determined from the test data in order only to compare with the experimental ultimate strengths. Having confirmed that the theory has been well correlated with the test of Reference<sup>11)</sup>, the theoretical strength now is evaluated for the design purpose based on the initial deflection and the effective width relevant to the design values adopted in current practice, and then is compared with the existing design formulae from the JRA and AISC specifications. Yield stress  $\sigma_y=235$  MPa is used hereinafter in the simplified analysis, unless otherwise stated.

The following design formulae are adopted in this paper for the overall column strength  $\sigma_g$ , the local buckling stress  $\sigma_{lc}$  and ultimate stress  $\sigma_{tu}$  of plate components, the first two of which are relevant to the JRA specification, as

$$\frac{\sigma_g}{\sigma_y} = \begin{cases} 1.0 & (0.0 < \lambda_g \leq 0.2) \\ 1.0 - 0.545(\lambda_g - 0.2) & (0.2 < \lambda_g \leq 1.0) \dots\dots\dots (11) \\ 1.0 / (0.773 + \lambda_g^2) & (1.0 < \lambda_g) \end{cases}$$

with

$$\lambda_g = \frac{1}{\pi} \sqrt{\frac{\sigma_y}{E}} \frac{L}{r}$$

for the overall column strength without local buckling<sup>3)</sup>, in which  $r$  is the radius of gyration with respect to the weak axis,

$$\frac{\sigma_{lc}}{\sigma_y} = \begin{cases} 1.0 & (0.0 < \lambda_l \leq 0.7) \\ 0.49 / \lambda_l^2 & (0.7 < \lambda_l) \dots\dots\dots (12) \end{cases}$$

for the local buckling stress<sup>3)</sup>, and

$$\frac{\sigma_{tu}}{\sigma_y} = \begin{cases} 1.0 & (0.0 < \lambda_t \leq 0.7) \\ 0.7 / \lambda_t & (0.7 < \lambda_t) \dots\dots\dots (13) \end{cases}$$

for the ultimate stress of plate components. Based on the above independent column and plate formulae, the interactive column strengths with local buckling are given as follows in conjunction of the JRA and AISC interactive formulae and the present simplified analysis.

( 1 ) JRA interactive formula

JRA specification stipulates the interactive strength denoted by  $\sigma_{uJ}$  as

$$\sigma_{uJ} = \sigma_g \sigma_l / \sigma_y \dots\dots\dots (14)$$

in which  $\sigma_g$  and  $\sigma_l$  are the relevant independent overall and local strengths of columns. The appropriate selection of the local strength depends on whether the local buckling stress or ultimate stress of component plates influences more on the interactive column strength. The current JRA specification adopts the local buckling stress  $\sigma_{lc}$  only for a safer approximation. Here in this paper, both cases are considered as

$$\sigma_{uJ1} = \sigma_g \sigma_{lc} / \sigma_y \dots\dots\dots (15)$$

with the adoption of local buckling stress  $\sigma_{lc}$  for the local strength and

$$\sigma_{uJ2} = \sigma_g \sigma_{tu} / \sigma_y \dots\dots\dots (16)$$

with the ultimate stress  $\sigma_{tu}$  of plate components.

( 2 ) AISC interactive formula

AISC specification stipulates the interactive strength denoted by  $\sigma_{uA}$  as

$$\sigma_{uA} = [\sigma_g]_{\sigma_y = \sigma_l} \dots\dots\dots (17)$$

which indicates that yield stress  $\sigma_y$  appeared in the formula of  $\sigma_g$  is replaced simply by the local strength  $\sigma_l$ . Although an alternative selection of  $\sigma_{lc}$  and  $\sigma_{tu}$  is possible in a similar way as for the JRA formula, only the case of  $\sigma_{tu}$  is considered here as

$$\sigma_{uA} = [\sigma_g]_{\sigma_y = \sigma_{tu}} \dots\dots\dots (18)$$

( 3 ) Simplified analysis relevant to design formulae

The equivalent initial deflection and the effective width for the simplified analysis are determined consistent with the basic independent design formulae as given in Eqs. (11-13). The square box shaped

columns of equal thickness are assumed for computations.

The ultimate column strengths without local buckling obtainable from the beam-column Eq. (3) for the initial deflections of  $\delta/L=0.001-0.005$  are shown in Fig. 3 in comparison with the overall column strength formula of Eq. (11). Noting from the figure that the beam-column curve of  $\delta/L=0.003$  is most fitted with the design curve of Eq. (11), the equivalent initial deflection is taken as  $(\delta/L)_e=0.003$ . As for the effective width, Eq. (10) is used with the reduction factor  $C$  equal to 0.7 consistent with the local ultimate stress formula of Eq. (13).

The results of simplified analysis using Eq. (4) with those parameters determined above are shown for the plate slenderness of  $\lambda_t=0.7, 0.9$  and  $1.1$  in Fig. 4 which also includes the results of JRA and AISC interactive formulae. It is observed from the figure that the AISC formula ② most coincides with the simplified analysis ① with some deviations in the inelastic range of  $\lambda_g=0.5-1.0$ . The JRA counterpart ③ is located at a considerably safer side from the AISC formula and is deviated more from the simplified analysis. In contrast to the formulae ② and ③ both of which are based on the ultimate stress of plate components of Eq. (13), the current JRA provision ④ of Eq. (15) with the local buckling stress of Eq. (12) deviates much from the simplified analysis particularly for larger plate slenderness, reaching half the results of simplified analysis. Considering the above results coupled with the fact that neither JRA nor AISC interactive formula has coherent rationale from the theoretical standpoint, the proposed simplified analysis can be a substitute to the existing design procedure for the design of the interactive column strength with local buckling.

Fig.5 summarizes the results of the simplified analysis for the plate slenderness of  $\lambda_t=0.7-1.1$ , which have been proved to predict the experimental ultimate strength with engineering accuracy. With the aid of curve-fitting procedure, the curves shown in Fig.5 have been represented by the proposed design formula in Reference<sup>12)</sup>. It should be remembered, however, that the proposed formula in Reference<sup>12)</sup> may not be appropriate for the design purpose because of the rather complicated appearance of design formula based on the numerical computation only for square box

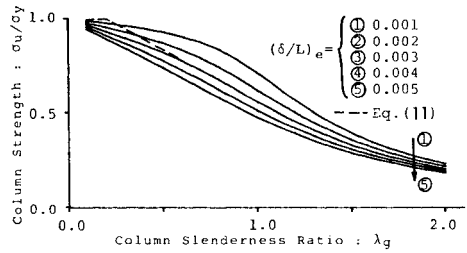
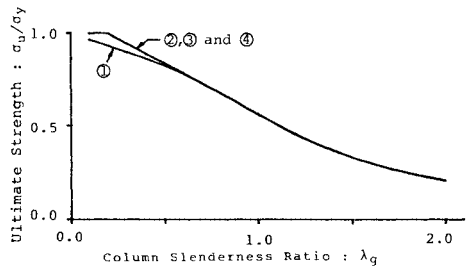
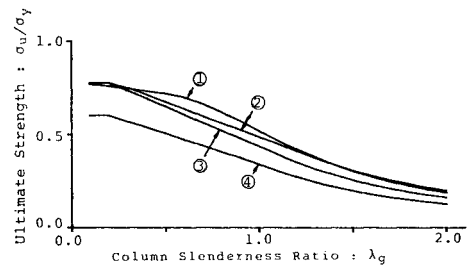


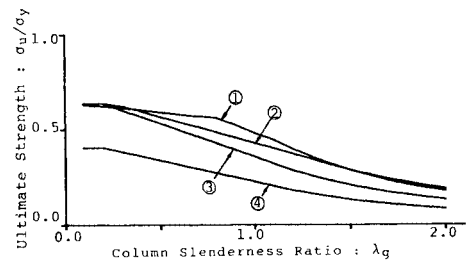
Fig.3 Column Strength without Local Buckling.



(a)  $\lambda_t = 0.7$



(b)  $\lambda_t = 0.9$



(c)  $\lambda_t = 1.1$

Fig.4 Results of Present Analysis and Design Curves.

Note : ① Simplified Analysis, ② AISC Eq. (18).

③ JRA Eq. (16), ④ JRA Eq. (15)

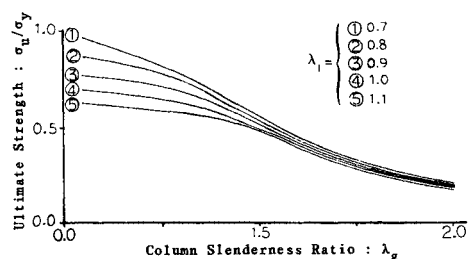


Fig.5 Summary of Present Analysis.

columns. Simplification may be required for the expression of design formula even at some sacrifice of accuracy. Meanwhile, just changing the standpoint, direct use of the revised Perry-Robertson formula of Eq. (4) with local buckling may preferably be left to individuals for respective design procedures, taking the circumstances into account that the recent proliferation of micro computers drastically facilitates to handle this sort of rather simple equation in design offices.

5. OPTIMIZATION STUDY

As described in INTRODUCTION, it has been reported<sup>5)</sup> that allowing the occurrence of local buckling may not necessarily lead to efficient and economical design, as far as the existing JRA and AISC interactive formulae are concerned. Since the present simplified analysis has given somewhat different results compared with those existing formulae, optimality for allowing local buckling should also be examined for the adoption of the present simplified analysis.

(1) Ultimate strength

Consider square box shaped columns of equal thickness again. Load maximization technique is used for optimization, in which the maximum load carrying capacity under constant weight of materials gives the optimum configuration of an interested structure<sup>5)</sup>. In order to facilitate the computational efficiency and versatility, the following non-dimensionalized quantities are introduced as

$$R = \frac{L^2}{A} = \frac{L^2}{4bt}; \quad x = \frac{b}{t}; \quad g_y = \frac{E}{\sigma_y} \dots \dots \dots (19)$$

Assuming thin-walled sections, the functional form of the present analysis can be reduced to

$$\sigma_u = \sigma_y f(R, g_y, x) \dots \dots \dots (20)$$

Considering that the column length be prescribed beforehand in ordinary design, constant value of  $R$  is equivalent to the constant volume of material. Given the steel grade of material indicating  $g_y = \text{const}$ , the optimality condition can be reduced to the very simple unconstraint optimization with a single unknown ( $x$ ) and two prescribed parameters ( $R$  and  $g_y$ ) as

$$\sigma_{u\text{max}} = \sigma_y \text{Max}_x [f(x)] \dots \dots \dots (21)$$

Numerical computations with the range of  $R=1\ 000-20\ 000$  have been made for the steel grades of SS 41 ( $\sigma_y=235$  MPa) and SM 53 ( $\sigma_y=352$  MPa) whose results are shown in Fig.6 in a similar manner as presented by Usami and Fukumoto<sup>10)</sup>. Fig. 6 indicates the relation between the interactive column strength of the present analysis and the plate slenderness  $\lambda_1$  for the range of  $R=1\ 000-20\ 000$ . Naturally the interactive strength is observed to decrease as the value of  $R$  increases, that is, the total volume of column decreases. The maximum interactive strengths at optimum for the respective constant volumes ( $R=\text{const.}$ ) are indicated by circles, which give the relation between the maximum strength and the optimal plate slenderness. It is noted that the value of optimal plate slenderness not greater than 0.7 does not cause local buckling as clear from Eq. (12). The values of  $R$  beyond which the optimal plate slenderness is found

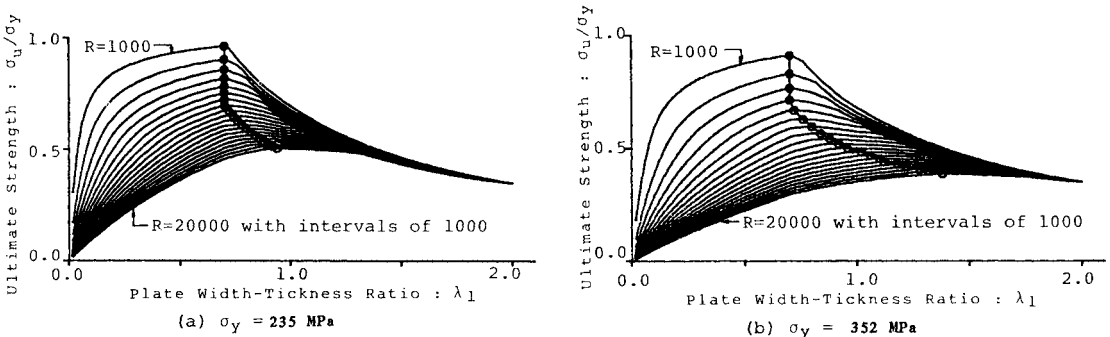


Fig.6 Optimality in Ultimate Strength.

in the range with the occurrence of local buckling are 8 000 for SS 41 steel and 4 000 for SM 53 steel. It should be remembered, however, that the interactive strength obtained for this range with local buckling at optimum is less than the value of  $0.7 \sigma_y$ , while the maximum plate slenderness of  $0.7$  which does not cause local buckling is always obtained for the interactive strength greater than  $0.7 \sigma_y$ .

As explained, there exists the range where allowing local buckling may produce more economical benefits for the design of square box columns, perhaps in the range of the column strength less than  $0.7 \sigma_y$ , when the present analysis is applied. For ordinary design circumstances, however, it is not frequent that the column design is made in the range of its strength less than  $0.7 \sigma_y$ . Moreover, even for this restricted range of  $\sigma_u < 0.7 \sigma_y$ , it is observed from the figure that the interactive strength does not change so much even when the plate slenderness varies in the neighborhood of its optimum value.

(2) Energy absorption

In addition to the optimal characteristics from the view of ultimate strength, it is also worthwhile to examine its optimality in terms of energy absorption to be reflected on the earthquake-resistant design. In order to grasp the general features of the energy absorption of steel columns up to the maximum load, the following non-dimensionalized value is introduced in the sense of relative magnitude as

$$E = \int_0^{P_u} P dw / (P_y L) \dots \dots \dots (22)$$

in which  $w$  is defined as mid-section deflection of column, and is given by

$$w = \frac{\delta}{(1 - PL^2 / \pi^2 EI_c)} \dots \dots \dots (23)$$

which is consistent with the beam-column solution of Eq. (4).

The energy absorption of Eq. (22) is computed for a variety of column slenderness  $\lambda_g$  and plate slenderness  $\lambda_t$ , and is shown in Fig. 7. The results indicate that the energy absorption  $E$  increases as  $\lambda_g$  increases with the maximum between  $\lambda_g = 1.5$  and  $2.0$ . This trend results from the fact that the maximum deflection sharply increases as  $\lambda_g$  increases, although the ultimate strength itself decreases. It is also observed that the energy absorption does not depend on  $\lambda_t$  for  $\lambda_t \leq 0.7$  without local buckling.

Next, choosing the energy absorption as the objective function, optimization is performed in a similar way as the aforementioned load maximization. The results are given in Fig. 8, indicating that the optimal width-thickness ratios with the maximum energy absorption for the respective constant volumes are found clearly in the range of  $\sigma_t < 0.7$  which does not cause local buckling. It is also noted that increasing the width-thickness ratio beyond this range monotonically decreases the energy absorption.

Combining the optimization in the sense of energy

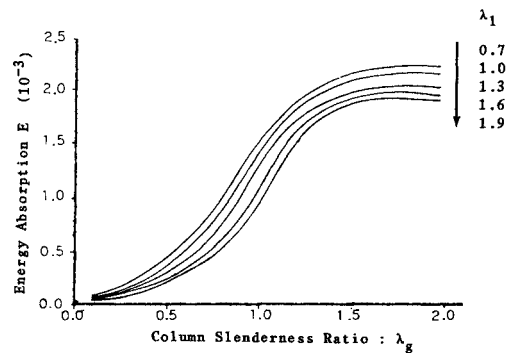


Fig. 7 Energy Absorption of Square Box Columns.

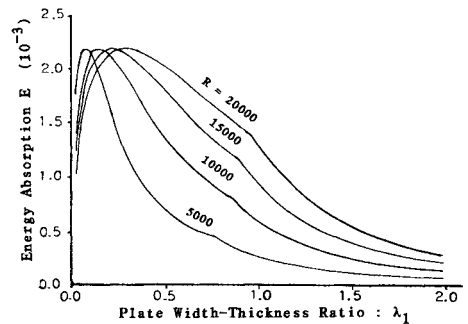


Fig. 8 Optimality in Energy Absorption.

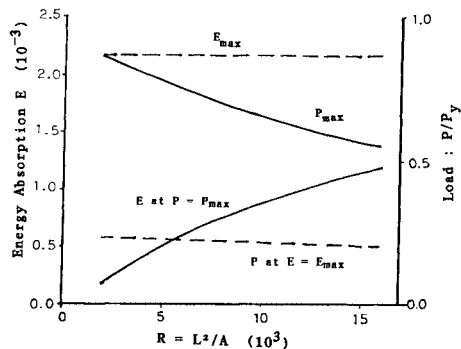


Fig. 9 Comparison of  $E_{max}$  and  $P_{max}$ .



absorption with the previous results of strength optimization, one can give the comparison between the maximum energy absorption  $E_{\max}$  and its corresponding applicable load  $P$  at  $E=E_{\max}$  as well as that between the maximum applicable load  $P_{\max}$  and its corresponding energy absorption  $E$  at  $P=P_{\max}$ , all of which are examined with respect to constant  $R$ , and the results are shown in Fig. 9. From the figure, it is said that the ratios not only of  $P$  (at  $E=E_{\max}$ )/ $P_{\max}$  but also of  $E$  (at  $P=P_{\max}$ )/ $E_{\max}$  are apparently less than 0.5 in the practical range of computations. It is remembered from this fact that one can not obtain the optimal solutions which simultaneously satisfy both the maximum strength capacity and the maximum energy absorption, although the respective optimum solutions have been found basically in the range of width-thickness ratio which does not cause local buckling.

All the facts described above may imply that the disadvantage of prohibiting the use of larger plate slenderness with local buckling is not remarkable. Since the design procedure incorporating the interactive column behavior with local buckling tends to become rather complicated, the width-thickness ratio requirements which prohibit the occurrence of local buckling may preferably be used for ordinary civil engineering structures. It should be noted, however, that the practical importance of allowing the occurrence of local buckling may appear in the design of large scale and/or specialty-oriented steel structures and components.

## 6. CONCLUDING REMARKS

The interactive column strength with local buckling has been investigated using a simplified analysis which utilizes the explicit solution of the elastic beam-column theory of Perry-Robertson type, incorporating all the imperfections such as residual stresses and geometrical crookedness by the equivalent initial deflection, and also reflecting the influence of local buckling through the use of the effective width concept.

The comparison of the present analysis with the test data available has indicated the sufficient accuracy with engineering satisfaction for the ultimate interactive strength of concern.

The results of the present analysis have been compared with the existing design formulae of the JRA and AISC specifications. It is found that the AISC formula with the local buckling effect evaluated by the ultimate stress of plate components, not by the local buckling stress, coincides most with the present analysis with some deviations in the inelastic range of  $\lambda_p=0.5-1.0$ . The use of the local buckling stress for the effects of locally buckled plates on the interactive column strength is found to be inadequate, as indicated by much deviations of the JRA original formula from the present analysis.

The optimality of square box shaped columns for allowing the occurrences of local buckling has been examined in terms not only of ultimate strength but also of energy absorption based on the proposed simplified analysis. The numerical study indicates that there exists the range where allowing local buckling may produce more economical benefits for ultimate strength, perhaps in the larger slenderness range of the column strength less than  $0.7 \sigma_y$  of less frequent use of ordinary structures. Noting further that the interactive strength does not change so much even when the plate slenderness varies in the neighborhood of its optimum value in this restricted range, the conventional width-thickness ratio requirements which prohibit the occurrence of local buckling may preferably be used for ordinary civil engineering structures. This opinion has also been confirmed from the view of energy absorption to benefit on the earthquake-resistant design, suggesting in the present study that the optimal width-thickness ratios with possible maximum energy absorption are found clearly in the range which does not cause local buckling.

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