論文

Development of an Effective Numerical Model for Fatigue Analysis of RC Bridge Slabs

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This study presents an effective numerical method to predict the fatigue behaviors of RC bridge slab due to moving wheel load. The bridging stress degradation concept is employed as a fundamental root of the propagation of cracks that prompts the fatigue failure of RC bridge slab. The smeared model approach is adopted in the numerical model established and finite element method (FEM) is used in order to solve the model. The fatigue behaviors of RC slab, centre displacement evolution and propagation of cracks, predicted by the numerical method are compared with those of experimental results. The numerical results show a good agreement with the experimental results, and this numerical method can be used for the fatigue analysis of RC slabs under moving wheel load.

Keywords：RC slab, fatigue, bridging stress degradation, moving wheel load

1. Introduction

Reinforced concrete (RC) bridge slab is amongst the most critical part in bridge susceptible to fatigue failure since it directly sustains repetitive moving wheel loads. Thus, it is very important to be able to accurately evaluate the load-carrying capacity and fatigue behaviors of RC bridge slab.

In the past, many researchers have performed experiments to examine the fatigue behaviors of RC slabs. Recently, RC structural members subjected to repetitive loading have also been numerically analysed to predict the fatigue life. Ueda et al. used finite element method (FEM) to analyze the fatigue characteristics of steel-concrete sandwich beams [1].

RC beams without shear reinforcement were analysed and the fatigue crack propagation concept was proposed to predict the fatigue flexural performances of RC beams [2]. A study on a numerical fatigue simlution of RC slabs under moving load was carried out and a direct path-integral method with fatigue constitutive models for concrete in tension and compression was used [3].

For the first time, the bridging stress degradation concept was introduced as a fundamental root of fatigue crack propogation in RC and fiber reinforced concrete (FRC) beams [4].

The bridging stress degradation was employed in fatigue analysis of RC slabs reinforced with plain bars. The model captured the cracking pattern of RC slab and strain in plain reinforcement bars [5].

This study presents a numerical model to predict the fatigue characteristics of RC bridge slab subjected to repetitive loading. The experimental study by Civil Engineering Research Institute (CERI) for Cold Region is used to verify the analytical results. The numerical model of RC slab is established using MSC/MARC software by smeared model methodology and FEM is used to get the solution of the model. The bridging stress degradation concept is considered as a major mechanism for the propagations of cracks in RC slab. Fatigue life, centre displacement evolution and propogation of cracks are presented and compared with experimental ones in this study. The numerical results show satisfactory agreement with the experimental results.

2. Methodology

2.1 Concrete model

Nonlinear behavior of concrete both in tension and compression is used as shown in Table 1. The bridging stress degradation concept, the most important part in this current fatigue analysis, is introduced to represent the cracking behavior of concrete.

Table 1 Nonlinear constitutive laws of concrete [6]

Compression		Tension*	
$0 \geq \varepsilon \geq \varepsilon_m$	$\sigma = f_c \frac{\varepsilon}{\varepsilon_m}\left(2 - \frac{\varepsilon}{\varepsilon_m}\right)$	$\varepsilon_t \geq \varepsilon \geq 0$	$\sigma = E_c \varepsilon$
$\varepsilon_m \geq \varepsilon \geq \varepsilon_u$	$\sigma = f_c \frac{\varepsilon_u - \varepsilon}{\varepsilon_u - \varepsilon_m}$	$\varepsilon \geq \varepsilon_t$	$\sigma = f_t \left(\frac{\varepsilon_t}{\varepsilon}\right)^{0.4}$

E_c = modulus of elasticity of concrete,

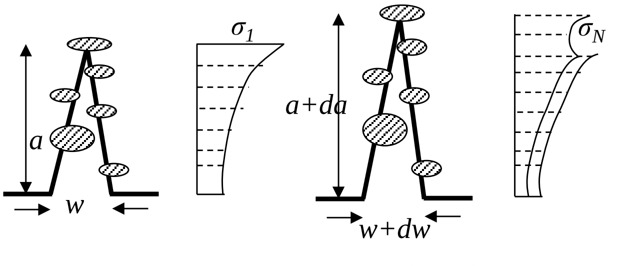
f_c' = concrete compressive strength,

$\varepsilon_m = f_c/2E_c$ =concrete strain corresponding of f_c'

f_t = tensile strength and $\varepsilon_t = f_t/E_c$ = strain at tensile strength

*The multiplier 0.4 can be used for the concrete with deformed bar

In bridging stress degradation concept, a crack begins with length (a) and width (w) as displayed in Figure 1(a). The crack goes through a process of opening and closing which causes in the reduction of bridging stress in repetitive moving loads. Thus, the existing crack progresses with increased length (da) and increased width (dw) as shown in Figure 1(b). This crack propogation and reduction in bridging stress is defined as bridging stress degradation which depends on maximum tensile strain (ε_t) and number of cycles (N) [4].



1st cycle Nth cycle

Figure 1 Crack propagation due to bridging stress:
(a) first cycle (b) after N cycles

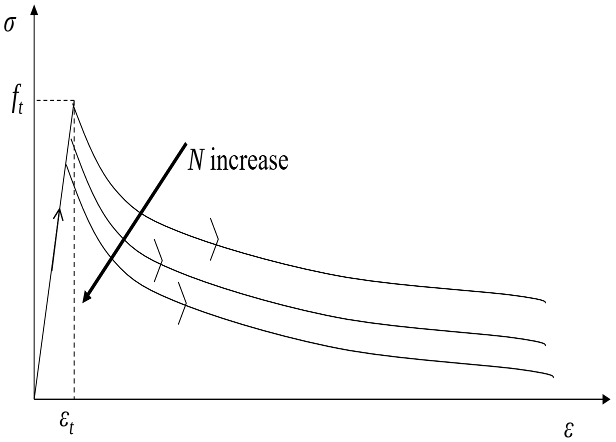


Figure 2 Hysteretic behavior of concrete under cyclic loading

The equation of bridging stress degradation proposed by Zhang et al. [7]:

$$\frac{\sigma_N}{\sigma_1} = 1 - \left(0.08 + 4 \times \delta_{\max}\right)\log\left(N\right)$$

Where δ_{max}, is rewritten in the form of tensile strain (ε_t) by considering unit element length ($\delta_{max} = \varepsilon_t \times 1$).

The stress strain behavior under cyclic loading is displayed in Figure 2. Since loading and reloading paths are same, no plastic strain is occurred.

2.2 Reinforcement bar model

The smeared model is adopted for reinforcement bar in the RC bridge slab. A bilinear curve with yield stress (f_y) is used for representation of stress strain behavior of the reinforcement bar. The Giuffré-Menegotto-Pinto model [8] is utilised to represent the hysteretic behavior of reinforcement bar under repetitive loading as following:

$$\frac{\sigma}{f^*_y} = H\frac{\varepsilon}{\varepsilon^*_y} + \frac{\left(1 - H\right)\frac{\varepsilon}{\varepsilon^*_y}}{\left[1 + \left(\frac{\varepsilon}{\varepsilon^*_y}\right)^R\right]^{1/R}}$$

$$R = R_o - \frac{a_1 \xi_{\max}}{a_2 - \xi_{\max}}$$

Where H is hardening parameter, R_o, and R are transition

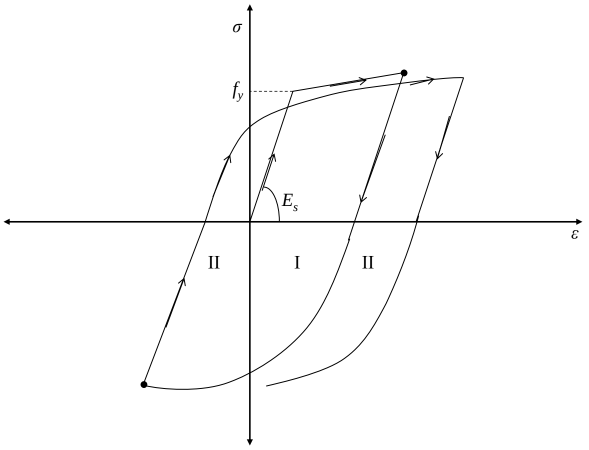


Figure 3 Hysteretic behavior of reinforcement bar under cyclic loading

parameters between elastic hardening for the first and *Nth* cycle ($R_o = 20$), ζ_{max} is the maximum excursion in the plastic range, a_1 and a_2 are the parameters for change of *R* with repetitive load history, equal to 18.5 and 0.00015, respectively. The yield strength reduces as number of cycles increases shown in Figure 3.

2.3 Slab details

The slab used in this study is haunched RC slab with dimensions 3300 × 2650 × 230 mm. In tension zone, the slab is reinforced with D19@125 mm along transverse direction and D16@125 mm along longitudinal direction. Similarly, D19@250 mm along transverse direction and D16@250 mm along longitudinal direction are provided in compression zone.The slab details are shown in Table 2.

Table 2 Slab Details

Slab Dimension (L × W)	3300 × 2650 mm	
Slab thickness	230 mm	
Transverse reinforcement	D19@125mm (Bottom)	
	D19@250mm (Top)	
Longitudinal reinforcement	D16@125mm (Bottom)	
	D16@250mm (Top)	
Concrete	$E_c = 21727$ MPa	$f_c' = 32.4$ MPA
Steel	$E_s = 200000$ MPa	$f_y = 345$ MPA

The moving load zone was of 2000 × 190 mm and the load was applied along the longitudinal direction of the slab. The geometry of slab, reinforcement details and moving load zone is displayed in Figure 4.

The step loading type was adopted as shown in Figure 5. At the start, 150 kN load was applied and then the load was increased up to 240 kN with the interval of 30 kN.

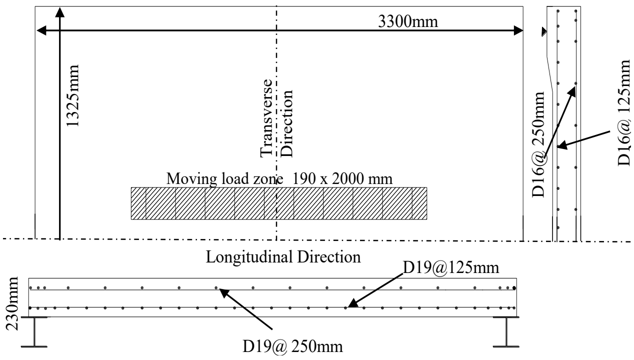


Figure 4 Slab geometry

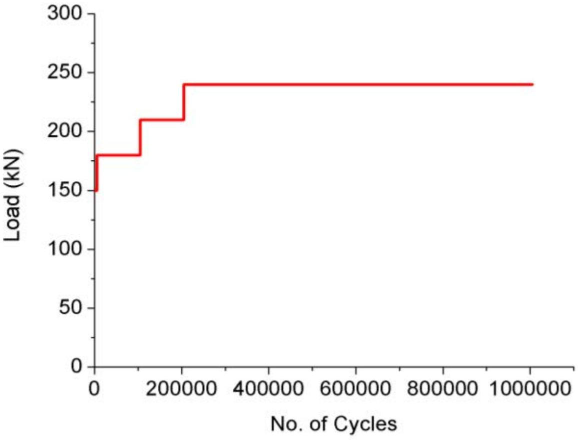
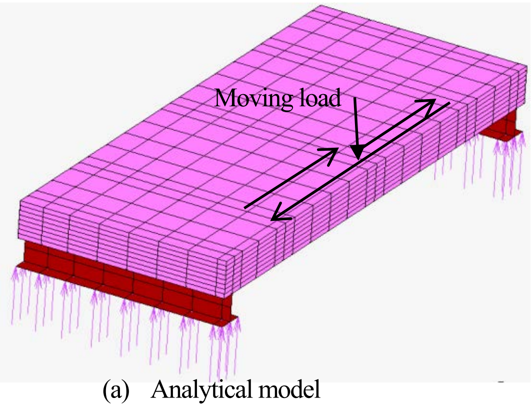


Figure 5 Loading pattern

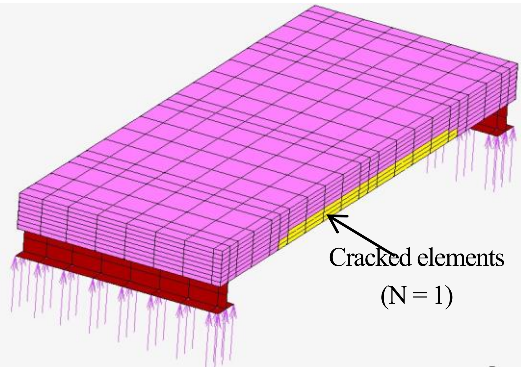
2.4 Analytical procedure

The finite element software MSC/MARC was used to model the slab, a 3D model using solid elements, as displayed in Figure 6 (a). Due to symmetry in loading and boundary conditions, the half slab was analysed. Firstly, the moving load was applied at the central elements of slab. Then those elements were unloaded and the adjacent elements on the right side were loaded. The same procedure was continued until the completion of one cycle of the moving load and one cycle of moving load was considered when the load is again applied on the central elements of slab.

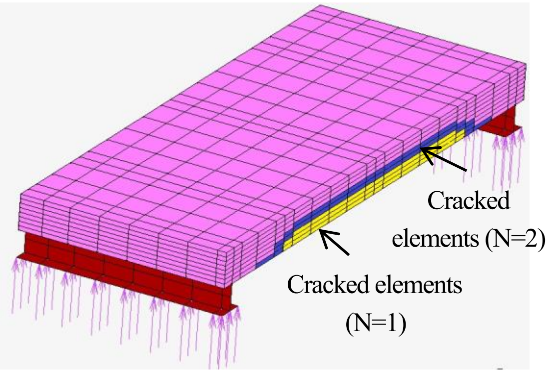
After completion of one cycle of moving load, a number of cracked elements were appeared as shown in Figure 6 (b). According to bridging stress degradation concept, the constitutive law for the cracked elements was modified. It led to the loss in tensile strength in those cracked elements causing a stress concentration at crack tip and new cracked elements were apperead around the stress concentration as displayed in



(a) Analytical model



(b) 1st cycle (N



(c) 2nd cycle (N=2)

Figure 6 Analytical procedure

Figure 6 (c).

The overall RC slab stiffness was decreased with the increase of cycles of moving load due to bridging stress degradation and crack propogation. The procedure was continued until fatigue failure occurred and the numerical results were recorded in each cycle of moving load.

3. Results and discussions

3.1 Center displacement evolution

The displacements at centre of the RC slabs obtained during fatigue analysis at different loading cycles are compared with the displacements observed in the experiment. As number of cycles is increased, the crack elements are increased which result into reduction of RC slab stiffness. The reduction in RC slab stiffness leads to increase in centre displacement of RC slab. The RC slab center displacement evolution shows acceptable agreement with the experimental result as shown in Figure 7.

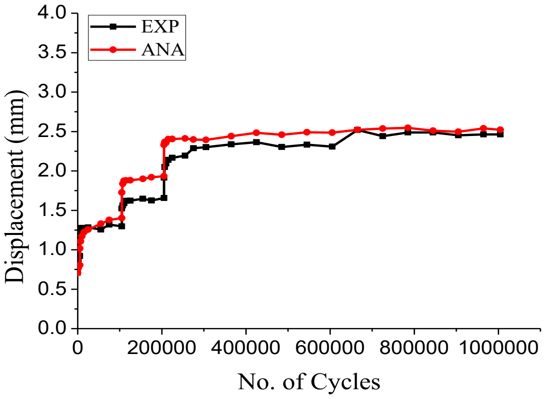


Figure 7 Centre displacement evolution

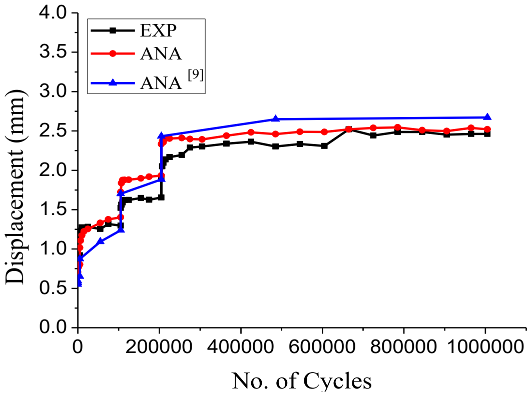


Figure 8 Centre displacement evolution

3.2 Effect of calculation step

Calculation step plays a significant role in numerical model and FEM analysis. In previous study, a comparison is made between experimental and numerical results for centre displacement evolution [9].

In this study, smaller calculation step is used, and centre evolution displacement is compared with experimental results as well as the numerical model results having larger calculation step. In the Figure 8, it can be seen that the numerical model using smaller calculation step is showing better agreement with the experiment work as compared with the model having larger calculation step.

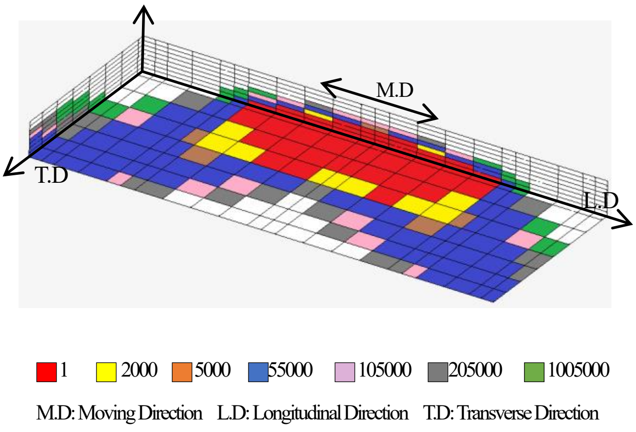


Figure 9 Propogation of cracked elements

3.3 Propagation of crack elements

This numerical model considers primary aspect that the propagation of cracked elements and degradation of bridging stress are the core reasons of fatigue failure. Thus, it is significant to show the propagation of cracked elements.

Uncracked elements are shown by white color. The cracked elements after first cycle of moving load are displayed by red color. Similarly, the cracked elements caused by different number of cycles are indicated with different colors as shown in Figure 9.

From the figure, it can be observed that the cracked elements are spread all over the longitudinal direction of RC slab, wheras the cracked elements are distributed from mid span of RC slab towards transverse direction with less degradation. Since the moving load effect was transferred along the longitudinal direction, the cracked zone is larger in the longitudinal direction as compared to the transverse direction. The cracked volume of RC slab after first cycle of moving load, shown by red color, is 5.75% and the total cracked volume after final loading cycle, 1005000, is 44.2% as presented in Table 3. In the analysis, the average degradation ratio of RC slab is 442.3 mm3/cycles. Furthermore, the cracked elements are propagated in vertical direction up to 0.63 times the thickness of the RC slab.

Table 3 Cracked elements volume

Slab	Cracked Volume (%)		Average degradation ratio (mm^3/cycles)
	1st cycle	Last cycle	
RC Slab	5.75	44.2	442.3

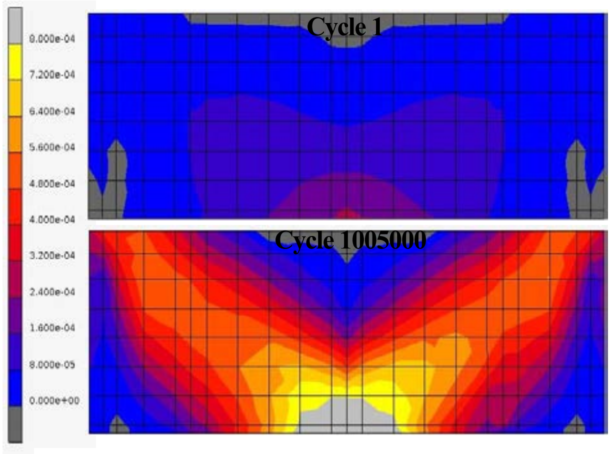


Figure 10 Maximum principal strain distribution

3.4 Maximum principal strain distribution

The comparison of maximum principal strain distribution on bottom surface of RC slab after 1st cycle and last cycle is displayed in Figure 10. The strain is increased in diagonal direction from loading point towards the supporting corner.

The increase in tensile strain in diagonal direction provides justification of propagation of cracked elements in diagonal direction as shown in Figure 9.

4. Conclusions

In this paper, bridging stress degradation concept is used to predict the fatigue behavior of RC slab under a moving load.The propagation of cracked elements due to bridging stress degradation is considered as the main cause of fatigue failure.

The cracked zone is larger in the longitudinal direction as compared to the transverse direction of the RC slab.

This method was verified using the experimental results, and it showed a good agreement. This numerical model using smaller calculation step is showing closer results with the experiment work as compared with the model having larger calculation step. Thus, the fatigue behaviors of RC slabs under moving load can be predicted by this numerical method.

Acknowledgement

We are grateful to Dr. K. Kakuma from Civil Engineering Research Institute (CERI) for Cold Region for sharing the experimental data.

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（2018 年 7 月 20 日受付）