論文

Life Prediction of RC Bridge Deck Slabs in a Punching Shear Failure Mode

Pengru DENG*，Takashi MATSUMOTO**

*博（工），北海道大学助教，大学院工学研究院（〒060-8628 札幌市北区北 13 西 8）
**Ph.D.，北海道大学教授，大学院工学研究院（〒060-8628 札幌市北区北 13 西 8）

Focusing on the propagation of critical cracks, a theoretical life prediction method for RC bridge deck slabs in a punching shear failure mode is developed based on fracture mechanics. In this method, the crack propagation is considered as a result of a concrete bridging degradation and a rebar/concrete interface bond slip degradation. From the analysis, the growth of the critical punching shear cracks can be obtained, which is then employed in determining the fatigue life following some shear failure criteria. Applicability and reliability are verified with a good accuracy achieved in the fatigue analysis of RC slabs tested in CERI.

Keywords：RC slabs, wheel running load, punching shear, fatigue

1. Introduction

In last few decades, a brittle and catastrophic punching shear failure mode has been usually reported around the world on RC bridge deck slabs. From existing researches, it has been confirmed that this detrimental phenomenon stems from a neglection of a shear capacity, especially a fatigue shear capacity, of RC slabs in previous design codes. Aoordingly, RC bridge slabs have been designed lacking rebars and thickness.

So far, extensive experimental works have been conducted to uncover this punching shear failure mode with a relatively simple experimental moving load set up as shown in Figure 1. The punching shear failure mode and shortened real fatigue life have been reproduced under this loading. According to the experimental observations like the failure crack pattern, an empirical life prediction equation was formulated through fitting a huge set of experiment data statistically [1]. In addition, taking advantage of FEM, the fatigue behaviors of RC slabs have been investigated through numerical analysis for the cyclic moving load [2]. From these numerical analyses, it was concluded that the modeling of tension fatigue of concrete and rebar/concrete interface bond slip characteristic should be considered carefully.

However, the existing researches possess their own disadvantages. As the empirical life prediction equations were from statistically fitting experiment data, the inner degradation mechanisms cannot be reflected. As for the numerical approaches, they are very time-consuming due to the widely distribution cracked elements. The computing time can even reach 1~2 days/cycle before failure.

Therefore, in this study, an analytical fatigue life prediction method is proposed focusing on the propagation of punching shear cracks of a critical RC beam simplified from an RC slab under cyclic moving loads according to the degradation process and failure crack pattern. From this method, fatigue crack growth which is assumed as a result of concrete bridging degradation and rebar/concrete interface bond-slip degradation can be obtained as well. Compared with existing empirical and numerical approaches, this method can not only account for the degradation mechanisms but also save a lot of computing time (about 10 minutes/cycle). This time-saving characteristic makes it extremely suitable for parameteric study and futher design code improvement.

2. Problem simplification

Referencing to a typical failure crack pattern shown in Figure 1, the cracking process can be explained as follows: (1) The moving load leads to an initiation of cracks along the direction vertical to the bridge axis because RC bridge slabs are normally with a smaller reinforcement ratio along bridge axis direction; (2) With continuing cycles of moving load, the cracks vertical to the bridge axis become more dense and fully developed as observed from a *b-b* cross-section in Figure 1 These cracks propagate almost parallelly to each other; (3) The parallel cracks cut down the shear transferring capability dramatically. Each region isolated by a couple of parallel cracks resembles an RC beam. Obviously, the beam located in the midspan is the most critical one; (4) In the critical beam, a large shearing stress concentrates around the loading area. This shear stress leads to propagation of a couple of punching shear cracks and the final brittle punching shear failure. Hence, the failure of an RC slab under cyclic moving loads is determined by the failure of a critical RC beam.

In addition, according to the cracking process and failure mode, Matsui proposed a method of determing the geometries of the critical RC beams as shown in Figure 2, where h and l are the depth and span of the RC beam, respectively; l_w and b are the length and width of the wheel/beam contact area; d_e is the effective depth for tensile rebar [1]. And then some empirical life prediction equations have been formulated using the punching shear capacity of this critical beam as a normalized parameter. In these empirical equations, the only parameter used is the punching shear capacity of the critical beam, which indicates that the fatigue life of the RC slab depends on the fatigue life of the critical RC beam.

Therefore, according to the theoretical bases of existing researches and the fatigue behavior of RC slabs under cyclic moving loads, the life prediction of an RC bridge slab is simplified into the life prediction of a critical RC beam focusing on the propagation of the punching shear cracks as shown in Figure 2.

3. Methods

For the critical RC beam subjected to loads, Figure 3 shows sectional forces and stresses acting on the punching shear crack cross-section. For every loading cycle, the cracking states can be calculated, and the failure moment can be determined. One can obtain the forces and stresses from actions (applied loads) and reactions (concrete and rebar stresses).

3.1 Basic assumptions

(1) Plane cross-section assumption: From this assumption, the strain distribution (ε_{II}) at the uncracked concrete and the normal strain of rebar on both compression (ε_{ru}) and tension zone (ε_{rb}) can be related to α and β as

$$\varepsilon_{II}(x) = \varepsilon_t \left(1 - \frac{x-\alpha h}{\beta h - \alpha h}\right) \tag{1}$$

$$\varepsilon_{ru} = -\varepsilon_t \frac{h-(c+d_b/2)-\alpha h}{(\beta-\alpha)h} \tag{2}$$

$$\varepsilon_{rb} = \varepsilon_t \frac{h-(c+d_b/2)}{(\beta-\alpha)h} \tag{3}$$

where c and d_b are the cover depth and rebar diameter, respectively. ε_t is cracking strain of concrete.

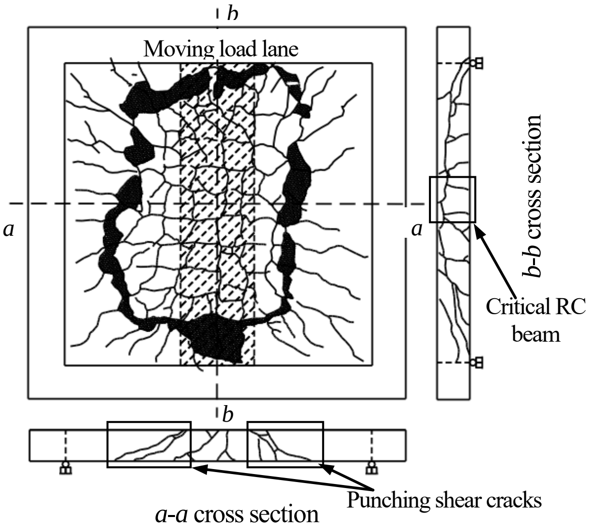


Figure 1 A typical failure crack pattern of an RC slab under cyclic moving load

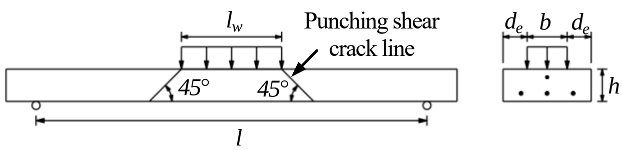


Figure 2 Geometry of the simplified RC beam and the punching shear cracks

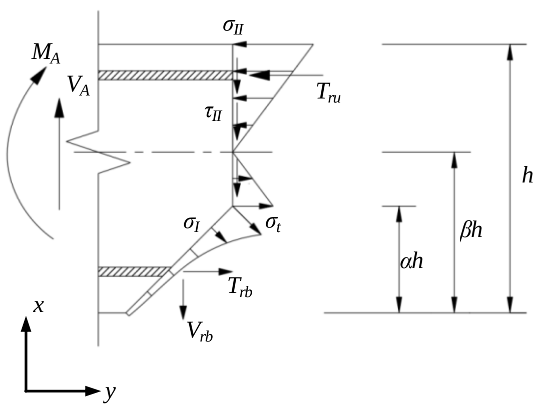


Figure 3 Sectional stresses and forces

(2) The cracked cross-section is assumed to rotate around the neutral axis [3]. From the tensorial consideration of strains at the crack location, the normal force (V_{rb}) of longitudinal rebars in the beam can be related to their shear force (T_{rb}) as:

$$V_{rb} = 0.4T_{rb} \tan\phi \tag{4}$$

(3) The crack has a linear crack opening profile [4]. From this assumption, the crack width ($w(x)$) at any location (x) along the crack can be expressed by the crack mouth opening displacement (CMOD, δ) and crack depth ratio (α) as

$$w(x) = \delta\left(1 - \frac{x}{\alpha h}\right) \tag{5}$$

With these material strains and crack opening displacments, materials stresses and concrete bridging stress can be expressed with cracking state parameters, i.e. α, β and δ following appropriate material stress-strain and concrete bridging models, respectively. Mathematically, to obtain α, β and δ after every loading cycle, at least three independent equations with unknown of α, β and δ should be formulated.

3.2 Governing equations with cracking state parameters

Referencing to the Figure 3, one can establish one force equilibrium equation along x axis and one moment equilibrium equation. In addition, a CMOD decomposition equation can be established following [5]. The three equations are given as:

$$\int_0^{\alpha h} \sigma_I(x)dx + \int_{\alpha h}^{h} \sigma_{II}(x)dx + T_{rb} + T_{ru} = 0 \tag{6}$$

$$\begin{aligned} &\int_0^{\alpha h} \sigma_I(x)\left[(h-x)+(\alpha h - x)\right]dx + T_{rb}\cdot(h-a_s) + \\ &\int_{\alpha h}^{h} \sigma_{II}(x)(h-x)dx + V_{rb}\cdot(\alpha h - a_s) + T_{ru}\cdot a_s = M_A \end{aligned} \tag{7}$$

$$\delta = \delta_A(\alpha) + \delta_I(\sigma_I(x), \alpha) + \delta_{rb}(T_{rb}, V_{rb}, \alpha) + \delta_s(T_{rb}) \tag{8}$$

where M_A is sectional moment due to applied loads. δ_A is CMOD due to applied load. δ_I are δ_{rb} are CMODs due to concrete bridging stress and rebar bridging forces, respectively. With crack face acting stresses due to applied loads, concrete and rebar bridging stresses, these three CMODs can be calculated employing a fracture mechanics based integral equation introduced in [5] and the crack face weight functions [6]. δ_s is CMOD due to bond slip which can be determined following an appropriate bond slip model [7].

Therefore, if all stresses and forces appeared in Eq. (6) to Eq. (8) can be expressed with α, β and δ, the cracking state parameters after every loading cycle are obtainable through solving Eq. (6) to Eq. (8).

3.3 Formulate sectional stresses and forces with cracking state parameters

(1) Sectional forces and stresses due to applied loads: According to the transferring characteristic of moment and shearing force, sectional moment (M_A) and shearing force in the critical RC beam can be obtained from FEM of the RC slab and analysis of the RC beam, respectively. And then, stresses can be easily expressed with α, β and δ from sectional analysis.

(2) Stresses from uncracked concrete can be expressed with α, β and δ through multiplying the concrete strain from Eq. (1) and elastic modulus of concrete. No degradation is assumed for uncracked concrete.

(3) Stresses from cracked concrete: In the first cycle, stresses from cracked concrete is expressed with α, β and δ following an empirical concrete bridging model [4]. From 2nd loading cycle to the final failure, the degradation due to a crack opening and closing process is accounted for based on a concrete bridging degradation model.

(4) Rebar stresses: To account for a tension stiffening effect, in this study, firstly the rebar stress-strain relation is modified incorporating multi-crack effect and bond-slip effect. And then, the rebar stresses are obtained through substituting rebar strain in Eq. (3) into this modified rebar stress-strain relation. From 2nd loading cycle to the final failure, the rebar stresses variation due to bond slip degradation is considered through introducing a bond slip degradation model.

3.4 Failure moment prediction with cracking state parameters

As concrete is a main source of sectional shear resistance and the stress transferring capability of concrete drops tremendously after cracking, the shearing capacity decreases remarkably due to the propagation of punching shear cracks. Consequetely, a localized shear failure was commonly observed in experiments. Therefore, the final failure moment of RC slabs is predicted following some shear capacity based failure criteria and using the obtained cracking state parameters.

As is generally accepted, the total shear resistance for a traditional RC slab is an integration of the vertical components of tensile stresses in concrete along the whole cross-section and the dowel action of flexural reinforcement. However, for the aimed problem, the shear resistance reduces due to follows: (1)

at failure moment, the shear stresses cannot be transferred effectively along moving load direction due to the existence parallel cracks perpendicular to moving load direction; (2) under cyclic moving loads, the crack opening and closing process reduce the stress transferring ability of concrete to almost zero; (3) post-moment inspections of tested specimens showed that a delamination along upper rebars occurred in the mid-span cross-section of specimens subjected to cyclic moving loads. This is probably a result of high-fatigue shear stresses at that depth due to the existence of upper rebars and a short distance from the neutral axis. Therefore, the dowel effect of upper rebars should be excluded in computing the punching shear capacity after the crack tip reaches. Therefore, the total shear capacity should only include concrete in the assumed failure area as shown in Figure 4 and rebar dowel action, which can be calculated following the equations from ACI [8] and Matsui [1], respectively. In summary, the punching shear capacity of the critical RC beams is given as

$$V_{pun} = \begin{cases} 2\left[b+(1-\alpha)h\right]\cdot(1-\alpha)h\cdot f_t + 4c\cdot B\cdot f_t \\ \quad \text{Before crack tip reaching upper rebars} \\ 2\left[b+(1-\alpha)h\right]\cdot(1-\alpha)h\cdot f_t + 2c\cdot B\cdot f_t \\ \quad \text{After crack tip reaching upper rebars} \end{cases} \tag{9}$$

The final failure moment is predicted based on this punching shear capacity in this study.

4. Numerical results and experimental verification

In this study, a fatigue analysis based on the proposed method is conducted in order to investigate crack propagation and fatigue life of an RC slab (C0) tested by Civil Engineering Research Institution for Cold Region (CERI) [9]. The method verification is done by comparing analysis results with results from experiments and some empirical equations. Four loading levels, i.e. 130 kN, 150 kN, 175 kN and 200 kN, are selected for fatigue analysis.

4.1 Prediction of fatigue crack growth

Under the selected loading levels, the crack depth/beam depth ratios are plotted with the number of loading cycles on the semi-logarithmic scale as shown in Figure 5(a). Generally, the fatigue growth of a bridged crack in a concrete structure involves of three distinct stages: initial decelerated growth, steady state growth and final accelerated growth. Most of the fatigue life is spent in the second stage. It is found that the theoretical simulation successfully captured the first two stages, whereas the third stage cannot be observed in the curves. This can be explained as follows [10]. To facilitate understanding, a crack-tip stress intensity factor (SIF) which reflects the crack-tip stress state is employed. For the punching shear cracks, the net crack-tip SIF is a resultant of SIF from external applied load and all bridging effects. The SIF amplitude due to external applied load increases with the length of the crack emanating from the tension face. At the same time, the magnitude of the negative crack-tip SIF due to crack bridging (rebar forces and concrete bridging stresses) increases with the propagation of the crack as well. The initial decelerated growth is caused by the effective reduction of the net crack-tip stress intensity factor amplitude due to the rapidly increased crack bridging related SIF. The steady state growth is caused by the constant value of net crack-tip SIF when the increase of SIF due to applied load is balanced with the increase of negative SIF due to bridging stress. The reason why the final accelerated crack growth stage cannot be observed is that the RC beam is assumed to fail in a brittle shear failure mode according to certain failure criterion.

However, the initial decelerated stage in the crack depth/beam depth ratio to number of loading cycles curve for higher load level is not as apparent as that for lower load levels. This is because the punching shear crack is more fully developed in the beginning several loading cycles under the higher load levels, manifested with higher crack depths and wider crack openings. Due to the wide crack opening, the bridging effect from nonlinear concrete bridging stress maybe small or even negligible compared with that from rebars. Thus, the structure degradation can be regarded as mainly from the

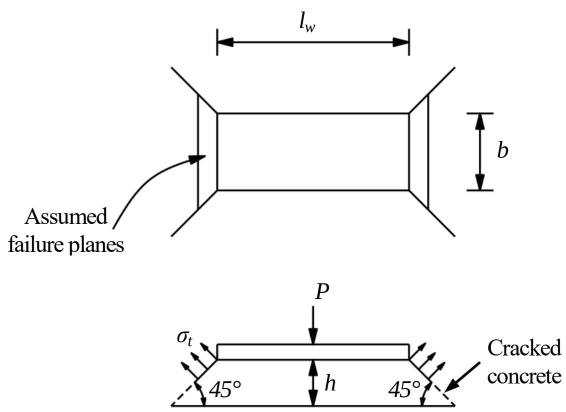


Figure 4 Schematic of failure plane combined fatigue degradation and experimental observations

degradation of bond slip, which follows a steady state growth as shown in Figure 5(a). As a result, the crack propagation curve seems to start from the steady state growth for higher load level as observed in Figure 5(a).

Similar to the characteristics exhibited in the crack depth/beam depth ratio to number of loading cycles relations, the two stages are also observed in Figure 5(b), where the relationships between tensile depth/beam depth ratio and the number of loading cycles for several fatigue loading levels are plotted on the semi-logarithmic scale.

The evolution of CMODs with respect to the increasing number of loading cycles for several fatigue loading levels are drawn on a semi-logarithmic scale Figure 5(c). It is noticed that the CMOD evolution of the punching shear crack depends on the load level. And, the CMOD increasing is expected to be mainly from the degradation of bond slip.

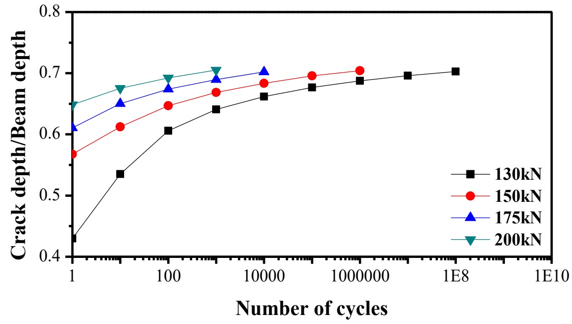
In summary, fatigue loading produces generally continuous increase of crack depth, tensile depth and CMOD.

4.2 Fatigue life prediction and comparison with experimental results and existing methods

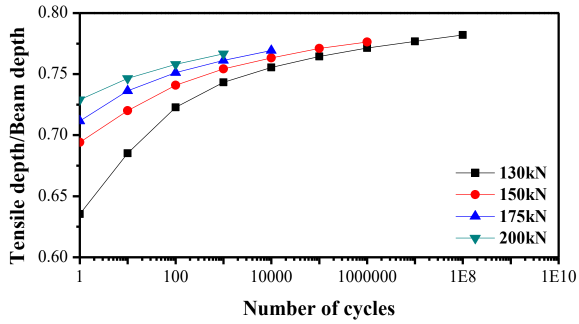
In this method, the final fatigue failure of RC slabs under cyclic moving load is determined following certain brittle shear failure criterion introduced in section 3.4. Substituting parameter values into Eq. (9), the punching shear load capacities for the moment of the crack tip reaching the upper rebars is determined as 230.63 kN. After the crack tip reaches the upper rebars, transverse shear cracks along the upper rebars initiate and propagate under repeated fatigue loads. Accodingly, the punching shear capacity reduces to 123.79 kN. Therefore, under the selected cyclic moving loads, fatigue punching shear failure occurs after the crack tip reaches the upper rebars with α equaling 0.7. Comparing this critical state with the obtained cracking state parameters vs. number of cycles curves shown in Figure 5, the fatigue life of the RC slab subjected to different load levels can be determined easily.

Moreover, the fatigue life can be calculated following some empirical life prediction equations derived by different research groups, such as Matsui [1] and Abe research teams [11], and institutions, such as Japan Society of Civil Engineers (JSCE) and Public Work Research Institute (PWRI). All the empirical equations were formulated as double logarithmic functions treating a punching shear capacity of the critical RC beam in [1] as a normalizing parameter. Since these empirical equations were obtained through statistically fitting a huge amount of experimental data on RC slabs under cyclic moving loads, each empirical equation represents a set of experimental results of RC slabs with different dimensions and loading levels. Thus, these equations implicitly reflect the internal mechanisms and can be employed in life prediction of similar problems as employed case in this study.

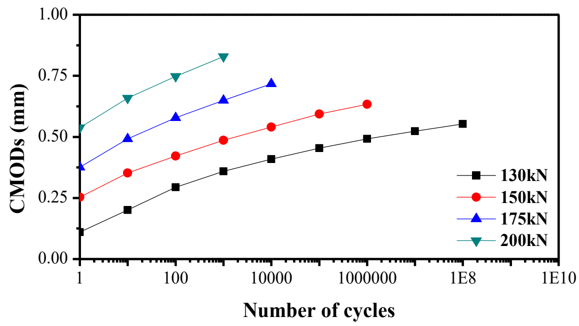
To illustrate the relationship between fatigue life and load condition, an S-N diagram is commonly employed. All the S-N diagrams from the proposed theoretical method and the empirical equations and experimental fatigue life are plotted on a double-logarithmic scale as shown in Figure 6 where the vertical and horizontal axes represent a load parameter normalized with a punching shear capacity determined by [1]



(a) α vs. number of loading cycles



(b) β vs. number of loading cycles



(c) CMOD vs. number of loading cycles

Figure 5 Cracking state parameters vs. number of cycles

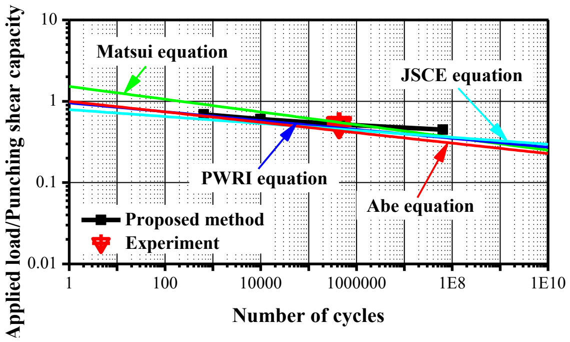


Figure 6 S-N curves from different approach

and number of loading cycles, respectively. It is found that the theoretical S-N relation agrees well with the fatigue life of the experimental RC slab and is almost the average results of the four reported equations derived statistically. These good agreements verify the reliability of the proposed theoretical method.

5. Conclusions

A theoretical fatigue life prediction method for RC bridge slabs which fail in a punching shear model under cycle moving loads has been proposed based on the nonlinear fracture mechanics for RC structures.

The method was established focusing on the propagation of punching shear cracks in a critical RC beam which was simplified from a RC slab under cyclic moving loads according to the cracking process and failure crack pattern and referencing to the theoretical bases of existing researches. From this method, the fatigue crack growth of the critical punching shear cracks was predicted. With the obtained crack growth results, the final failure moments or fatigue life were predicted according to certain punching shear failure criterion combining with experimentally observed failure modes. Comparisons between method predictions to results from experiment and some existing empirical life prediction equations indicate a reasonably good agreement, which verified the applicability and reliability of the proposed method. In addition, a time-saving characteristic of this method makes it extremely suitable for parametric studies and analysis on the effect of environment actions on the occurrence of punching shear failure.

Acknowledgement

We are grateful to Dr. K. Kakuma from Civil Engineering Research Institute for Cold Region for providing us with helpful suggestions and unpublished experimental data.

References

1) Maeda, Y., & Matsui, S. : Punching shear load equation of reinforced concrete slabs. Doboku Gakkai Ronbunshu, pp.133-141, 1984.
2) Drar, A. A. M., & Matsumoto, T. : Fatigue analysis of RC slabs reinforced with plain bars based on the bridging stress degradation concept. Journal of Advanced Concrete Technology, Vol.14, No.1, pp.21-34, 2016.
3) Zararis, P. D., & Papadakis, G. C. : Diagonal shear failure and size effect in RC beams without web reinforcement. Journal of structural engineering, Vol:127, No.7, pp.733-742, 2001.
4) Zhang, J., Stang, H., & Li, V. C. : Fatigue life prediction of fiber reinforced concrete under flexural load. International Journal of Fatigue, Vol.21, No.10, pp.1033-1049, 1999.
5) Deng, P., & Matsumoto, T. : Estimation of the rebar force in RC members from the analysis of the crack mouth opening displacement based on fracture mechanics. Journal of Advanced Concrete Technology, Vol.15, No.2, pp.81-93, 2017.
6) Deng, P., & Matsumoto, T. : Weight function determinations for shear cracks in reinforced concrete beams based on finite element method. Engineering Fracture Mechanics, Vol.177, pp.61-78, 2017.
7) Maekawa, K., Okamura, H., & Pimanmas, A. : Non-linear mechanics of reinforced concrete: CRC Press, 2003.
8) ACI Committee 318. : Building code requirements for structural concrete (ACI 318-99) and commentary (318R-99). American Concrete Institute, Farmington Hills, Michigan, pp.399, 1999.
9) Mitamura, H., Syakushiro, K., Matsumoto, T. & Matsui, S. : Experimantal study on fatigue durability of RC deck slabs with overlay retrofit. Journal of Structural Engineering A, Vol.58, pp.1166-1177, 2012.
10) Li, V. C., & Matsumoto, T. : Fatigue crack growth analysis of fiber reinforced concrete with effect of interfacial bond degradation. Cement and Concrete Composites, Vol.20, No.5, pp.339-351, 1998.
11) Abe, T.: Study on punching shear load-carrying capacity of RC slab. Journal of College of Industrial Technology Nihon University, Vol.38, No.2, pp.11-21, 2005.

（2018 年 7 月 20 日受付）