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PREDICTION OF LIQUEFACTION OCCURRENCE OF SANDY DEPOSITS DURING EARTHQUAKES BY A STATISTICAL METHOD

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1. INTRODUCTION

It is very important to predict the occurrence of liquefaction of saturated sandy deposits during an earthquake. Many researches have been made on liquefaction phenomena and the method of predicting the occurrence of liquefaction. As the results, proposed were the four types of methods of using past experiences, standard blasting, ground response analyses and laboratory test procedures, and design acceleration level of ground surface and laboratory test procedures. Most of these methods are based on the mechanism of liquefying process. On the other hand, the method presented in this paper is derived from a statistical treatment of only basic data of earthquake and soil conditions, although the data available are limited at present. In this method the liquefaction potential, defined as a function of some basic factors, expresses a measure of susceptibility to liquefaction and makes it possible to predict easily liquefaction occurrence at any site for given soil conditions and assumed earthquake data. The merit of this method is neither to request any assumptions in analyses nor to include any errors which may be caused in laboratory tests. Furthermore, this method is very simple and considerably good in discrimination between two categories of liquefaction and no liquefaction, and is applicable to this branches of problems.

2. BRIEF REVIEW

For discussing whether soil deposits under investigation will liquefy during an assumed earthquake, it is desirable to establish a criterion which can exactly predict the occurrence of liquefaction. The predictive methods proposed for recent years are as follows:

(1) The method of using past experiences

The effect of soil gradation, relative density and effective overburden pressure on liquefaction potential was studied based on the past experiences of liquefaction occurrence. A criterion chart for liquefaction potential was given by Fig. 1,[1),2),3)] which is simple and convenient for practical use. However, because of no explicit description of ground motion, it may be difficult to predict liquefaction occurrence during any particular earthquake.

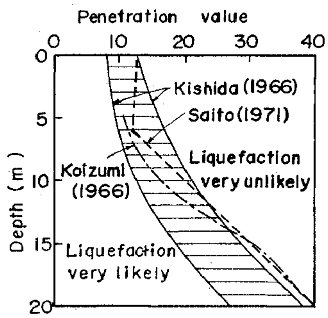

Fig. 1 Liquefaction criterion chart.

(2) The method of using standard blasting tests

Florin and Ivanov[4)] proposed a method which provided a measure of resistance to liquefaction from the measurement of settlements of a test soil deposit due to repeated blasting. Though this is very practical, it may not be easy to correlate the amplitude of the ground motion due to blasting with that of earthquake motion likely to be developed at the site.

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(3) The method of using response analyses and laboratory test procedures

In this method proposed by Seed et al.,[5] the prediction of liquefaction occurrence is done by comparing the shear stresses induced within the deposits during an assumed earthquake with the shear stresses to cause liquefaction.

The response analyses are carried out to estimate the shear stresses induced in soil element by assuming shear wave propagating upwards from bedrock, the motion of which is determined by earthquake magnitude and epicentral distance. By means of loboratory soil tests conducted under various confining pressures, the stresses which have to be developed to cause liquefaction in the same number of stress cycles as that in response analyses are determined. Cyclic loading triaxial tests or, in direct means, cyclic loading simple shear tests[6),7] are used for this purpose. Then, by comparing the stresses induced during an earthquake with those to cause liquefaction, the depth where liquefaction occurs may be determined.

The method supplies a means for considering the effects of ground motion during an earthquake, the in-situ soil conditions, the variation of overburden pressure with depth and the level of water table.

Although it is reported that this reasonable method was successful in the analyses of the Niigata[5] and the Santa Barbara[8] earthquakes, a few problems remain unsolved in some assumptions involved in the response analyses and the similarity of soil samples to field soils.

(4) The method of using design acceleration level of ground surface and laboratory test procedures

Earthquake resistant design of structures is often carried out by assuming acceleration level of ground surface during an earthquake. Therefore it is desirable for consistent design to evaluate liquefaction potential using the acceleration level of ground surface.

Seed and Idriss[9] investigated liquefaction occurrence and maximum acceleration of ground surface in many past earthquakes, and proposed a method of using design acceleration level of ground surface and laboratory test procedures, described in some detail later in 7, to estimate critical penetration value which might be likely to occur liquefaction at the design acceleration level.

3. LIQUEFACTION POTENTIAL

The present study is concerned with statistical approaches to predict whether any site under investigation is likely to liquefy during an assumed earthquake. Such approaches may be carried out by statistically treating known data of soil and earthquake conditions in past experiences. To conduct discrimination among some categories, it may be possible to use statistical analyses, for example, quantification theory, linear discriminant function[10] and so on. In this paper, the linear discriminant function is employed because of its simplicity and applicability to such problems.

The predictive procedure with the linear discriminant function was first introduced in the previous paper[11] and let it be reviewed in detail here. The comparison of shear stresses induced during an earthquake and those required to cause liquefaction will determine whether liquefaction occurs at any depth within soil deposit. Therefore the factors treated in the statistical method should be related to two stresses in the above.

Liquefaction potential described herein is considered as a measure of susceptibility to liquefaction. It is assumed that the liquefaction potential Z is a linear function of factors x_i ($i=1, 2, \cdots, k$) which are closely related to the conditions of liquefying soil deposits, for example, relative density, earthquake magnitude, epicentral distance, as is explained in detail later.

Thus, the expression:

$$Z=l_1x_1+l_2x_2+\cdots+l_kx_k \qquad (1)$$

is assumed, where l_i ($i=1, 2, \cdots, k$) are coefficients to be properly determined.

A policy of determining l_i is that the function Z should be a good indication of susceptibility to liquefaction based on past experiences. The function Z consists of $Z^{(1)}$ and $Z^{(2)}$; $Z^{(1)}$ is for the group of liquefaction and $Z^{(2)}$ is for the group of no liquefaction. For good discrimination of two groups from each other, it may be necessitated that (i) the mean values of $Z^{(1)}$ and $Z^{(2)}$ are separated from each other as far as possible and (ii) the variance of Z in each group is as small as possible.

The above conditions are satisfied if a function:

$$G=\frac{(\bar{Z}^{(1)}-\bar{Z}^{(2)})^2}{\sum_{p=1}^{2}\sum_{j=1}^{n_p}(Z_j^{(p)}-\bar{Z}^{(p)})^2} \qquad (2)$$

takes its maximum, where

$\bar{Z}^{(p)}$: mean value of $Z^{(p)}$ $(p=1, 2)$,
$Z_j^{(p)}$: value of Z for j-th case belonging to group p $(p=1, 2)$,
n_p : number of cases belonging to group p $(p=1, 2)$.

From eq. (1),

$$\bar{Z}^{(1)}-\bar{Z}^{(2)}=(l_1\bar{x}_1^{(1)}+l_2\bar{x}_2^{(1)}+\cdots+l_k\bar{x}_k^{(1)})-(l_1\bar{x}_1^{(2)}+l_2\bar{x}_2^{(2)}+\cdots+l_k\bar{x}_k^{(2)})=l_1(\bar{x}_1^{(1)}-\bar{x}_1^{(2)})+l_2(\bar{x}_2^{(1)}-\bar{x}_2^{(2)})+\cdots+l_k(\bar{x}_k^{(1)}-\bar{x}_k^{(2)}), \quad \cdots\cdots(3)$$

where $\bar{x}_i^{(p)}$ is the mean value of i-th factor for all cases belonging to the group p $(p=1, 2)$.

In the same manner,

$$Z_j^{(p)}-\bar{Z}^{(p)}=l_1(x_{1j}^{(p)}-\bar{x}_1^{(p)})+l_2(x_{2j}^{(p)}-\bar{x}_2^{(p)})+\cdots+l_k(x_{kj}^{(p)}-\bar{x}_k^{(p)}), \quad \cdots(4)$$

where $x_{ij}^{(p)}$ is the value of i-th factor for j-th case belonging to the group p $(p=1, 2)$.

The numerator of eq. (2), the square of so-called Mahalanobis' generalized distance, is transformed into the expression:

$$(\bar{Z}^{(1)}-\bar{Z}^{(2)})^2=\left\{\sum_{n=1}^{k} l_n(\bar{x}_n^{(1)}-\bar{x}_n^{(2)})\right\}^2=\left\{\sum_{n=1}^{k} l_n d_n\right\}^2=\sum_{m=1}^{k}\sum_{n=1}^{k} l_m l_n d_m d_n, \quad \cdots\cdots(5)$$

where

$$d_n=\bar{x}_n^{(1)}-\bar{x}_n^{(2)} \quad (n=1, 2, \cdots, k). \quad \cdots\cdots(6)$$

Also, the denominator of eq. (2) can be rewritten as

$$\sum_{p=1}^{2}\sum_{j=1}^{n_p}(Z_j^{(p)}-\bar{Z}^{(p)})^2=\sum_{p=1}^{2}\sum_{j=1}^{n_p}\left\{l_1(x_{1j}^{(p)}-\bar{x}_1^{(p)})+l_2(x_{2j}^{(p)}-\bar{x}_2^{(p)})+\cdots+l_k(x_{kj}^{(p)}-\bar{x}_k^{(p)})\right\}^2$$
$$=\sum_{p=1}^{2}\sum_{j=1}^{n_p}\left\{\sum_{m=1}^{k}\sum_{n=1}^{k} l_m l_n(x_{mj}^{(p)}-\bar{x}_m^{(p)})(x_{nj}^{(p)}-\bar{x}_n^{(p)})\right\}=\sum_{m=1}^{k}\sum_{n=1}^{k} l_m l_n\left\{\sum_{p=1}^{2}\sum_{j=1}^{n_p}(x_{mj}^{(p)}-\bar{x}_m^{(p)})(x_{nj}^{(p)}-\bar{x}_n^{(p)})\right\}=\sum_{m=1}^{k}\sum_{n=1}^{k} l_m l_n S_{mn}, \quad \cdots(7)$$

where

$$S_{mn}=\sum_{p=1}^{2}\sum_{j=1}^{n_p}(x_{mj}^{(p)}-\bar{x}_m^{(p)})(x_{nj}^{(p)}-\bar{x}_n^{(p)}). \cdots(8)$$

Substitution of eqs. (5) and (7) into eq. (2) leads to

$$G=\frac{\sum_{m=1}^{k}\sum_{n=1}^{k} l_m l_n d_m d_n}{\sum_{m=1}^{k}\sum_{n=1}^{k} l_m l_n S_{mn}}=\frac{A}{B}, \quad \cdots\cdots(9)$$

where A and B denote the numerator and the denominator of the middle term of eq. (9), respectively, and it is assumed that B is not zero.

To determine l_i which make the function G maximum, the partial derivatives of G with respect to l_i are put zero, i.e.,

$$\frac{\partial G}{\partial l_i}=\frac{B\dfrac{\partial A}{\partial l_i}-A\dfrac{\partial B}{\partial l_i}}{B^2}=0 \quad (i=1, 2, \cdots, k) \quad \cdots\cdots(10)$$

or

$$\frac{\partial B}{\partial l_i}=\frac{1}{G}\frac{\partial A}{\partial l_i} \quad (i=1, 2, \cdots, k). \quad \cdots\cdots(11)$$

It is easy to derive the relations:

$$\left.\begin{aligned}\frac{\partial B}{\partial l_i}&=2(l_1S_{i1}+l_2S_{i2}+\cdots+l_kS_{ik})=2\sum_{n=1}^{k} l_nS_{in} \quad (i=1, 2, \cdots, k),\\ \frac{\partial A}{\partial l_i}&=2(l_1d_1+l_2d_2+\cdots+l_kd_k)d_i=2d_i\sum_{n=1}^{k} l_nd_n \quad (i=1, 2, \cdots, k).\end{aligned}\right\} \quad \cdots\cdots(11)$$

Hence, eq. (11) becomes

$$l_1S_{i1}+l_2S_{i2}+\cdots+l_kS_{ik}=\sum_{n=1}^{k} l_nS_{in}=\frac{d_i}{G}\sum_{n=1}^{k} l_nd_n=cd_i \quad (i=1, 2, \cdots, k), \quad \cdots\cdots(13)$$

where

$$c=\frac{1}{G}(l_1d_1+l_2d_2+\cdots+l_kd_k)=\frac{1}{G}\sum_{n=1}^{k} l_nd_n. \quad \cdots\cdots(14)$$

Since eq. (13) gives a set of solutions l_i for any value of c, the following simultaneous equations for $c=1$ are adopted for the determination of l_i.

$$\left.\begin{aligned}S_{11}l_1+S_{12}l_2+\cdots+S_{1k}l_k&=d_1,\\ S_{21}l_1+S_{22}l_2+\cdots+S_{2k}l_k&=d_2,\\ &\vdots\\ S_{k1}l_1+S_{k2}l_2+\cdots+S_{kk}l_k&=d_k.\end{aligned}\right\} \cdots\cdots(15)$$

4. COMPUTATION OF LIQUEFACTION POTENTIAL-PROCEDURE 1

The most important point in calculating liquefaction potential may be in the selection of basic factors x_i and the input values for them. The

Table 1 Summary of data (Seed and Idriss).

Case number	Earthquake	Date	Site	Magnitude	Epicentral distance (km)[a]	Soil type	Depth of water table (m)[a]	Depth under study (m)[a]	Penetration value at depth under study	Max. ground surface acceleration (g)	Duration of shaking (sec)	Liquefaction
1	Niigata	1802	Niigata	6.6	39	Sand	1.0	6.0	6	0.12	20	No
2	Niigata	1802	Niigata	6.6	39	Sand	1.0	6.0	12	0.12	20	No
3	Niigata	1887	Niigata	6.1	47	Sand	1.0	6.0	6	0.08	12	No
4	Niigata	1887	Niigata	6.1	47	Sand	1.0	6.0	12	0.08	12	No
5	Mino-Owari	1891	Ogaki	8.4	32	Sand	1.0	5.0[b]	4[b]	0.35	75	Yes
6	Mino-Owari	1891	Ginan West	8.4	32	Sand	2.0	9.0	10	0.35	75	Yes
7	Mino-Owari	1891	Unuma	8.4	32	Sand & gravel	2.0	7.5	19	0.35	75	No
8	Mino-Owari	1891	Ogase Pond	8.4	32	Sand	2.5	4.0[b]	10[b]	0.35	75	Yes
9	Santa Barbara	1925	Shefield Dam	6.3	11	Sand	4.5	7.5	(3)	0.20	15	Yes
10	El Centro	1940	Brawley	7.0	8	Sand	4.5	4.5	(9)	0.25	30	Yes
11	El Centro	1940	All-Am. Canal	7.0	8	Sand	6.0	7.5	(4)	0.25	30	Yes
12	El Centro	1940	Solfatara Canal	7.0	8	Sand	1.5	6.0	(1)	0.25	30	Yes
13	Tonankai	1944	Komei	8.3	161	Sand	1.5	4.0	4	0.08	70	Yes
14	Tonankai	1944	Meiko St.	8.3	161	Silt & sand	0.5	2.5	1	0.08	70	Yes
15	Fukui	1948	Takaya	7.2	6	Sand	3.5	4.0[b]	12[b]	0.30	30	Yes
16	Fukui	1948	Takaya	7.2	6	Sand	1.0	7.0	28	0.30	30	No
17	Fukui	1948	Shonenji Temple	7.2	6	Sand	1.0	3.0	3	0.30	30	Yes
18	Fukui	1948	Agr. Union	7.2	6	Sand & silt	1.0	6.0	5	0.30	30	Yes
19	San Francisco	1957	Lake Merced	5.5	6	Sand	2.5	3.0	7	0.18	18	Yes
20	Chile	1960	Puerto Montt	8.4	113	Sand	3.5	4.5	6	0.15	75	Yes
21	Chile	1960	Puerto Montt	8.4	113	Sand	3.5	4.5	8	0.15	75	Yes
22	Chile	1960	Puerto Montt	8.4	113	Sand	3.5	6.0	18	0.15	75	No
23	Niigata	1964	Niigata	7.5	52	Sand	1.0	6.0	6	0.16	40	Yes
24	Niigata	1964	Niigata	7.5	52	Sand	1.0	7.5	8[b]	0.16	40	Yes
25	Niigata	1964	Niigata	7.5	52	Sand	1.0	6.0	12	0.16	40	No
26	Niigata	1964	Niigata	7.5	52	Sand	3.5	7.5	6	0.16	40	No
27	Alaska	1964	Snow River	8.3	97	Sand	0.0	6.0	5	0.15	180	Yes
28	Alaska	1964	Snow River	8.3	97	Sand	2.5	6.0	5	0.15	180	Yes
29	Alaska	1964	Quartz Creek	8.3	113	Sandy gravel	0.0	7.5	35	0.12	180	No
30	Alaska	1964	Scott Glocier	8.3	89	Sand	0.0	6.0	10	0.16	180	Yes
31	Alaska	1964	Valdez	8.3	56	Sand	1.5	6.0	13	0.25	180	Yes
32	Tokachi-Oki	1968	Hachinohe	7.8	172	Sand	1.0	3.5	14	0.21	45	No
33	Tokachi-Oki	1968	Hachinohe	7.8	172	Sand	1.0	3.5	6	0.21	45	Yes
34	Tokachi-Oki	1968	Hachinohe	7.8	172	Sand	1.5	3.0	15	0.21	45	No
35	Tokachi-Oki	1968	Hakodate	7.8	283	Sand	1.0	4.5	6	0.18	45	Yes

Note: [a] Changed to meter unit, [b] Revised from original papers, ( ) Estimated from relative density.

basic factors, in the first trial, are employed as follows:

x_1: position of water table below ground surface (m),

x_2: depth under study below ground surface (m),

x_3: penetration value at the depth under study, and

x_4: maximum acceleration of ground surface (g),

and the input values are taken from the basic data in Table 1, summarized by Seed and Idriss.[9]

The solutions l_i ($i=1, 2, 3, 4$) of eq. (15) give the expression of liquefaction potential:

$$Z=x_1-0.28x_2-1.09x_3+0.37x_4 \ . \quad \cdots\cdots\cdots(16)$$

The vulues of Z calculated by eq. (16) for the 35 cases in Table 1 are plotted in Fig. 2, in which the open circles and the solid circles show the cases of liquefaction and no liquefaction in past experiences, respectively.

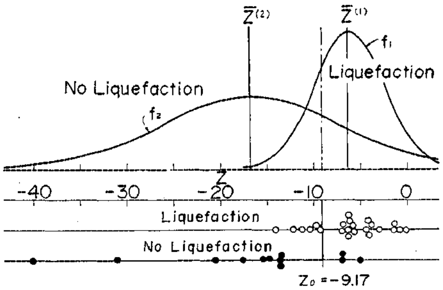

Fig. 2 Results of the computation—Procedure 1.

The curves f_1 and f_2 in Fig. 2 are drawn by normal curve approximation of the plots for liquefaction and no liquefaction, respectively.

For the prediction of liquefaction occurrence, it is assumed that a case with any value of Z liquefies if Z is larger than Z_0, a critical liquefaction potential to be determined in the following, and does not liquefy if Z is smaller than Z_0. Then, the ratio of successful discrimination P_r, defined as the ratio of the number of cases predicted correctly to the number of all cases, is given by

$$(P_r)_{\text{liq.}}=\frac{\int_{Z_0}^{\infty} f_1(Z)\,dZ}{\int_{-\infty}^{\infty} f_1(Z)\,dZ} \quad \cdots\cdots\cdots\cdots\cdots\cdots(17)$$

for the group of liquefaction, and

$$(P_r)_{\text{no liq.}}=\frac{\int_{-\infty}^{Z_0} f_2(Z)\,dZ}{\int_{-\infty}^{\infty} f_2(Z)\,dZ} \quad \cdots\cdots\cdots\cdots\cdots\cdots(18)$$

for the group of no liquefaction.

If the value of Z_0 is taken so as to give an equal ratio of successful discrimination to each group, i.e.,

$$(P_r)_{\text{liq.}}=(P_r)_{\text{no liq.}} \ , \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(19)$$

then $Z_0=-9.17$ is obtained for the data in Fig. 2. The ratio of successful discrimination with this value of Z_0 is calculated as $P_r=78.5\%$.

The results of the computation in the above are somewhat unsatisfactory for a predictive method, probably because the factors x_i ($i=1, 2, 3, 4$) have different dimensions and variances from one another.

5. COMPUTATION OF LIQUEFACTION POTENTIAL WITH MODIFIED FACTORS—PROCEDURE 2

To improve the computation, a new set of variables y_i ($i=1, 2, \cdots, k$) instead of x_i ($i=1, 2, \cdots, k$) are introduced to define a function:

$$L=l_1'y_1+l_2'y_2+\cdots+l_k'y_k \ . \quad \cdots\cdots\cdots\cdots(20)$$

The variables y_i are assumed to have its j-th component given by the relation:

$$y_{ij}=\frac{x_{ij}-\bar{x}_i}{s_i} \ , \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(21)$$

where
y_{ij}: j-th value of y_i ($i=1, 2, \cdots, k$)
x_{ij}: j-th value of x_i ($i=1, 2, \cdots, k$),
$\bar{x}_i$: mean value of x_i ($i=1, 2, \cdots, k$) and
s_i : standard deviation of x_i ($i=1, 2, \cdots, k$) or $s_i^2=\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_i)^2/(n_i-1)$, where n_i is number of cases.

The determination of the coefficients l_i' ($i=1, 2, \cdots, k$) can be conducted by the same procedure as that of l_i ($i=1, 2, \cdots, k$).

(1) The expression with four factors

Four factors y_i ($i=1, 2, 3, 4$) are taken from x_i ($i=1, 2, 3, 4$) which are employed in the preceding section 4. The results give the expression:

Table 2 Summary of factors.

	Factors	$\bar{x}_i$	s_i
x_1	Position of water table below ground surface (m)	1.84	1.42
x_2	Depth under study below ground surface (m)	5.50	1.61
x_3	Penetration value at the depth under study	9.40	7.11
x_4	Maximum acceleration of ground surface (g)	0.20	0.08
		for the data in Table 1	

$$L = y_1 + 4.05y_2 - 31.93y_3 + 22.14y_4 \cdots\cdots\cdots(22)$$

and the mean values and standard deviations of x_i ($i=1, 2, 3, 4$) are shown in Table 2.

The values of L calculated from eq. (22) are plotted in Fig. 3, from which $L_0 = -2.36$ and $P_r = 80.5\%$.

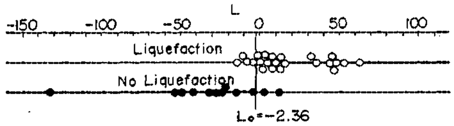

Fig. 3 Results of the computation—the expression with four factors of Procedure 2.

(2) The expression with six factors

In the next trial, six factors y_i ($i=1, 2, \cdots, 6$) are taken from x_i ($i=1, 2, \cdots, 6$) listed in Table 3. The results are as follows:

$$\left.\begin{aligned} L &= y_1 - 1.15y_2 - 0.14y_3 - 1.30y_4 \\ &\quad -4.39y_5 + 5.37y_6\,, \\ L_0 &= -2.46\,,\quad P_r = 83.4\%\,, \end{aligned}\right\} \cdots\cdots(23)$$

and the discrimination chart is given by Fig. 4.

Table 3 Summary of factors.

	Factors	$\bar{x}_i$	s_i
x_1	Magnitude of earthquake	7.57	0.81
x_2	Epicentral distance (km)	71.00	65.99
x_3	Position of water table below ground surface (m)	1.84	1.42
x_4	Depth under study below ground surface (m)	5.50	1.61
x_5	Penetration value at the depth under study	9.40	7.11
x_6	Duration of ground motion (sec)	63.20	53.53
		for the data in Table 1	

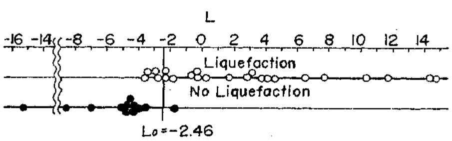

Fig. 4 Results of the computation—the expression with six factors of Procedure 2.

From the computation of Procedures 1 and 2 in the above, some expressions of liquefaction potential and the ratios of successful discrimination by them have been presented. It is noted from these results that the ratio of successful discrimination is considered to depend on a combination of factors employed and eq. (23) with six modified factors gives the best ratio of suc-

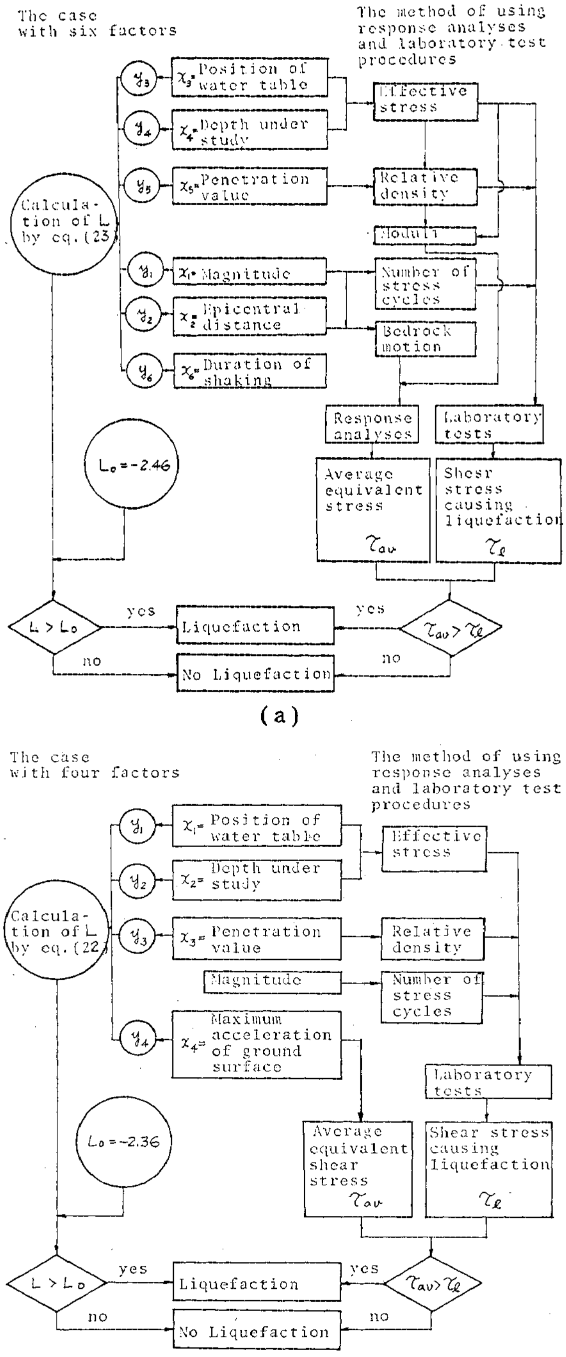

Fig. 5 Comparison of flow charts; (a) the case with six factors and 2.(3), (b) the case with four factors and 2.(4).

cessful discrimination, as far as the basic data given in Table 1 and the computations conductedd herein are concerned.

The comparison of the flow charts of the present method with those of the methods (3) and (4) in 2 is shown in Fig. 5 (a) and (b), respectively.

6. APPLICATION TO OTHER CASES

(1) The applicability of eqs. (22) and (23) to other data in past experiences

Fig. 6 is the discrimination chart for the data in Table 4, summarized by Whitman[12]. It is seen from this figure that the ratios of successful discrimination calculated by the number of individual cases are 66.6% (6/9) by eq. (22) and 88.9 % (8/9) by eq. (23). This result also proves that eq. (23) is better one.

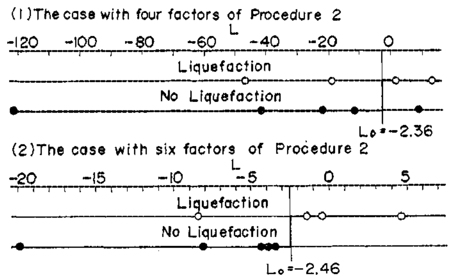

Fig. 6 The discrimination chart of the data in Table 4.

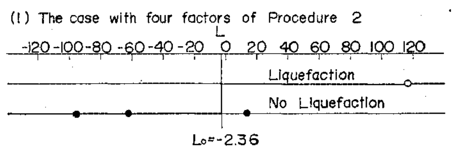

Fig. 7 The discrimination chart of the data in Table 5.

Table 5 is the data summarized by Christian and Swiger[13]. Although the data are short of some factors, only eq. (22) is available to the application. The discrimination chart is given by Fig. 7, from which the ratio of successful discrimination is 75% (3/4).

(2) The applicability of eq. (22) to the evaluation of critical penetration value

The liquefaction potential evaluation charts for sands were proposed by Seed et al.[9] as shown in Fig. 8. The parameters used there are depth below ground surface, position of ground water table, maximum acceleration level of ground sur-

Table 4 Summary of data (Whitman)

Case number	Earthquake	Date	Site	Magnitude	Epicentral distance (km)[a]	Soil type	Depth of water table (m)[a]	Depth under study (m)[a]	Penetration value at depth under study	Max. ground surface accleeration (g)	Duration of shaking (sec)	Liquefaction
1	Chile	1960	Conception	8.4	113	Fluvial	3.5	7.0	10	0.15	20	No
2	Chile	1960	Huachipato	8.4	113	Beach	3.5	8.0	35	0.15	20	No
3	Niigata	1964	Niigata (zone C)	7.5	52	Fluvial	1.0	6.0	7	0.16	20	Yes
4	Niigata	1964	Niigata (with fill)	7.5	52	Fluvial	3.5	8.5	7	0.16	20	No
5	Niigata	1964	Niigata (zone B)	7.5	52	Older-Fluvial	1.0	6.0	12	0.16	20	No
6	Alaska	1964	River Bridges	8.3	97	Fluvial	0.0	4.5	16	0.15	120	Yes
7	Tokachi-Oki	1968	Hachinohe	7.8	172	Beach	1.5	3.5	18	0.21	20	No
8	Tokachi-Oki	1968	Hachinohe	7.8	172	Fill	1.5	3.5	6	0.21	20	Yes
9	Tokachi-Oki	1968	Hakodate	7.8	283	Fill	1.0	3.5	13	0.19	20	Yes

Note: [a] Changed to meter unit.

Table 5 Summary of data (Christian and Swiger)

Case number	Earthquake	Date	Site	Magnitude	Epicentral distance (km)[a]	Soil type	Depth of water table (m)[a]	Depth under study (m)[a]	Penetration value at depth under study	Max. ground surface acceleration (g)	Duration of shaking (sec)	Liquefaction
1	Chile	1960	Huachipano	—	—	—	3.0	9.0	(35)	0.25	—	No
2	Chile	1960	Huachipano	—	—	—	3.0	22.5	(35)	0.25	—	No
3	San Francisco	1971	Jensen Plant	—	—	—	16.5	16.5	(6)	0.35	—	Yes
4	Kern County	1952	Kern Station	—	—	—	3.5	6.0	(9)	0.25	—	No

Note: [a] Changed to meter unit, ( ) Estimated from relative density, Nos. 1, 2 and 3 from G. Castro and 4 from Swiger.

face and critical penetration value. This corresponds to eq. (22) with four factors.

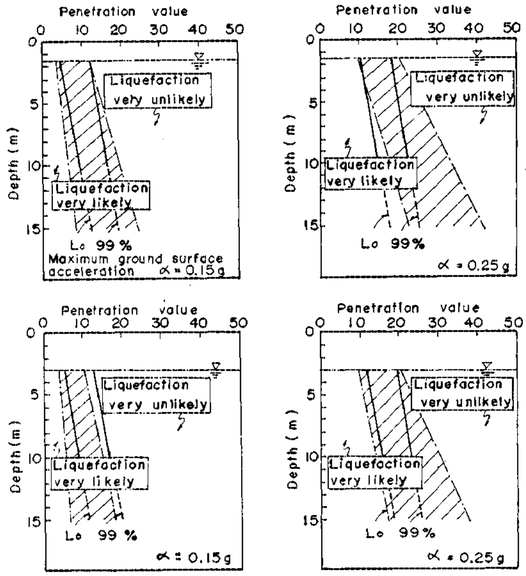

Fig. 8 Liquefaction potential evaluation charts.

Two solid lines calculated from eq. (22) are drawn in Fig. 8. The L_0 line shows the critical condition and the 99% line is the line of no liquefaction with probability of 99% which are calculated from the data in Table 1. The shaded zones by Seed et al.[9] are generally a little wider than the zones between the L_0 and 99% lines. The ratio of successful discrimination in the statistical method can be selected in accordance with the importance of projects planed at any site under investigation.

7. EXAMPLE OF EVALUATION OF LIQUEFACTION POTENTIAL

Let us discuss whether a sandy deposit shown in Fig. 9 will liquefy or not during an earthquake of 0.2 g, using a customarily used method and eq. (22).

(1) Evaluation of liquefaction potential by a customarily used method

To discuss the liquefaction potential of the given deposit, we use the method (2.(4)) of using design acceleration level and laborotory test procedures. This analysis is carried out in the following steps.

i) Determination of shear stresses developed in soil deposit during an earthquake

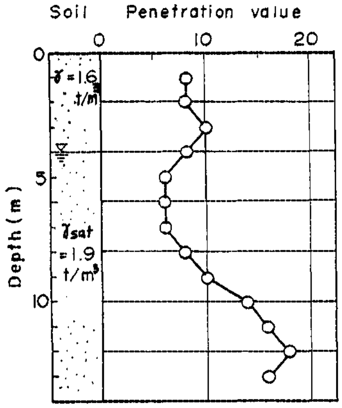

Fig. 9 Example soil profile.

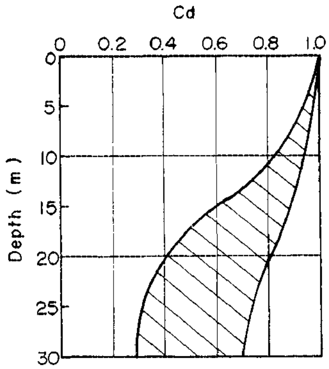

Fig. 10 Range of value C_d for different soil profiles (Seed and Idriss).

The maximum shear stress $\tau_{\max}$ in soil element at a depth below ground surface is given by the equation $\tau_{\max}=\frac{\gamma z}{g}\cdot\alpha\cdot C_d$, where α is maximum acceleration of ground surface, g acceleration by gravity, γ unit weight of soil, C_d stress reduction coefficient. The values of C_d for a wide variety of earthquake motions and soil condition are given in Fig. 10. Since the actual time-history of shear stress during an earthquake has an irregular form, the average equivalent uniform shear stress τ_{av} is proposed to be about 65% of the maximum stress $\tau_{\max}$, i.e.,

$$\tau_{av}=0.65\,\tau_{\max}=0.65\cdot\frac{\gamma z}{g}\cdot\alpha\cdot C_d\,. \quad\cdots\cdots\cdots(24)$$

ii) Determination of cyclic shear stresses causing liquefaction

If shear stress and initial effective overburden pressure on a horizontal plane of a soil element are denoted by τ_l and σ_v', respectively, the stress ratio τ_l/σ_v' causing liquefaction is obtained from corresponding triaxial test results, as follows:

$$\frac{\tau_l}{\sigma_v'}=\left(\frac{\sigma_{dp}}{2\sigma_0'}\right)_{50}\cdot\frac{D_r}{50}\cdot C_r\,, \quad\cdots\cdots\cdots\cdots\cdots\cdots(25)$$

where σ_{dp} is the cyclic deviator stress and σ_0' is the initial ambient pressure under which the sample was consolidated in the triaxial test. D_r is relative density in percentage and C_r is the correction factor shown in Fig. 11.

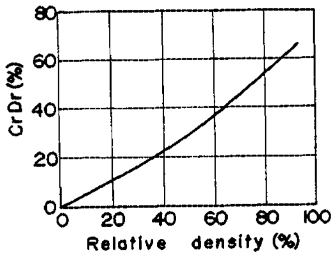

Fig. 11 Relationship between relative density and the correction factor (Seed and Idriss).

iii) Prediction of liquefaction occurrence

After the calculation of τ_{av} and τ_l by eqs. (24) and (25), it is predicted that the soil deposit liquefies at depth where the condition $\tau_{av}>\tau_l$ is satisfied.

In solving the example problem, we use relative density-penetration value correlation in Fig. 12, proposed by Gibbs and Holtz[14]. The term

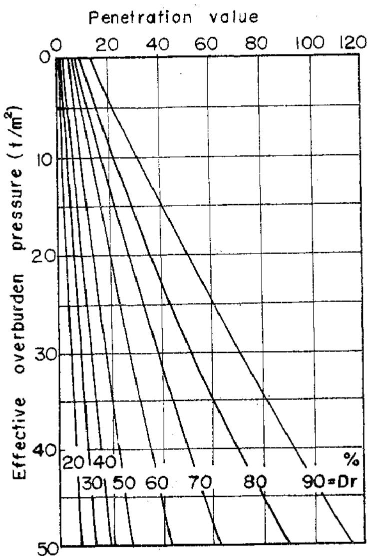

Fig. 12 Relationship between penetration value, relative density and effective overburden pressure (Gibbs and Holtz).

$\left(\frac{\sigma_{dp}}{2\sigma_0'}\right)_{50}$ depends on grain size of soil and the number of cycles to cause liquefaction. If the grain size of 50% passing is 0.4 mm and the number of cycles is 20 are assumed in this problem, $\left(\frac{\sigma_{dp}}{2\sigma_0'}\right)_{50} \fallingdotseq 0.23$ is obtained from previous data[9),15)].

The results of the calculation is shown in Fig. 13. It is noted from this figure that liquefaction may occur at the depth between 4.2 m and 10.5 m below the ground surface.

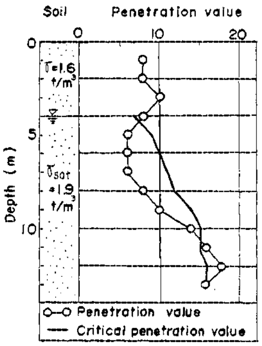

Fig. 13 Results of liquefaction potential evaluation by the method 2.(4).

(2) Evaluation of liquefaction potential by eq. (22)

For this example, eq. (22) with four factors can be used. The results are shown as the L_0 and the 99% lines in Fig. 14. The L_0 line shows the critical condition with which 80.5% of the cases in Table 1 can be discriminated successfully. It is noted from the L_0 line that liquefaction may occur at the depth between 4.0 m and 9.5 m below the ground surface which is a similar result to that in the above (1). The 99% line shows the boundary by which no liquefaction is predicted with the probability of 99%, as far as the basic data are concerned.

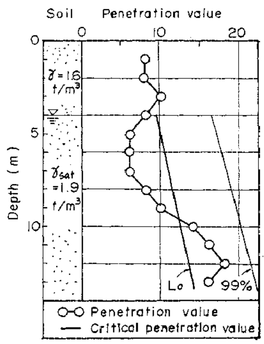

Fig. 14 Results of liquefaction potential evaluation by eq. (22).

8. CONCLUSIONS

A statistical analysis of known data for earthquakes during which liquefaction had occurred was carried out to establish a predictive method for the occurrence of liquefaction of sandy deposit. Liquefaction potential was defined herein as a function of some factors related closely to liquefaction process, and some combinations of such factors were employed to find the best one.

As the results, it is made clear that eq. (23) with six modified factors can well discriminate the two groups of liquefaction and no liquefaction in past experiences. Because of simplicity and convenience in calculation, eq. (23) can be used for a predictive method. Though less successful in discrimination, eq. (22) with four modified factors is also practical and more convenient to discuss the susceptibility to liquefaction if a design acceleration level of ground surface is given.

However, the methods introduced in this paper may have such problems as in the following. The expression of liquefaction potential varies with the combination of basic factors and input data for them. Accordingly, eqs. (22) and (23) naturally change, if another combination of basic factors is employed or some other data are added to the basic data in Table 1. As for the basic factors, the method of linear discriminant function easily allows the inclusion of the other factors, such as gradation and relative density of soils, provided that the sufficient data are available. Since eqs. (22) and (23) lack the term of gradation, they should be applied to sandy soils only, as seen in the basic data.

There may be another problem as to whether or not the basic data are suitable for statistical analysis. The data are not random samplings from possible cases existing in nature. It is very important to accumulate more informations to add to the present limited data. In this standpoint, the method in this paper is a temporary one.

It is considered that a shortcoming of statistical method is in its independency of the mechanism of liquefying process. However, the method using only basic and objective factors is, first of all, simple and practical, and also makes it possible to avoid theoretical assumptions and experimental errors which may exist in the study of the mechanism.

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